TEXT-BOOKS OF SCIENCE,

MECHANICAL AND PHYSICAL.

ADAPTED FOR THE USE OF STUDENTS IN PUBL
AND SCIENCE SCHOOLS.

THE

ELEMENTS OF MECHANISM

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CONTENTS.

ELEMENTS OF MECHANISM.

CHAPTER I.

INTRODUCTORY.

A MACHINE is an assemblage of moving parts, constructed for the purpose of transmitting motion or force, and of modifying, in various ways, the motion or force so transmitted.

In order to form a definite idea of the meaning which attaches to the word 'machine,' it may be useful to refer to an example commonly met with—such as an ordinary sewing machine. The apparatus is rightly called a machine, as being capable of doing work of one definite kind, under the simple condition that some natural source of energy shall bear upon it and set the working parts in motion. Upon looking into its construction we should find a fixed framework supporting combinations of movable parts, whereof some are employed in actuating a needle and while others carry forward the material which is to be stitched. The movable parts are constrained to take certain definite which are arranged beforehand, while some natural force, such as the power of the hand or the foot, is applied to the proper recipient, and then the machine does work as a necessary consequence of the action of the motive power.

In commencing the systematic study of machinery, it will be readily understood that certain simple relations of motion are traceable between the prime mover which starts the machinery

and the pieces which execute the work; and it is also clear that, in practice, relations governing the transmission of force must exist as certainly as those which govern the transmission of motion. The considerations relating to force may often occupy the mind of the mechanic in a greater degree than those which refer to motion; but in reducing the subject to analysis it will be found convenient to separate the two points of view, and to confine our attention in the first instance mainly to *Theoretical Mechanism*—that is, to an examination of the various contrivances and arrangements of parts in machinery whereby motion is set up or modified—and to disregard or postpone any enquiry into the mechanical laws which control the forces concerned in these movements. But as the present work is intended for use and study by practical men, the author will to a small extent break through this general rule, and will take occasion, where the enquiry would be useful, of pointing out also the manner in which certain pieces of mechanism have served a compound object in transmitting exact and definite amounts of motion, while dealing at the same time with refined and subtle distinctions as to the method of transmitting force.

We have now to consider and arrange the method according to which our enquiries are to be carried on, and if we were to pause for a moment and look back upon that rapid creation of machinery which followed so closely upon the splendid invention of the steam engine by Watt, we should naturally expect that some uniform arrangement for applying steam power would be adopted by common consent, and that this arrangement would powerfully influence the art of constructive mechanism. Accordingly we find that in applying the power derived from steam for the purpose of driving machinery in our mills and factories, it is the practice to connect the engine with a heavy fly wheel, the rotation of which is made as uniform as possible, and then to carry on, by lengths of shafting, the uniform motion of the fly wheel to each individual machine in a factory.

Suppose, for example, that we were visiting a cotton mill, and were examining and endeavouring to comprehend the action of a complete piece of machinery, such as a power loom for weaving calico. We should at once see that every moving part, acting to produce the required result, derived its motion from the uniform

and constant rotation of a disc or pulley, outside the machine itself, and communicating by means of a band with the shafting driven by the engine, and thus it would become obvious that the problem of making a machine resolved itself mainly into a question of the resolution or transfer of circular motion in every variety of manner, and subject to every possible modification.

We shall therefore commence, in the present chapter, with some general observations on the conversion and transfer of motion in the simple primary forms under which it is to be regarded at the outset of the study of mechanism, reserving a more complete discussion of the different divisions of the subject for the remaining chapters, each of which will treat of movements of a particular class.

It will soon become apparent that, by combining, transferring or modifying simple modes of motion, an almost endless variety of mechanisms may spring into existence, and our object will be to classify and arrange these mechanisms in such a manner that the reader may acquire a fair knowledge of what has been already accomplished, and may trace the principles which have been developed in the construction of many well-known machines.

To the geometrician a straight line or a plane surface are creations of the mind. Euclid, more than 2,000 years ago, had as complete a conception of a straight line or a plane as we have at the present day, but he could not realise his conceptions even approximately, by reason that accuracy of surface was at that time a thing unknown.

And even now how few among young mechanics are aware of the exact conditions under which truth of surface has been originated, or that a difference of length of $\frac{1}{10000}$ of an inch is a quantity which can be palpably and unmistakably measured by a workshop instrument, without the aid of a microscope or magnifying lens of any kind. It is not enough to describe machines on paper, and to say that they will effect such and such results. The science of mechanism is a practical science; it must be more than a speculative creation; the principle of each movement must be embodied in shaped pieces of suitable material, and there must be some method of testing the exact form or dimensions of the several parts. It follows that a knowledge of the steps which have

given to the mechanician the two aids upon which he mainly relies, viz. truth of surface and the power of measurement, will form an essential portion of the subject-matter upon which we have to treat.

ART. I.—To commence with a few enquiries relating to the motion of a point in space—a point being that ideal thing called a *material particle*, which is defined in a Cambridge text-book as being 'a portion of matter indefinitely small in all its dimensions, so that its length, breadth, and thickness are less than any assignable linear magnitude'—we shall treat the motion of such a point as a simple matter of geometry, all its movements being exact of their kind.

There are three primary cases :—

I. The point may move in a straight line. In such a case the direction of its motion remains constant, being that of the line in which it moves.

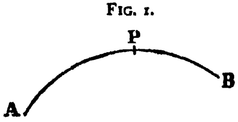
II. The point may move in one plane, but may continually change the direction of its motion.

III. The point may change its direction so as to move in a curved line of any kind.

In the second case the point is said to move in a *plane curve*, for, according to geometers, a *curve* is a line traced out by a moving point, which is continually changing the direction of its motion, and a plane curve is one which lies in a given plane.

Conceive that a point is describing a plane curve AB; then the straight line in which the point would move at P, if it there ceased to change the direction of its motion, is called the *tangent* to the curve AB at the point P—that is to say, the direction of the motion of a moving point is at each instant the tangent to its path when the path is a curve.

FIG. 1.



It should be borne in mind that we have spoken of a material particle as the moving thing, because a material particle has the property of mass, and cannot change its velocity or direction abruptly unless it be subjected to the action of an infinite force, though of course it may be moved abruptly in the sense of being set in motion from a position of rest.

In the third case the point may move in a curve traced upon the surface of a cylinder, and not lying in one plane, as, for example, if it followed the outline of the banister of a cylindrical staircase.

The definition of a tangent previously given applies generally to all curves, and if a point be moving in a straight line at any instant it must also be moving in some definite plane; hence we may describe the motion now under consideration by saying that the direction of the curve at any instant is continually changing by the twisting of the plane in which the point is then moving about its tangent line.

ART. 2.—We have next to speak of the *velocity* of the moving point.

So long as a point is moving continuously, we can form an idea of the rate at which it is changing its position relatively to other points which are assumed to be at rest. This rate of change of position is the velocity of the moving point.

Velocity may be either *uniform* or *variable*.

The word 'uniform' indicates that the lengths of path described by the moving point in equal times are always the same, and the word 'variable' is applied when the rate of change of position of the moving point is *continuously* altering. The latter term would not apply to a step-by-step movement.

When the velocity of a point is uniform, it is measured by the length of path passed over in a unit of time—that is, in technical language, by the space described in the unit of time.

The unit of space is usually one foot, and the unit of time is one second.

Hence if a point be moving *uniformly* with a velocity v, it will describe v feet in any second, and will describe tv feet in t seconds.

Let s be the space described in t seconds, then

$$s = tv, \quad \text{or} \quad v = \frac{s}{t}.$$

Def. A *foot-second* of velocity is the velocity which would cause a point to move uniformly through a foot in every second. If the velocity of a point be n foot-seconds the point will move uniformly through n feet in each second of time.

ART. 3.—If the velocity of the point be variable we must look to the rate at which s changes as t flows on, or to the ratio between the so-called fluxions of s and t. The word 'fluxion' was introduced at the time of Newton.

Suppose that in time Δt the point describes a space Δs, and that its velocity in the same time increases or decreases continuously and becomes $v + \Delta v$. The space it actually describes lies between the spaces it would describe if its initial and final velocities were continued uniform during time Δt, or

$$v\Delta t, \quad \Delta s, \quad (v+\Delta v)\Delta t$$

are in order of magnitude, and so are

$$v, \quad \frac{\Delta s}{\Delta t}, \quad v+\Delta v.$$

Now let Δt, and consequently Δs, Δv be diminished indefinitely, in which case let $\frac{\Delta s}{\Delta t}$ become $\frac{ds}{dt}$, then the first and third terms become equal to that lying between them, or $v = \frac{ds}{dt}$.

Hence $v = \frac{ds}{dt}$ for variable motion.

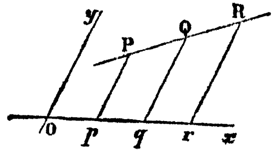
In other words, velocity, when uniform, is measured by the space described in a unit of time; when variable it is measured by the space which would be described in a unit of time, if the point retained throughout that unit the velocity which it has at the instant considered. The above statements apply equally whether the point be moving in a straight line or in a curved line of any kind.

It is further apparent that the velocity of a point at any instant may be represented by a straight line, for the direction of motion of the point will be the direction of the line, and the numerical measure of the velocity will determine the number of units of length in the line.

Inasmuch as a straight line can be drawn in any direction from a point, and since it is usual to describe the straight line as *positive* when it is drawn in one direction from a point, and *negative* when it is drawn from the same point in the opposite direction, so *velocities* can be similarly described as *positive* or *negative*, according to their directions in the same straight line.

ART. 4.—We pass on to the resolution and composition of velocities. Conceive that a point is moving uniformly in the straight line PQR with a velocity v, and let P, Q, R be three positions of the moving point.

FIG. 2.

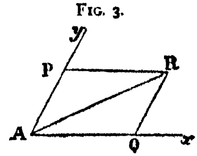


Take any two straight lines Ox, Oy inclined at a given angle, and lying in a plane passing through PQR; draw Pp, Qq, Rr respectively parallel to Oy; then $pq : pr :: \text{PQ} : \text{PR}$, and the motion of p along Ox will be in a constant ratio to the motion of P along PR.

The point p is called the *projection* of P on Ox, and the velocity of p is said to be the *velocity of* P *resolved* along Ox.

In like manner, if a straight line AR be taken to represent the velocity existing in a point at any instant, and Ax, Ay be any two straight lines intersecting in A, and if the parallelogram APRQ be completed, the sides AP, AQ will represent the resolved velocities of the point in directions Ay, Ax. It is usual to speak of AP, AQ as the components of the velocity AR; and, again, AR is called the resultant of the component velocities. Hence the following proposition:—

FIG. 3.



Prop. If there be impressed simultaneously on a particle at A two velocities which would separately be represented by the adjacent sides AP, AQ (fig. 3) of the parallelogram APRQ, the actual velocity of the point will be represented by that diagonal AR which passes through the point A.

Cor. 1. Let the angle PAQ be a right angle, and let RAQ$=\alpha$,

FIG. 4.

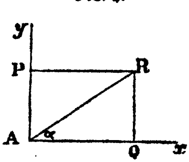
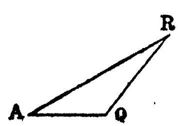


FIG. 5.



AR $= v$. Then will AQ $= v \cos \alpha$, AP $=$ QR $= v \sin \alpha$.

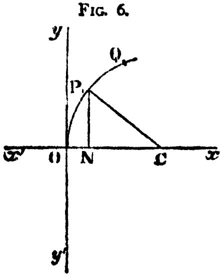
Cor. 2. Let velocities represented by the three sides of a triangle taken in order—viz. AQ, QR, RA—be impressed at the same instant on a point; then no motion will ensue, or the point will remain at rest; for the velocities AQ, QR are equivalent to a single velocity AR, and the velocities AR, RA are equal and opposite, and therefore destroy each other.

ART. 5.—It has been stated that the most simple case of motion is that of a point describing a straight line with a uniform velocity, also that the motion of a point in a straight line may be the *aggregate of two movements in lines at right angles to each other, and that this is true whether the motion of the point in a straight line be uniform or variable.*

But one imperative condition must be observed, viz. that the amounts of the corresponding movements in the two perpendicular lines must be in a fixed ratio to each other. When this condition fails, the point will describe a curve and not a straight line.

The whole learning of analytical geometry proceeds on the doctrine of the composition of motion. If we wish to represent a curve by means of a relation between symbols, called an *equation to the curve*, we begin by drawing two straight lines xOx', $y'Oy$ at right angles to each other, and employing them as lines of reference.

FIG. 6.



For example, let the curve pass through point O; take P any point in the curve, and draw PN perpendicular to Ox; let ON$=x$, NP$=y$; then, if y be to x in a fixed ratio, the point P must of necessity lie in a straight line passing through O.

Whereas if the ratio between y and x varies for every position of the moving point P, that point will describe a curve.

Let P describe a circle whose centre is C, and let CO$=a$; join CP: then NC$=$OC$-$ON$=a-x$.

But $$CP^2=PN^2+NC^2.$$

$$\therefore a^2=y^2+(a-x)^2$$
$$=y^2+a^2-2ax+x^2.$$
$$\therefore y^2=2ax-x^2.$$

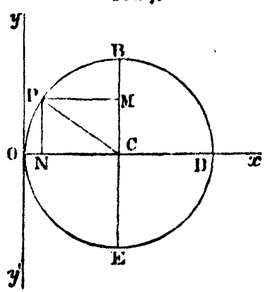
The above relation is satisfied only by points lying in the circle, and gives an analytical representation of the particular curve to which reference has been made.

The lines x, y are called the co-ordinates of the point P, and the axes xOx', yOy' are the axes of co-ordinates; also the signs $+$ and $-$ are employed to indicate the position of P in any particular quadrant; thus if P were situated anywhere in the quadrant $x'Oy'$, the corresponding values of x and y would both be negative.

ART. 6.—We are now in a position to discuss the nature of circular motion, and may premise that the belief held by the ancients with regard to it was fanciful in the extreme, and is obviously untenable. It was said that the motion of a point in a circle was *simple*, in the sense that it was not made up by putting together other separate movements, a doctrine in direct opposition to that just laid down.

The modern belief is that the point P, while describing the arc OP of the circle OBDE, may have been the subject of two independent movements, one from O to N in the direction of the diameter OD, and the other from N to P in a perpendicular direction.

FIG. 7.



Thus let OCD, BCE be two diameters of a given circle at right angles to each other, P any point in the circumference, PN, PM perpendiculars on OC and BC respectively.

Let P describe the circumference with a *uniform* velocity; then the point N will travel to and fro along OCD with a *variable* velocity, while at the same time the point M moves at a varying rate up and down through BCE.

The motion of the point N is distinguished by a technical name, according to the following definition:—

Def. When a point P moves *uniformly* in a circle, the extremity N of the perpendicular PN let fall from P upon a fixed diameter OD has a *simple harmonic motion.*

It appears that this is nearly the case with such bodies as the satellites of Jupiter when seen from the earth.

The term 'harmonic motion' has been chosen for designating one component of circular motion because it represents approximately the motion of a particle in the various media in which waves of sound, light, and heat are propagated. Thus a point at the end of the leg of a vibrating tuning-fork has a simple harmonic motion very approximately.

We conclude that circular motion is of a compound character, and is capable of resolution into its elements. If it be thus resolved, and if one equivalent be suppressed, so that the motion of N is substituted for that of P, we obtain the fundamental case of the conversion of circular into straight-line motion.

And, further, we regard circular motion as compounded of two simple harmonic motions in lines at right angles to each other, but so related that one component comes to rest when the other is in the middle of its swing.

It will be found that harmonic motions enter into the analysis of many forms of mechanism, and that no progress can be made without some knowledge of the laws here sketched out.

The following technical terms are introduced in order to state the nature of the movement correctly.

Def. The *amplitude* of a simple harmonic motion is half the distance between two extreme positions. In other words, it is the radius of the auxiliary or bounding circle.

Def. The *period* is the interval of time between two successive passages through the same position in the same direction—that is, it is the time of describing the complete circle.

Def. The *phase* is that fraction of the period which has elapsed since the moving point was at its extreme position in the positive direction.

In applying these definitions we should say that M and N have the same amplitude, that they have the same period, and that they differ in phase by $\frac{1}{4}$ of the period; whence we finally conclude that uniform circular motion is compounded of two simple harmonic motions of equal period and amplitude, taking place in lines at right angles to each other, and differing in phase by one-quarter of the whole period.

ART. 7.—In order to express the relation between the positions P and N we proceed as follows :—

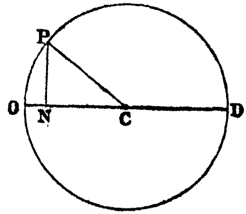
Let $CP=a$, $PCO=C$.

Then $ON=OC-CN$

$=OC-CP\cos C$.

Or $ON=a(1-\cos C)$.

FIG 8.



Ex. To find the position of CP when N is half-way between O and C.

Here $ON=\frac{OC}{2}=\frac{a}{2}$,

$\therefore \frac{a}{2}=a(1-\cos C)$,

whence $1-\cos C=\frac{1}{2}$, $\cos C=1-\frac{1}{2}=\frac{1}{2}$,

or $C=60°$.

It follows from this that N describes the first half of OC in double the time which it takes to describe the other half.

In further analysing the movement we have to ascertain the relation which exists between the actual velocities of P and N at any instant.

Suppose P to sweep round with a uniform velocity, and we see at once that N begins to move slowly at O, comes to rest gradually at D, and that its greatest speed occurs when just passing through C.

The motion of N is not uniform, and its rate of advance at any given instant may be deduced from the so-called triangle of velocities.

Thus take PC, which is perpendicular to the direction of motion of P, to represent its velocity in magnitude ; then PN, NC represent the components of the velocity of P in directions NC, PN respectively.

The sine of C has all values from 0 to 1 which are registered in a table of natural sines, and by substitution from this table we can find how much the velocity of P differs from that of N at any period of the motion.

When $C=45°$, $\sin C = \cdot 7071068$,

$\therefore$ vel. of $N = \frac{707}{1000}$ vel. of P, very nearly.

When $C=90°$, $\sin C = 1$, $\therefore$ vel. of N=vel. of P,

which is evidently true, because the point P is then moving in a direction parallel to OD.

We observe also that the motion may be divided into four equal portions, and that the advance of N from O to C has an exact counterpart on the return from D to C, and so for the other divisions.

ART. 8.—It happens that the composition of two harmonic motions in lines at right angles to each other can be readily verified by experiment. For this purpose two tuning-forks, each four or five times as large as an ordinary tuning-fork, are required. A small polished steel mirror is attached to the outside of the leg of one fork in such a manner that the vibration of the leg would cause the image of a small luminous spot to describe a straight line. The fork is then mounted so that the line in question becomes vertical. The second fork is provided also with a small reflecting mirror, and is caused to vibrate in a horizontal plane, whereby the image of the luminous point, as taken from the first mirror, describes a horizontal line when the first fork is at rest.

We have thus two separate motions in lines at right angles to each other, and it has been stated that the motion of a point in the leg of a tuning-fork is a simple harmonic motion.

When the two forks vibrate together, the spot of light describes a path which represents the result of compounding the two separate harmonic motions. That path is found to change from a straight line to an oval, which presently swells out to a complete circle, and then sinks down again to a straight line. The circle and the straight line are the two extreme cases between which the intermediate curves are to be found.

The fact is that the forks are not exactly in unison, and that one is continually gaining upon the other. If they start together, a straight line is the result. If one has gained upon the other so far that it is half a swing in advance, the straight line has changed into a circle.

ART. 9.—The velocity of N may be found by analysis. Thus let $ON=x$, $PCN=\theta$, $CP=a$. Then $x=a\,(1-\cos\theta)$.

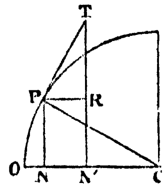
$\therefore$ vel. of $P = \frac{dOP}{dt} = a\frac{d\theta}{dt}$, and vel. of $N = \frac{dON}{dt} = a \sin\theta\frac{d\theta}{dt}$,

$$\text{or} \quad \frac{\text{vel. of N}}{\text{vel. of P}} = a\sin\theta\frac{d\theta}{dt} \div a\frac{d\theta}{dt} = \sin\theta.$$

Another easy proof is the following :—

Assume that P retains its present velocity, and let it move in any time to T. Draw TN′ perpendicular to CN, and PR perpendicular to TN′, and let

FIG. 9.

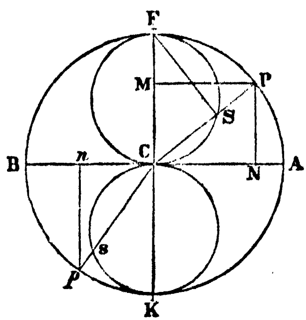


$$\text{then } \frac{\text{vel. of N}}{\text{vel. of P}} = \frac{NN'}{PT}$$

$$= \frac{PR}{PT} = \frac{PN}{PC} = \sin\theta.$$

ART. 10.—The changes which occur in the velocity of N may be set out in a diagram.

FIG. 10.



Let C be the centre of the circle described by P; take FK, AB, two diameters at right angles; and draw PM, PN, respectively perpendicular to them. Also take CS=CM=PN, and join FS.

Then in the triangles PMC, FCS the sides CM, CP are respectively equal to CS and CF, and the angle FCP is common to both; therefore the triangles are equal,

and angle FSC=angle CMP.

But CMP is a right angle; therefore CSF is also a right angle, and S lies in the circumference of a circle described on CF as a diameter.

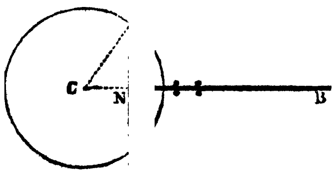
It follows that if two circles be described on CF, CK as diameters, and any straight line CSP be drawn as in the diagram, the ratio of CS to CP will be the ratio of the velocities of the points N and P respectively.

At the points F and K we have CS=CP, and the velocities of N and P become equal, which is evidently true.

Art. 11.—The motion of N may be derived from that of P by a simple mechanical arrangement.

Let P represent a small pin set in a circular plate, which is movable about C as a centre of motion, and let the pin work in a groove EF whose direction is at right angles to that of the sliding bar BN. The bar is, of course, rigidly attached to EF, and is constrained by guides to move in a line pointing to C.

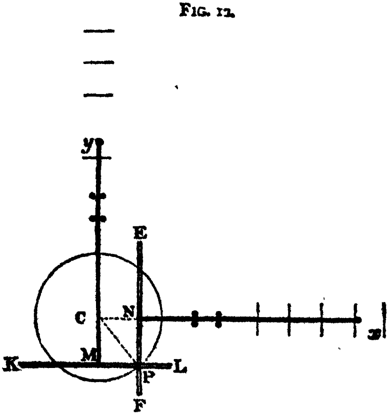
Fig. 11.



Of the two equivalents which combine to produce the circular motion of P that which occurs in the direction FE is rendered inoperative, and the whole of the other equivalent is imparted to the sliding bar; in this way, then, the bar moves to and fro as the disc rotates upon its centre, and any point in it reproduces exactly the motion of N.

Referring to the drawing, which is sketched from a model intended to exhibit simultaneously the motions of the points M and N, it will be seen that a pin P passes through the grooved bars EF and KL, which overlap each other and are connected by slender rods sliding between guides. Also x and y are small balls or index fingers, each of which traverses over a graduated scale, and indicates the movement of the corresponding bar. As the point P travels round a circular groove, it is apparent that x and y respectively reproduce the movements of N and M.

Fig. 12.



Art. 12.—Having satisfied ourselves as to the nature of circular

motion, the next step is to consider the manner in which it may be modified or transferred.

There are two fundamental cases :—

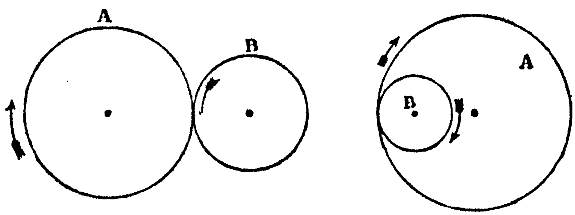
1. We may draw the circular motion from the circumference of the circle, as we should draw off a piece of string from a reel on which it has been wound.

2. We may suppress one of the two components of the circular motion, and may take the other in the form of a complete harmonic motion.

Each method leads to a variety of useful mechanisms.

As to (1), when two circles touch each other, the direction of each curve at the point of contact is that of their common tangent, and hence a moving point may be readily transferred from one circle into another touching it.

FIG. 13.



Thus a point describing the circle A with an uniform velocity may be passed into the circle B, and will describe that also with the same linear velocity.

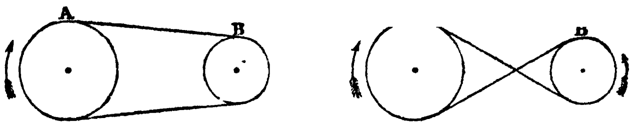
If the circle B touch the circle A externally, the point will appear to whirl round in the opposite direction as it passes from one circle to the other ; whereas, if the circle B touch the circle A internally, the motion is the same as before, except that the direction of the whirl does not change.

As a particular case let the radius of one circle become infinite, and we pass from circular into rectilinear motion, and conversely. In this case the point describing the circle goes off in a tangent, or the point travels along a tangent into its circular path.

In like manner, if two circles be connected by a pair of common tangents, as in the sketch, the point travelling round one

circle may pass off in a tangent, describe a portion of the second circle, and so return by a second tangent to its primary path.

FIG. 14.

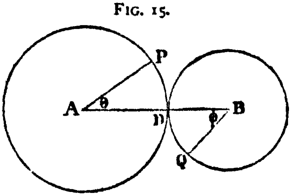


It is apparent that the direction of the whirl in B will be the same as that in A when the tangents do not cross, and will be in the opposite direction when they do cross.

ART. 13.—When a point passes from the circle A to the circle B in any of the previous cases, and is moving uniformly, the angles described in any given time are inversely as the radii of the circles.

Let a point moving uniformly round the circumference of the circle whose centre is A pass on at D into the circumference of the circle whose centre is B without change of velocity.

FIG. 15.



Let PD, DQ be arcs described in equal times, and let $AD=a$, $DB=b$, $PAD=\theta$, $DBQ=\phi$.

Then $DP=a\theta$, $DQ=b\phi$.

But $DP=DQ$, because the velocity of the moving point remains unaltered,

which proves the proposition.

ART. 14.—Hitherto the motion of a point has been regarded under three aspects, viz.—

I. The motion in a straight line.

II. The motion in a circle or in any other plane curve.

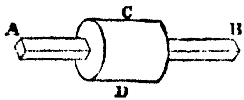
III. The motion in a curve of any kind not lying in one plane.

The like motions, when applied to material bodies, give rise to elementary combinations or *pairs*, as they are termed, which are repeated in an endless variety of forms in complete machines. At present we confine our attention to cases I and II.

Case I. A body may be moved in such a manner that every point of it is constrained to describe a straight line, and that all such straight lines are parallel to each other.

To effect this in the most simple manner we require a pair of elements, viz. a sliding bar and an enveloping guide. Let the sliding bar be rectangular in section, as AB in the diagram, and have four plane faces. Also let it be enclosed in an enveloping block, with like internal plane surfaces, such as CD. The bar and the block may be conveniently made of cast iron, the latter being formed in two parts, and the surfaces being prepared by the operation of scraping. It is clear that the prism can only move in the direction of its axis, and that the motion of AB is one of simple translation, each point in the solid describing a straight line in the same direction. Also it is essential to have a pair of elements in order to provide that the motion shall take place in one direction only.

Fig. 16.



Case II. The body may be constrained to move in such a manner that any point in it describes a plane curve. Here, again, we may begin with a simple case, and suppose that the body is a pulley or wheel having a fixed axis on which it turns, and that any point in the rotating body describes a circle.

Note.—The term *axis* denotes the central line of a cylinder, and is a mathematical phrase: an engineer distinguishes a heavy cylindrical piece of metal as *shafting*, or a *shaft*, and designates smaller cylindrical bars as *spindles*; a wheelwright speaks of the *axle* of a wheel, and a watchmaker calls the same thing an *arbor.*

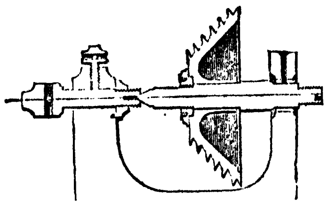
To obtain one motion of the second case a pair of elements is again necessary; there is (1) the axle of the pulley or wheel, (2) the bearing or block in which the axle rotates.

The axle of a wheel is commonly a cylindrical bar running completely through the wheel, and supported at both ends on cylindrical hollow bearings which fit the shaft and envelope it closely, any motion endways being prevented by collars or shoulders upon the shaft.

The drawing shows the poppet head of a lathe; the speed pulley which takes the driving cord is shown in section, and is

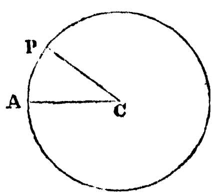
grooved in steps, the spindle or *mandrel* which supports the pulley being coned at one end in a steel point, and resting on a parallel bearing with a shoulder at the other extremity. A small hole is truly drilled in the direction of the axis of the mandrel, to receive the point of the cone, and to preserve the truth of the geometrical axis as the point wears. The two supports form essentially one bearing, for it is apparent that a single cylindrical bearing, if sufficiently lengthened, would answer the purpose so far as rotation is concerned.

FIG. 17.



ART. 15.—In order to apply measurement to movements of rotation it is necessary to introduce a new method of estimating velocity. Thus let a plane circular disc rotate upon an axis passing through its centre C and perpendicular to its plane, and let the radius CA come into the position CP at the end of a given interval of time.

FIG. 18.



Taking CA as a fixed line of reference, the rate of change of the angle PCA is called the *angular velocity* of the disc.

Angular velocity, when *uniform*, is measured by the angle described in a unit of time. The unit of time is always one second, unless the contrary be expressed.

Thus, if PCA be the angle described by CP in one second, the angular velocity of CP is expressed by the angle PCA.

This angle is not estimated in degrees, but in circular measure —that is, by the ratio of AP to AC—and it has been customary to designate it by the Greek letter ω (*omega*).

$$\text{Hence } \omega = \frac{AP}{AC}.$$

Let r be the radius of the circle APC, v the linear velocity of the point P,

$$\text{then } v = AP,\ \omega = \frac{AP}{AC} = \frac{v}{r}.$$

$$\therefore v = \omega r,$$

which is the equation connecting the uniform linear velocity of the point P with the angular velocity of the disc or rotating body. It follows that when a body is rotating uniformly, the linear velocity of any point of it increases directly as the distance from the axis of rotation.

Ex. 1. A wheel 6 feet in diameter turns uniformly on its centre 20 times in a minute: what is the linear velocity of a point in its circumference?

We shall now apply the notation of foot-seconds, which are written f.s. for brevity.

Here the angular velocity $= \frac{40\pi}{60} = \frac{2\pi}{3}$,

and the linear velocity of a point at a distance of 3 feet from the centre of rotation

$$= \frac{2\pi}{3} \times 3 \text{ foot-seconds,}$$
$$= 2\pi \text{ f.s.}$$

Ex. 2. How far from the centre will a point lie which is moving at the rate of one mile per hour?

The linear velocity of this point $= \frac{1760 \times 3}{60 \times 60}$ f.s.

The angular velocity of this point $= \frac{2\pi}{3}$

$$\therefore \text{ required distance} = \frac{1760 \times 3}{60 \times 60} \times \frac{3}{2\pi} \text{ feet,}$$
$$= \frac{22}{10\pi} = \frac{7}{10} \text{ feet nearly.}$$

ART. 16.—Again, if θ be the angle described by CP in t seconds, we have $\theta = \omega t$, or $\omega = \frac{\theta}{t}$.

If the angular velocity be variable, it may be proved by reasoning precisely similar to that adopted in Art. 3, that the angular velocity ω is given by the equation

$$\omega = \frac{d\theta}{dt};$$

that is to say, $\frac{d\theta}{dt}$ represents the angular velocity of a rotating body when the motion is variable.

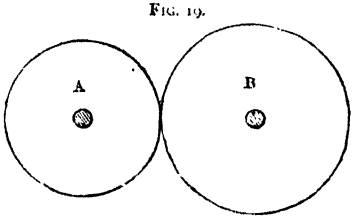
ART. 17.—Of two moving pieces that which transmits motion is termed the *driver*, and that which receives it is the *follower.*

Conceive that the driver and follower have each a simple motion, either of translation or rotation ; then the ratio of their comparative velocities is called the *velocity ratio* between them.

ART. 18.—It will now become necessary to consider the manner in which a motion of rotation of a solid body may be transferred from one axis to another, and it is apparent that the most simple case occurs when a circular disc or plate moves another in its own plane by rolling contact.

In such a case the uniform motion of the axis A conveys a perfectly even and uniform motion to the other axis B.

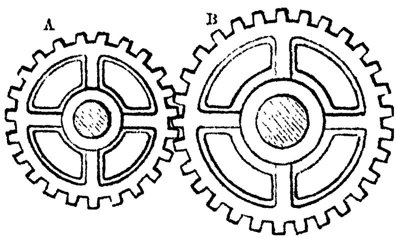
FIG. 19.



If A and B were circular plates with flat edges, and very accurately adjusted, it would be quite possible for A to move B by friction alone, the two plates rolling smoothly and evenly upon each other without any slipping of the surfaces in contact ; but we could not expect A to overcome any great resistance to motion in B ; or, in other words, we could not in practice convey any considerable amount of force by the action of one disc upon the other.

The transmission of energy being an essential condition in machinery, the discs A and B are provided with teeth, as in the annexed figure, and the mechanist endeavours so to form and shape the teeth that the motion shall be exactly the same as if one circle rolled upon another.

FIG. 20.

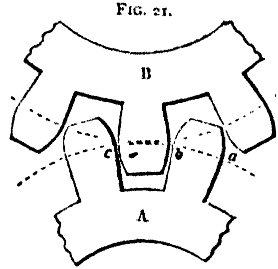


Herein consists the perfection of wheelwork : a perfectly uniform motion of the axis A is to be conveyed by means of teeth to the axis B ; and the motion of B, when

tested with microscopic accuracy, is to be no less even and uniform than that of A.

Since, then, it appears that the motions of A and B are exactly the same as those of two circles rolling upon each other, such imaginary circles may always be conceived to exist, and are called the *pitch circles* of the wheels in question. They are represented by the dotted lines in fig. 21.

FIG. 21.



The pitch circle of a toothed wheel is an important element, and determines its value in transmitting motion.

Suppose that two axes at a distance of 10 inches are to be connected by wheelwork, and are required to revolve with velocities in the proportion of 3 to 2. Two circles, centred upon the respective axes, and having radii 4 and 6 inches, would, by rolling contact, move with the desired relative velocity, and would, in fact, be the pitch circles of the wheels when made. So that whatever may be the forms of the teeth upon the wheels to be constructed, the pitch circles are determined beforehand, and must have the proportion already stated.

It appears also that when the number of teeth upon a wheel is indefinitely increased, the wheel itself degenerates into the pitch circle.

So much of the tooth as lies within the pitch circle is called its *root* or *flank*, and the portion beyond the pitch circle is called the *point* or *addendum*.

The *pitch of a tooth* is the space *ac* upon the pitch circle cut off by the corresponding edges of two consecutive teeth.

Spur wheels are represented in fig. 20, and are those in which the teeth project radially along the circumference.

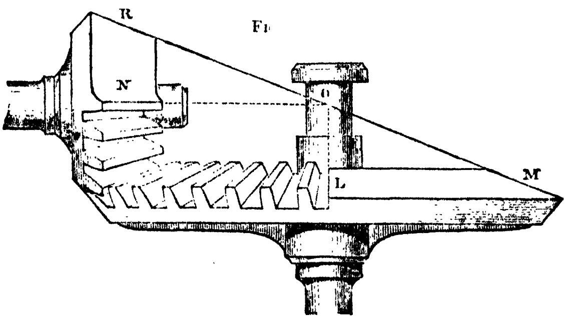
In a *face wheel*, cogs or pins, acting as teeth, are fastened perpendicularly to the plane of the wheel; in a *crown wheel* the teeth are cut upon the edge of a circular band; and *annular wheels* have the teeth formed upon the inside of an annulus or ring, instead of upon the outer circumference.

A straight bar provided with teeth is called a rack, and a wheel with a small number of teeth is termed a pinion.

Gearing and *gear* are the words used to indicate the combination of any number of parts in a machine which are employed for a common object.

Toothed wheels are said to be *in gear* when they are capable of moving each other, and *out of gear* when they are shifted into a position where the teeth cease to act.

ART. 19.—The spur wheels, before described, are suited to convey motion only between parallel axes ; it often happens, however, that the axes concerned in any movement are not parallel, and as



a consequence they may, or may not, meet in a point. If the axes do not intersect we proceed by successive steps, and continually introduce intermediate intersecting axes, and thus we are led to the use of inclined wheels whose axes meet each other, and which are known as *bevel wheels.*

It is easily proved in geometry that two right cones which have a common vertex will roll upon each other, and the same would be true of the frusta of two cones such as LM and NR, which are represented as having a common vertex in the point O.

The rolling of the cones will allow us to consider any pair of circles in contact and perpendicular to the respective axes as the pitch circles of the frusta, and teeth may accordingly be shaped upon them so as to produce the same even motion as that which exists in the case of spur wheels.

This fact about the rolling of two cones becomes very clear

when enquired into, and it is evident that if one of the cones be flattened out into a plane table, by increasing its vertical angle up to 180°, the property of rolling will not be interfered with. But in that case the common vertex will be a fixed point in the table, and, accordingly, if we roll a cone upon a table, the vertex ought not to move in the least degree as the cone runs round.

It is quite easy to test the matter in this way, and if the table be smooth and level the apex will remain perfectly stationary, although the cone itself is free to run in any direction.

The principle under discussion is sometimes applied in the construction of machinery; there is a large circular saw in the arsenal at Woolwich which is driven by the rolling contact of the frusta of two cones, and upon examination it will be found that the directions of the axes of the two frusta meet exactly in the centre of the revolving circular saw.

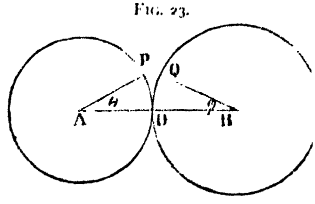
Equal bevel wheels whose axes are at right angles are termed *mitre wheels.*

ART. 20.—*Prop.* When two circles roll together, their uniform angular velocities are inversely as the radii of the circles.

This proposition is exactly analogous to that which obtains when a point describing the circumference of one circle passes off into another circle of different diameter, and the proof is the same.

Let the circles centred at A and B move by rolling through the corresponding angles PAD and QBD.

FIG. 23.



Let $\left.\begin{array}{l}AD=a\\BD=b\end{array}\right\}\left.\begin{array}{l}PAD=\theta\\QBD=\phi\end{array}\right\}$

then $PD=a\theta$, $QD=b\phi$,

but $PD=QD$

$\therefore\ a\theta=b\phi,$

$\therefore\ \dfrac{\theta}{\varphi}=\dfrac{b}{a}.$

But the angular velocities of the circles, being uniform, are as the angles described by each of them in the same given time,

$$\therefore\ \frac{\text{angular vel. of A}}{\text{angular vel. of B}}=\frac{b}{a}=\frac{BD}{AD},$$

which proves the proposition.

ART. 21.—Two simple questions relating to the transfer of motion by wheelwork remain to be determined.

1. Let two axes be parallel, and let $\frac{m}{n}$ be the velocity ratio to be communicated between them.

If a be the distance between the axes, and r, r' be the radii of the two pitch circles A and B,

the condition of rolling gives us $\frac{r}{r'}=\frac{n}{m}=\frac{\text{vel. of B}}{\text{vel. of A}}$.

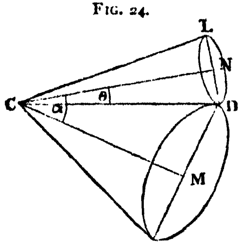
$$\text{Also}\quad r+r'=a,\ \therefore\ r+\frac{rm}{n}=a,$$

$$\therefore\ r=\frac{na}{m+n},\ \text{and}\ r'=\frac{ma}{m+n};$$

whence r and r' are known in terms of m, n, and a.

2. Let the axes meet in a point, and let it be required to construct two *cones* which shall communicate the same velocity ratio by rolling contact.

FIG. 24.



We now refer to fig. 24, and assume that DN, DM are the radii of the bases of the cones LCD, HCD, whose angular velocities are as the numbers m and n respectively.

$$\text{Let}\quad MCN=\alpha,\ NCD=\theta,$$

$$\text{then}\quad DN=CD\sin\theta,$$

$$DM=CD\sin(\alpha-\theta);$$

$$\therefore\ \frac{DM}{DN}=\frac{\sin(\alpha-\theta)}{\sin\theta}.$$

But $\frac{DM}{DN}=\frac{m}{n}$, since the inverse ratio of the radii of the bases of the cones is the velocity ratio between the axes,

$$\therefore\frac{m}{n}=\frac{\sin(\alpha-\theta)}{\sin\theta}=\sin\alpha\cot\theta-\cos\alpha,$$

$$\text{whence}\ \tan\theta=\frac{n\sin\alpha}{m+n\cos\alpha}.$$

If ϕ be an angle such that $n\cos\alpha=m\cos\phi$, $(m>n)$,

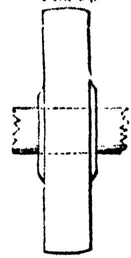
$$\text{we have}\ \tan\theta=\frac{n\sin\alpha}{m(1+\cos\phi)}=\frac{n\sin\alpha}{2m\cos^2\frac{\phi}{2}},$$

whence θ is expressed in a form adapted for logarithmic computation.

Cor. If $a=90$, we have $\tan\theta=\frac{n}{m}$.

ART. 22.—*Belts* or *straps*, otherwise called *bands*, are much used in machinery, in order to communicate motion between two axes at a distance from each other. In this case an endless band is stretched over the circumference of a disc or pulley upon each axis, and the motion is the same as if the discs rolled directly upon each other. The usual form of the pulley is shown in fig. 25.

FIG. 25.

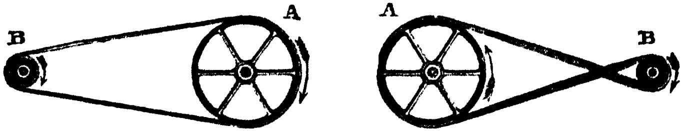


It is a common practice to convey steam power by means of shafting and wheelwork to the various floors of a mill, and then to distribute it to the separate machines by the aid of straps or belts.

These straps adhere by friction to the surfaces of pulleys, and work with a smooth and noiseless action; but they are subject to two principal objections, which may or may not be counterbalanced by their other advantages. The friction of the axes upon their bearings is increased by the double pull of the strap, arising from its tension, and there is a liability to some change in the exactness of the transmission of motion by the possible stretching or slipping of the band.

The drawing shows the method of communicating motion from one axis to another at a distance. The diameters of the large and small pulleys A and B are respectively as 3 to 1, and

FIG. 26.



the result is that when A makes 40 revolutions B makes 120 revolutions. The velocity ratio is precisely the same as if A moved B by rolling contact.

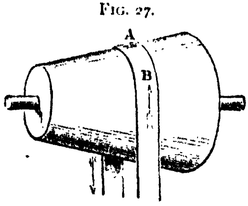
The strap may be *open* or *crossed.* In the former case A and B rotate in the same direction, and in the latter case they rotate in opposite directions, as indicated by the arrows.

ART. 23.—The term *band* is applied either to a flat strap or a round cord indifferently. The best material for round bands, such as are used in light machinery, is no doubt catgut, and then the band is fitted with a hook and eye to make it continuous. It must work in a pulley with a grooved rim, or it would slip off, and this groove prevents our shifting it easily from one pulley to another. The power of readily shifting a driving band is often an indispensable condition, and can be obtained at once by the use of a flat belt, which will hold on to its pulley with perfect security if we only take care to make the rim slightly convex, as shown in fig. 25, instead of being concave. No groove is necessary, or indeed admissible ; and, upon entering a workshop where steam power is employed, we see each machine driven by a flat belt riding upon one of a set of two or more pulleys with perfectly smooth edges.

The belt has no tendency to slip off, and it is shifted with the greatest ease from one pulley to another when pressed a little upon the advancing side by a fork suitably placed.

The reason for making the rim of a pulley slightly convex will be apparent if we examine the case of a tight belt running upon a revolving conical pulley. The belt embraces the cone, and tends to lie flat upon the slant surface, thus becoming bent into the form AB, the portion B being somewhat nearer to the base of the cone than the portion A.

FIG. 27.



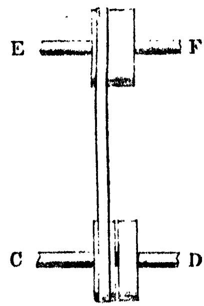
The cone, during its revolution, exerts an effort to carry B onward in a circle parallel to its base, and the consequence is that the belt tends to remain upon the slant surface of the cone, and to rise higher rather than to slip off.

In like manner, if a second cone of equal size were fastened to the one shown in the drawing, the bases of the two cones being joined together, the belt would, if its length were properly adjusted, work its way up to the part where the bases met, and

would ride securely upon the angular portion formed by their junction; but this is the same case as that of a slightly convex pulley, for it is evident that a little rounding off of the angle at the junction of the bases would convert this portion of the double cone into a convex pulley. Thus the action becomes perfectly intelligible.

ART. 24.—The *fast and loose pulleys* are an adjunct of the driving belt. They consist of two pulleys placed side by side, as in fig. 28, whereof one, A, is keyed to the shaft, CD, to which motion is to be conveyed, and the other, B, rides loose upon it. When the strap is shifted from the loose to the fastened pulley the shaft will begin to rotate, otherwise it remains at rest, the loose pulley alone turning round.

FIG. 28.



The shaft EF is the driver, and carries one broad pulley keyed upon it.

The band is shifted by a fork, which, as before stated, is made to press laterally upon its *advancing* side.

The advancing portion of the band must always lie in the plane of the pulley round which it is wrapped, but the retreating portion may be pulled on one side without causing the band to leave the pulley. This rule applies whether the band is round or flat.

ART. 25.—It is by observing this condition that a band may be used to communicate motion between two axes which are not parallel, and which do not meet in a point.

Problems such as these are interesting, as presenting difficulties to be overcome by a knowledge of principles.

Suppose that we are required to arrange that a band working over a pulley upon one given axis shall drive another pulley upon an axis at right angles to the first.

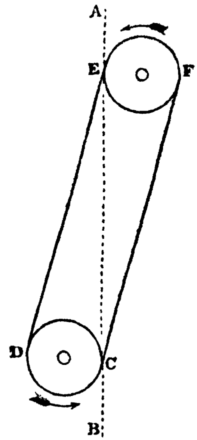
Here we intend that the pulleys should be placed one above the other as in the sketch. As the band goes round we have to provide that its advancing portion shall always lie in the plane of the pulley upon which it works. The easiest way of proceeding is to draw a straight line, AB, upon paper, and to place circles

EF and DC representing the pulleys in contact with AB upon each side of it.

Draw now the lines ED, CF, to represent bands passing round the circles, and however you may bend the two planes containing

Fig. 29.

Fig. 30.



the pulleys by folding the paper about AB as an edge, it is clear that the advancing portion of the strap will continue to lie in the plane of its pulley so long as the motion occurs in the direction of the arrows.

Reverse the motion, and the strap will leave the pulleys at once.

Art. 26.—It may be useful here to enquire how the necessary size or strength of the strap is ascertained when energy is transmitted, and we take the following example :—

Suppose that a force capable of doing work which is technically estimated at 5 horse-power is to be carried on by a strap moving with a velocity of 600 feet per minute over a suitable pulley. The work done by 5 horses is 5×33000 foot pounds per minute, and the work done by the strap must be the same.

Let P be the pull upon the strap in pounds, then $P \times 600$ is the work done by the strap in one minute,

$$\therefore P \times 600 = 5 \times 33000,$$

$$\therefore P = \frac{5 \times 330}{6} = 5 \times 55 = 275 \text{ lbs.}$$

If the velocity of a point in the strap had been reduced to 300 feet per minute, P would have been 550 lbs.; if it had been increased to 3,000 feet per minute, P would have been 55 lbs.; and thus we recognise the well-known mechanical principle, that the slower the movement by which any given amount of energy is transmitted, the greater must be the strength with which the moving parts are constructed.

In carrying out this principle successful attempts have recently been made to transmit the driving power of turbines or water-wheels to considerable distances, by means of a slender wire rope moving at a high velocity, and the method is called the *telodynamic transmission* of power.

The first experiment was made in 1850 by Mr. C. F. Hirn, at Loyelbach, near Colmar, Alsace. A band of steel 172 yards long, $\frac{1}{25}$ inch thick, and 2 inches broad was slung as an endless band over two pulleys, each $6\frac{1}{2}$ feet in diameter, which were placed at a distance of 84 yards, and made 120 revolutions per minute, giving a speed of 28 miles per hour in the band.

There were practical objections to the use of a flat band; nevertheless the plan was successfully adopted for a year and a half, and transmitted 12 horse-power to 100 looms.

Since that time the flat rope has been replaced by a round rope made of steel wire.

It may be interesting to refer to some operations carried on at Schaffhausen, on the Upper Rhine. Here the water-power is taken from three ordinary vertical-flow turbines, each $9\frac{1}{2}$ feet in diameter, and driven by a fall of water varying from 12 to 16 feet. Each turbine makes about 48 revolutions per minute, and the whole can develope collectively about 750 horse-power.

The wire rope is $\frac{3}{4}$ inch in diameter, made of the best Swedish iron, and having 72 wires in the rope. It starts by running upon pulleys driven by the turbines, and these pulleys are each 15 feet in diameter, and make 100 revolutions per minute, giving a linear velocity to the rope of about 53 miles per hour. No less than 17 factories in different positions have been supplied with motive

power from one set of turbines, and it is stated that the total length of transmission is 3,300 feet.

For a more homely illustration we may mention the locomotive workshops of the London and North-Western Railway at Crewe, where a cotton rope $\frac{5}{8}$ths of an inch in diameter, and weighing about $1\frac{1}{2}$ oz. per foot, has been carried along the length of a workshop with a velocity of 5,000 feet per minute, and so employed for actuating a traversing crane which is adapted for lifting a weight of 25 tons. The velocity of 5,000 feet per minute would be reduced, by suitable mechanism, to that of 1 foot $7\frac{1}{2}$ inches per minute, and the requisite work would be done by subjecting the whole cord to no greater strain than that of 109 lbs.

ART. 27.—*Guide pulleys* are sometimes used, and they are constructed as follows :—

Conceive that a band moving in the direction of AB is to be diverted into another direction, CD. There are two cases to be considered.

1. Let AB and CD meet in E. At the angle E place a small guide pulley whose plane is coincident with the plane AED. This pulley obeys the required condition, and will answer its purpose.

FIG. 31.

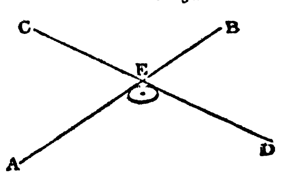
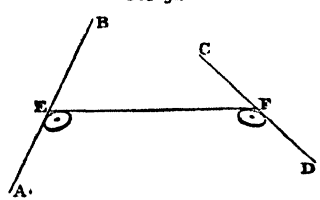


FIG 32.



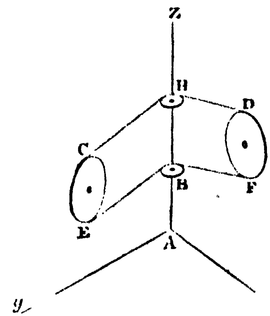
2. If AB and CD do not meet, or do not meet within a reasonable distance :—

Draw any straight line, EF, cutting both AB and CD. In the plane AEF, and at the angle E, place a guide pulley, E, and do the same thing at F by fixing a guide pulley in the plane EFD, and thus the strap will be carried on.

One advantage in the use of guide pulleys will be found in the fact that they enable us to overcome the inconvenience of not being able to reverse the motion when the planes of the pulleys are inclined to each other.

Thus, conceive that two pulleys work in the planes ZAx, ZAy inclined to each other at an angle xAy. In the line of the intersection of the planes, viz., AZ, take any two convenient points, H and B, place one guide pulley at H in the plane CHD, and another at B in the plane EBF, then the band CHDFBE will run round the two main pulleys securely in either direction.

FIG. 33.



This is evident, as we have done nothing to infringe the necessary condition, each advancing and retreating portion of the band will, in both cases, be found in the plane of the pulley upon which it rides.

Instead of bands we may employ chains to communicate motion from one axis to another, and there is one instance where a chain is always so used, viz., in the transfer of the pull of the spring from the barrel to the fusee of a watch. Here the form of chain is the type of most others of the heaviest construction, consisting of one flat plate or link riveted to two others, which are placed one above and the other below it, and thus the chain consists of one and two plate-links alternately. When a chain of this sort is used to transmit great force, it is called a *gearing chain*, and the open spaces formed by the two parallel links engage with projections on the wheel or disc over which it runs, rendering it impossible for the chain to slip.

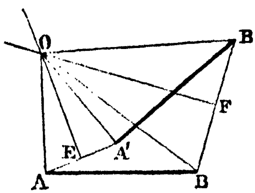
One practical objection to the use of chains, where great accuracy is required, consists in the fact that the links are liable to stretch, and that the pitch or spacing may lose its exactness, the result being to cause jar and vibration in the working.

ART. 28.—Instead of confining the motion of a body to simple translation in a straight line, or to simple rotation about a fixed axis, we shall now suppose that the body moves by sliding along a plane, in such a manner that any straight line in it, as AB, passes into the position A′B′, which lies in the same plane with AB, but is not parallel to it.

In such a case AB has a motion both of translation and rotation. Its centre describes some line straight or curved, which indicates a motion of translation, and at the same time the line itself changes the direction in which it points, or is subject also to a motion of rotation.

We shall now prove that the motion of AB may be represented by supposing it to be rotating at each instant about some point O, which point is continually changing its position, and is therefore called the *instantaneous* centre of motion.

FIG. 34.



The curve which the instantaneous centre describes is called the *centrode*, from two Greek words signifying 'the path of the centre.'

It will be understood that if the actual motion existing at the time considered were to be permanent, it would be a motion of rotation about an axis through O, which is therefore called the instantaneous axis.

In order to find the position of O, let the motion from AB to A′B′ be infinitesimal, and join AA′, BB′. Bisect AA′ in E, and draw EO perpendicular to it; bisect also BB′ in F, and draw FO perpendicular to it. O will mark the instantaneous axis concerned in the motion from AB to A′B′.

Join OA, OA′, OB, and OB′.

Then since OA=OA′, OB=OB′, and AB=A′B′

$\therefore$ angle AOB = angle A′OB′.

Take away the common angle A′OB and we have

angle AOA′ = angle BOB′.

Therefore while A is rotating about O into the position A′, the point B is also rotating about O into the position B′, or O gives the position of the instantaneous axis.

ART. 29.—In some simple cases, O is a fixed point, but it may still have the property of an instantaneous axis. This happens when a body is fixed to one end of a rotating arm, the centre of motion of the arm lying away from the body.

For example, let an arm placed horizontally be made to rotate about a vertical axis through one end, and let a wheel B be locked to the other end of the arm, in such a manner that its plane is horizontal.

As the arm goes round, a person inspecting the apparatus from a little distance will see the wheel B turning on its axis, and if he watches a mark upon the rim, he can entertain no doubt about this fact.

The truth is that although the wheel B does not move relatively to the arm, it is, nevertheless, the subject of two distinct motions, whereof one consists in a *rotation* about an axis through its centre, and the other is a motion of *translation*, whereby the centre of the wheel describes a circle whose radius is the distance between the centre of B and the axis of rotation of the arm.

This is an example of the resolution of a compound movement into its simple elements, and the instantaneous axis remains permanently in the axis about which the arm rotates.

If the wheel B were looked at from the centre about which the arm revolves, no motion of rotation could be recognised. B would appear to have a motion of translation which carried it round in a circle.

The very same thing happens in the case of the moon. Astronomers tell us that only about one-half of the face of the moon has ever been seen by those upon the earth's surface, and they explain the fact by saying that the moon turns once upon its axis during the period of a single revolution in its orbit round the earth; or, in other words, that it moves as if it were fixed to a rigid bar stretching from the earth to the moon.

ART. 30.—*Case* III. If the motion of rotation of a body about an axis be combined with a motion of translation of the axis itself in the line of its direction, any point in the body will describe a curve which cannot lie in one plane.

Such a movement may be obtained from a single pair of elements, as in the example of a nut on a screwed bolt.

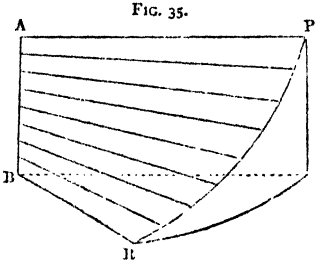
After toothed wheels, the screw plays the most important part in mechanical appliances, and indeed it is difficult to over-estimate its value or utility. The screw bolt and nut are used to unite the various parts of machinery in close and firm contact, and are

peculiarly fitted for that purpose; then, again, the screw is employed in the slide rest and in the planing machine to give a smooth longitudinal motion, the same purpose for which it aids the astronomer in measuring the last minute intervals which are recognisable in the telescope. In the screw press we rely upon it to transmit force, we use it in screw piles to obtain a firm foundation for piers or lighthouses, and as a propeller for ships it has given a new element of strength and power to our navy.

The definitions relating to the screw are the following:—

If a horizontal line AP, which always passes through a fixed vertical line, be made to revolve uniformly in one direction, and at the same time to ascend or descend with a uniform velocity, it will trace out a *screw surface* APRB, in the manner indicated in the sketch.

Fig. 35.



The points of intersection of this generating line with any circular cylinder whose axis coincides with AB, will form a *screw thread*, PR, upon the surface of the cylinder.

The *pitch of a screw* is the space along AB, through which the generating line moves in completing one entire revolution.

Also AB is called the *length* of the screw surface APRB, and the angle PRQ represents the *angle* of the screw.

In the diagram, AP is shown as describing a *right-handed* screw, if it revolved in the opposite direction during its descent, it would describe a *left-handed* screw.

Art. 31.—The *screw thread* used in machinery is a projecting rim of a certain definite form, running round the cylinder, and obeying the same geometrical law as the ideal thread which we have just described.

In practice the pitch of a screw bolt is usually estimated by observing the number of ridges which occur in an inch of its length; thus we speak of a screw of one-eighth of an inch pitch as being a screw with eight threads to the inch.

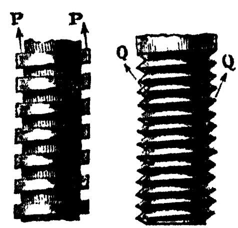
If a single thread were wound evenly round a cylinder, and the path of a thread marked out, we should have a *single-threaded* screw; whereas, if two parallel threads were wound on side by side, we should obtain a *double-threaded* screw.

The object of increasing the number of threads is to fill up the space which would be unoccupied if a fine thread of rapid pitch were traced upon a bolt, and thus to give the bolt greater strength in resisting any strain which tends to strip away the thread. Increasing the number of threads makes no difference in the pitch of the screw, which is dependent on any one continuous thread of the combination.

The ordinary screw-propeller is a double-bladed screw, and has sometimes three or even four blades, which correspond to the multiple threads here spoken of.

ART. 32.—The two principal forms of screw-thread used by engineers are the square and the V thread; they are given in the sketch, and in applying them we should understand that there are three essential characters belonging to a screw-thread, viz., its *pitch*, *depth*, and *form*; and three principal conditions required in a screw when completed, viz., *power*, *strength*, and *durability*.

FIG. 36.



It is easy to see that no one can declare exactly what power, strength, or durability is given by a screw-thread of a certain pitch, depth, or form, when traced out upon a given cylinder. The problem is indeterminate, and must remain so; we cannot lay down any rule for determining the diameter of a screw bolt required for any given purpose, nor can we say what should be the precise form of thread.

It is the province of practical men to determine any such questions when they arise, being guided in their judgment by experience and by certain general considerations which we propose now to examine.

1. The *power* of a screwed bolt depends upon the *pitch* and *form* of the thread.

If the screw-thread were an ideal line running round a cylin-

der, the power would depend solely on the pitch, according to the relation given in all books on mechanics, viz.:—

$$\text{weight} \times \text{pitch} = \text{power} \times \left\{ \begin{array}{l} \text{circumference of the circle described} \\ \text{by the end of the lever-handle.} \end{array} \right.$$

If the thread were square we should substitute for the ideal line a small strip of surface, being a portion of the screw surface shown in fig. 35, which would present a reaction P to the weight or pressure everywhere identical in direction with that which occurs in the case of the ideal thread. Hence, if there were no friction, we should lose nothing by the use of a square thread in the place of a line.

A square-threaded screw is, therefore, the most powerful of all, and is employed commonly in screw presses.

But if the thread were angular, the reaction Q which supports the weight or pressure would suffer a second deflection from the direction of the axis of the cylinder over and above that due to the pitch, by reason of the dipping of the surface of the angular thread, and we should be throwing away part of the force at our disposal in a useless tendency to burst the nut in which the screw works.

In this sense, the square thread is more powerful than an angular or V thread of the same pitch.

2. The *strength* depends on the *form* and *depth*.

This statement is obvious. In a square thread half the material is cut away, and the resistance to any stripping of the thread must be less than in the case of the angular ridges.

Again, if we deepen the thread we lessen the cylinder from which the screw would be torn if it gave way, and thus a deep thread weakens a bolt.

3. Finally, the *durability* of a screw-thread depends chiefly upon its depth, that is, upon the amount of bearing surface; and in the case of a screw which is in constant use, as, for example, in the slide-rest of a lathe, it would be well for the young mechanic to satisfy himself upon this point by ascertaining the amount of bearing surface given by the fine deep thread which is found upon the screw working in the slide-rest of a well-made lathe.

Probably the finest specimen of minute workmanship in screw-cutting will be found in the screws provided by Mr. Simms

for moving the cross wires or web across the field of view of a micrometer microscope.

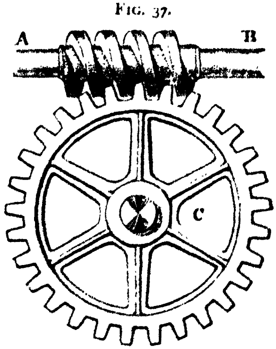
There are 150 threads to the inch, the diameter of the bolt being about $\frac{1}{8}$th of an inch; the head of the screw is a graduated circle read off to 100 parts, and the movement of the wires produced by turning the screw-head through the space of one graduation is quite apparent.

Upon examining the thread with a microscope, we should see a fine angular screw, consisting of a number of comparatively deep-cut ridges, having the sides a little inclined and the edges rounded off.

In the year 1841, Sir J. Whitworth proposed a uniform system of screw threads for bolts and screws used in fitting up steam engines and other machinery. This system has been adopted, and has given rise to the so-called Whitworth thread, about which it is only necessary here to say that tables are published giving the pitches for screws with angular threads on bolts of given diameter, and further that the angle of inclination of the sides of the thread is constant, being 55°, with one-sixth of the depth rounded off at the top and bottom.

ART. 33.—A *worm wheel* is a wheel furnished with teeth set obliquely upon its rim, and so shaped as to be capable of engaging with the thread of a screw; the revolution of the endless screw or worm AB will then impart rotation to the wheel C, and the wheel will advance through one, two, or three teeth, upon each revolution of AB, according as the thread thereon traced is a single, double, or triple thread.

FIG. 37.



This reduction of velocity causes the combination to be particularly valuable as a simple means of obtaining mechanical advantage, and, as we have stated, the number of threads upon the screw determines the number of teeth by which the wheel will advance during each revolution of AB.

In the transmission of force the screw is always employed to

wheel, one half of which is shown in either view of the apparatus, the remaining half being cut away in order that the disposition of other working parts may be better understood.

The worm consists of an endless screw on a spindle terminating at C, and rotated by a lever handle HH. It is apparent that the lever handle and worm drive the worm wheel, and further that the rotation of the worm wheel or nut imparts a longitudinal motion to the lifting screw.

In order that the apparatus may work, provision is made that the lifting screw shall not rotate, the nut in which it works is fixed in position in the casing, and can freely turn without shifting, the result being that S rises or falls slowly as the handle rotates.

An example, set out upon the diagram, and solved upon the principle of work, will give a better insight into the mechanical construction. The friction of the working parts is neglected in order to obtain a simple numerical result.

Let the pitch of the lifting screw be 1·25 inches,
let the worm wheel have 16 teeth, and
let the circumference described by H be 87·5 inches.
Then motion of handle after 16 turns $= 87{\cdot}5 \times 16$ inches,
$= 1400$ inches,
motion of screw at same time $= 1{\cdot}25$ inches.
$\therefore$ motion of H is to motion of S as 1400 to 1·25,
as 1120 to 1.
Let pressure on handle $= 20$ lbs.
$\therefore$ weight raised by screw $= 20 \times 1120$ lbs.
$= 22400$ lbs.
$= 10$ tons.

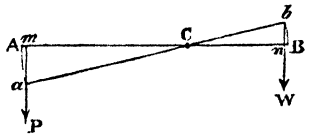
ART. 35.—In concluding this chapter we may mention that the doctrine of harmonic motion enables us to deduce the relation between two balancing forces when acting on the straight lever. This relation follows as a direct consequence of the principle of work.

Let ACB be a straight lever whose fulcrum is C, and let the forces P and W acting perpendicularly to the lever at the points A and B balance each other.

Let the lever be now tilted into the position *a*C*b*, and draw *am*, *bn*, perpendiculars on ACB. Then the harmonic motion *am*

represents the displacement of A in direction of P, while *nb* represents the displacement of B in direction of W.

FIG. 39.



But the forces P and W balance, and no work is done, therefore

$$P \times am - W \times bn = o.$$

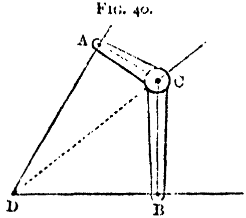
or $$\frac{P}{W} = \frac{bn}{am} = \frac{Cb}{Ca} = \frac{CB}{CA},$$

which is the well-known condition of equilibrium.

ART. 36.—*Bell-crank levers* serve to change the line of direction of some small motion, and are of universal application. They consist simply of two arms standing out from a fixed axis so as to form a bent lever.

1. Suppose it to be required to construct a bell-crank lever so as to change the direction of some small motion from the line BD into the line DA, where BD and DA meet in a point D.

FIG. 40.



Draw DC, dividing the angle at D into two parts whose *sines are in the ratio of the velocities* of the movements in the given directions.

This may be done by setting up perpendiculars anywhere on BD and DA in the required ratio, and drawing straight lines through their extremities parallel to BD and DA respectively. These parallel lines will intersect somewhere in DC, and will determine that line.

In DC take any convenient point C, and draw CA, CB, perpendicular to DA and DB respectively, then ACB will be the bell-crank lever required.

This is the construction, and it can be immediately verified, for the arcs described by the extremities A and B, when the lever ACB is shifted through a small angle α, will be represented by $AC \times \alpha$ and $CB \times \alpha$ respectively, and will measure the velocities of the points A and B.

$$\text{Hence } \frac{\text{velocity of A}}{\text{velocity of B}} = \frac{AC \times \alpha}{CB \times \alpha} = \frac{AC}{CB} = \frac{\sin CDA}{\sin CDB}.$$

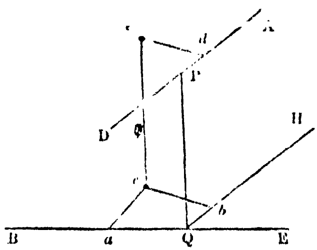
It is evident that the movements in DA and DB are very nearly rectilinear, and will become more so the further we remove C from the point D.

Any play which may be necessary at the joints A and B, by reason that the ends of the levers really describe small circular arcs, may be easily provided for in the actual arrangement.

2. To change the direction from one line to another not intersecting it.

Draw PQ, a common perpendicular to the lines AD and BE; through Q draw QH parallel to DA; construct a bell-crank lever, *a c b*, for the movements as transferred to the lines BQ, QH; draw *c e* parallel to PQ and equal to it, and further make *e d* parallel and equal to *c b*.

FIG. 41.



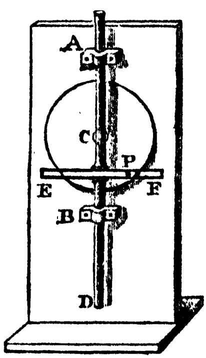
The piece *a c e d* will be the lever required; what has been done is this, a bell-crank lever *a c b* has been formed by the rule given above in order to transfer the motion from BE to QH, and then the motion in QH has been shifted into another line DA parallel to itself.

CHAPTER II.

ON THE CONVERSION OF CIRCULAR INTO RECIPROCATING MOTION.

ART. 37.—In discussing the nature of harmonic motion we have necessarily been led to consider the most elementary form of apparatus for converting the circular motion of a pin moving in one plane into the reciprocating motion of a guided bar.

FIG. 42.



The annexed sketch, taken from a model belonging to the School of Mines, shows an apparatus having a pin, P, fastened to a disc of wood, and capable of being rotated by a handle at the back. A horizontal slotted bar, EF, attached at right angles to a vertical guided bar, AD, completes the arrangement. Such an apparatus has already been referred to, and it is apparent that the bar rises and falls with a true harmonic motion as the pin, P, moves round uniformly in a circle.

We pass on to analyse the conversion of circular into reciprocating motion by means of the *crank and connecting rod.*

A crank is merely a lever or bar movable about a centre at one end, and capable of being turned round by a force applied at the other end; in this form it has been used from the earliest times as a handle to turn a wheel. When the crank is attached by a connecting rod to some reciprocating piece, it furnishes a combination which is extremely useful in machinery.

In the next chapter we shall see that the crank and connecting

rod is one of the principal contrivances for converting reciprocating into circular motion; the student will understand that any such distinction as to the effect of the contrivance is one of classification only, regard being had to the direction in which the moving force travels. The arrangement is often used under both aspects in one and the same machine; as in a marine engine, where the piston in the steam-cylinder actuates the paddle-shaft by means of a crank and a connecting rod, and the motion is then carried on, by a crank forged upon the same shaft, to the bucket or piston of the air-pump.

It was in the year 1769 that James Watt published his invention of 'A Method of Lessening the Consumption of Steam and Fuel in Fire-Engines,' the main feature of which was the condensation of the steam in a vessel distinct from the steam-cylinder. The steam-engine was at that time called a fire-engine, and was used exclusively in pumping water out of mines. The steam piston and the pump rods were suspended by chains from either end of a heavy beam centred upon an axis, the action of the steam caused a pull in one direction only, and the pump rods being raised by the agency of the steam were afterwards allowed to descend by their own weight.

In this shape the steam-engine was entirely unfitted for actuating machinery, and it was not until after the impulse given by Watt's invention was beginning to be felt that it became apparent that the expansive force of steam could be made available as a source of power in driving the machinery of mills.

While Watt was occupied with the great problem of the construction of double-acting engines, which eventually he fully solved, it happened that one James Pickard, of Birmingham, in the year 1780, took out a patent for a 'new invented method of applying steam or fire engines to the turning of wheels,' in which he proposed to connect the great working beam of the engine with a crank upon the shaft of a wheel by means of a spear or connecting rod, jointed at its extremities to the beam and crank respectively.

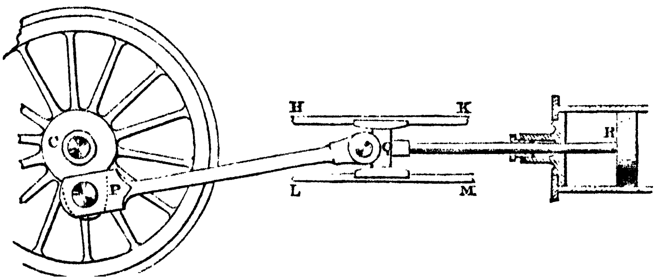
It is probable that Watt had foreseen this application of the crank as early as the year 1778, and had intended to apply the combination as a means of carrying on the power from the end of

the working beam to the fly-wheel. Being forestalled, however, by the patentee, he did not dispute the invention, and contented himself with patenting certain other methods of obtaining a like result, among which will be found the sun and planet wheels described in a subsequent chapter.

This latter invention served his purpose until the patent for the crank had expired, and then it was that the arrangement which we are now about to discuss came into general use.

The manner of employing the crank and connecting rod in the

FIG. 43.

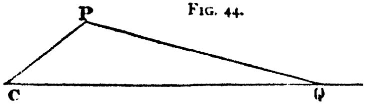


locomotive engine is shown in fig. 43. The crank CP is made a part of the driving wheel of the engine, the connecting rod PQ is attached to the end of the piston rod QR, and the end Q is constrained to move in a horizontal line by means of the guides HK, LM.

In this engine the crank is the follower and not the driver, but the combination is the same whether the circular motion of CP causes the reciprocating motion of Q, or whether the reciprocation of Q imparts a circular motion to CP.

ART. 38.—We now pass on to discuss the primary fundamental piece of mechanism derived from a simple triangle.

FIG. 44.

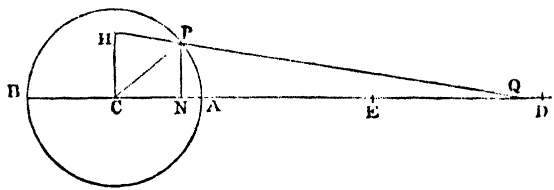


Since a triangle is an immovable figure, and will not 'rack,' as mechanics express it, some provision must be made for varying the dimensions of at least one side.

In our combination, the crank CP is of fixed length, as is also the connecting rod PQ, but the side CQ is of variable length.

When CP performs complete revolutions, it is clear that Q will reciprocate in the line CQ. We shall presently see that the motion of Q is of an *aggregate* character, or that Q is the recipient of two distinct movements which are simultaneously impressed upon it.

ART. 39.—To determine the relative positions of the crank and connecting rod during the motion.



Let CP be the *crank* centred at C, PQ the *connecting rod*, and let the point Q be constrained to move in the straight line CED.

Draw PN perpendicular to CD, and let $CP = a$, $PQ = b$, also let the angle $PCQ = C$, and $PQC = Q$.

Then $CQ = CN + NQ = a \cos C + b \cos Q$.

Also $\dfrac{\sin Q}{\sin C} = \dfrac{a}{b}$, $\therefore \sin Q = \dfrac{a}{b} \sin C$,

$$\text{and } \cos Q = \sqrt{1 - \frac{a^2 \sin^2 C}{b^2}}.$$

$$\therefore CQ = a \cos C + \sqrt{b^2 - a^2 \sin^2 C},$$

which gives the position of Q for any value of C, that is, for any given position of the crank CP.

Cor. 1. Let $C = 0°$, $\therefore CD =$

$C = 180°$, $\therefore CE$:

whence $DE = CD - CE = 2a$.

The space DE is called the *throw* of the crank.

Cor. 2. If the position of Q be estimated by its distance from D, we have

$$DQ = CD - CQ = a + b - (a \cos C + b \cos Q)$$
$$= a(1 - \cos C) + b(1 - \cos Q).$$

But we have just shown that an expression such as $a\,(1-\cos C)$ represents the resolution of circular into reciprocating motion, and we infer that the motion of the point Q is compounded of the resolved parts of two circular motions, one being that due to the motion of P through an angle C in a circle round C, the other, that resulting from the motion of P in a circular arc through an angle Q, and produced by the swinging of the rod PQ about one end Q as it moves to and fro.

Hence the connecting rod introduces an inequality, which prevents the motion of the point Q from retaining that evenness and regularity of change which was found in the motion of the point N (Art. 7). We now see by analysis that this inequality, whereby the motion of Q differs from that of N, is equal to $b\,(1-\cos Q)$.

Ex. Let CP = 10 inches, PQ = 5 feet, as in the engine in fig. 43; find the position of the piston when the crank is vertical.

$$\text{Here } \sin Q = \frac{CP}{PQ} = \frac{10}{60} = \frac{1}{6}.$$

$$\therefore \cos Q = \sqrt{1-\frac{1}{36}} = \sqrt{\frac{35}{36}} = \frac{1}{6}\sqrt{35}.$$

$$\therefore DQ = a(1-\cos C) + b(1-\cos Q)$$

$$= 10 + 60\left(1-\frac{1}{6}\sqrt{35}\right)$$

$$= 10 + \cdot 84 \text{ nearly,}$$

or Q is nearly six-sevenths of an inch in advance of the centre of its path when the crank has made a quarter of a revolution from the line CD.

As the connecting rod is shortened the inequality increases, and the motion becomes more unequal.

Take a very extreme case, where $PQ = CP = a$,

$$\therefore DQ = a(1-\cos C) + a(1-\cos Q).$$

Let now $C = 60°$, then $Q = 60°$ also, because the triangle CPQ is now isosceles,

$$\therefore \cos C = \frac{1}{2} = \cos Q$$

$$\therefore DQ = 2a - a = a,$$

or Q has moved through half its path, while CP has turned through an angle of only 60°.

When $C=90°$, the point Q comes to C, and there the motion ends, for the crank CP can now go on rotating for ever without tending to move Q.

Cor. 3. If the connecting rod could be prolonged until it became infinite, we should have PQ always parallel to itself, or $Q=0$, and in that case the travel of Q would be represented by the equation $DQ=a(1-\cos C)$.

A crank with a connecting rod of infinite length is an imaginary creation, but we shall presently see that an equivalent motion may be obtained in various ways.

ART. 40.—To determine the velocity ratio of P to Q :—

Let V be the velocity of P, v the velocity of Q. From C draw a straight line at right angles to CQ, and let it meet QP produced in H (fig. 45).

Then since PQ is a rigid rod, the resolved velocity of P along QP is equal to the resolved velocity of Q along QP.

$$\text{Let } PCQ=C,\ PQC=Q,\ CPH=\alpha.$$

Then P may be taken to move in a tangent at P, and therefore its resolved velocity in PQ is $V \sin \alpha$, also the resolved velocity of Q in the same direction is $v \cos Q$.

$$\text{Hence } V \sin \alpha = v \cos Q,$$

$$\therefore \frac{v}{V}=\frac{\sin \alpha}{\cos Q}=\frac{\sin \alpha}{\sin H}=\frac{CH}{CP}.$$

The same result may be obtained by analysis, for making $PCQ=\theta$, $PQC=\phi$, we have

$$V=\frac{ad\theta}{dt},\quad v=a\sin\theta\frac{d\theta}{dt}+\frac{a^2\sin\theta\cos\theta}{\sqrt{b^2-a^2\sin^2\theta}}\frac{d\theta}{dt}.$$

$$\text{Whence } \frac{v}{V}=\sin\theta+\frac{a\sin\theta\cos\theta}{\sqrt{b^2-a^2\sin^2\theta}}$$

$$=\sin\theta+\frac{a\sin\theta\cos\theta}{b\cos\phi}$$

$$=\sin\theta\frac{(b\cos\phi+a\cos\theta)}{b\cos\phi}$$

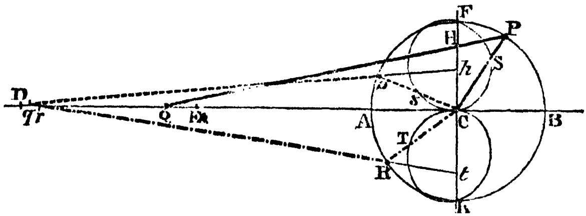
$$=\frac{\sin\theta\times CQ}{PQ\cos\phi}$$

$$=\frac{\sin\theta}{\cos\phi}\cdot\frac{\sin\alpha}{\sin\theta}=\frac{\sin\alpha}{\sin H}=\frac{CH}{CP}.$$

ART. 41.—The relative velocities of P and Q may be set out in a diagram.

Take CP : PQ :: 1 : 4, which will give a figure of convenient dimensions, and let Q, q be two positions of the end of the connecting rod. Produce QP, qp to meet the vertical diameter FCK

FIG. 46.



in H and h respectively, the line BCD being horizontal. Join CP, Cp, and measure off CS = CH, and Cs = Ch. Then, as before,

$$\frac{v}{V} = \frac{CH}{CP} = \frac{CS}{CP}.$$

In like manner, $\frac{\text{vel. of } q}{\text{vel. of } p} = \frac{Cs}{Cp}$, and so for all other values, whereby the curve CSs will indicate the comparative velocities of the crank pin and of the end of the connecting rod during one single stroke. Also, by a similar construction, the lower loop may be set out, preserving the proportion,

Vel. of r : vel. of R :: Ct : CR,

and making CT = Ct.

If PQ were kept parallel to itself throughout the motion we should have in effect a crank with an infinite link, and the looped curve would become a circle described on the radius FC as a diameter, as in the case of the simple harmonic motion. The inequality introduced by the obliquity of the connecting rod is thus rendered apparent by comparison of the distorted loop with the true circle.

Ex. To compare the positions of Q when CP makes a given angle of $x°$ with CA, CB respectively.

Let D and E be the extreme positions of Q, then

$$DQ = a(1 - \cos\theta) + b(1 - \cos\phi)$$
$$EQ = 2a - DQ = a(1 + \cos\theta) - b(1 - \cos\phi).$$

Also, let θ be the circular measure of x°, then ϕ can be found in terms of a, b, and θ.

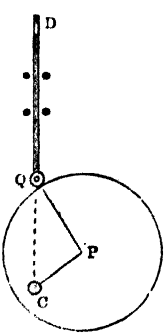
Take the case of a direct acting engine, where $a = 1$ foot, $b = 6$ feet, and let $x = 1$. Calling Q,Q′ the required positions of the end of the connecting rod, we have $\cos\theta = \cdot 9998477$, $\cos\phi = \cdot 9999958$,

$$\therefore DQ = \cdot 0001775 \text{ feet},$$
$$EQ' = \cdot 0001271 \text{ feet}.$$

ART. 42.—The *eccentric circle* supplies a ready method of obtaining the motion given by a crank and link, and we proceed to examine it with the intention of ascertaining by what expedients we are enabled to vary the lengths of the particular crank and link which exist in every form of the arrangement.

And, first, we notice that the length of the crank is in every case equal to the distance between the centre of motion and the centre of the eccentric circle; it is, in fact, the line CP in each of the annexed drawings.

FIG. 47.

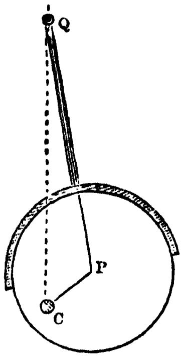


Let us consider the motion shown in fig. 47, where a circular plate, movable about a centre of motion at C, imparts an oscillatory movement to a bar, QD, which is capable of sliding between guides in a vertical line, DQ, pointing towards C. Since PQ remains constant as the plate revolves, it is evident that Q moves up and down in the line CD, just as if it were actuated by the crank, CP, and the connecting rod, PQ. The length of the connecting rod is in this case, therefore, equal to the radius of the rotating circle. It is obvious that an arrangement of this kind would be little used, by reason of the oblique thrust on the bar QD.

A second form which, however, is of the greatest possible value, is deducible at once from that last examined.

Instead of allowing the end of the bar, QD, to rest directly upon the circumference of the circle, suppose that bar to terminate in a half-hoop which fits the circle, as shown in the drawing; let the rod point to the centre of the circle, and let one end, Q, be compelled to move in a line pointing to C. As the circle revolves it is evident that we have a crank, CP, just as before; but we have, in addition, a link which is now represented by PQ, and which may extend as far as we please beyond the limits of the circular plate.

FIG. 48.



We thus obtain a combination which is sometimes described as a mechanical equivalent for the crank and connecting rod.

The form usually adopted in practice is derived at once from the arrangement just described. A circular plate is completely encircled by a hoop, to which a bar is attached; this bar always points to P, the centre of the plate, and its extremity drives a pin, Q, which is constrained to move in the line CQ.

The plate is movable about a centre of motion at C, and we have already explained that PQ remains constant during each revolution of the plate, or that the resulting motion impressed upon Q is that due to a crank, CP, and a link, PQ.

As before, the *throw of the eccentric* is the same as that of the crank, viz., a space equal to the diameter of the circle whose radius is CP.

We should remark that P, the centre of the plate, may be brought as near as we please to C, the centre of the shaft, and that the throw of the eccentric may be reduced accordingly; but that we are limited in the other direction, for the shaft must be kept within the boundary of the plate, and the plate itself must not be inconveniently large, considerations which are sufficient to prevent our increasing CP in any great degree.

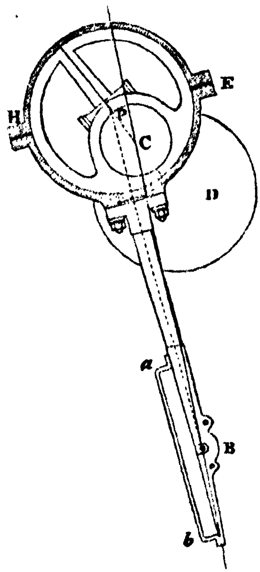
The eccentric circle may also be regarded as a simple form of *cam* (see Art. 58), but we have examined it here on account of its being identical in principle with the crank and connecting rod.

The object of the complete hoop is to drive Q in alternate

directions. In some cases Q is brought back by a spring, and then only half the hoop is required. An instance occurs in modern forging machines, where the motion is very small and rapid.

ART. 43.—Having thus explained the principle of construction adopted in the eccentric, it remains to show the contrivance as made and applied in an engine. In the annexed diagram, which is taken from a small oscillating engine, the circle C represents a section of the crank shaft, C being its centre. Upon the shaft are fitted two circular half-pulleys of cast iron, which are bolted together, and have a centre at P. Two half-hoops of brass, tinted in the sketch, and united together by bolts and double nuts at E and H, carry the eccentric bar, which actuates a pin at B connected with the valve lever. The engine being designed for a river boat, and therefore requiring to be reversed at pleasure, there is a strap, *ab*, to prevent the eccentric rod from falling away from the pin while the valve is being moved by hand. Also, in this case, the eccentric pulley rides loose upon the shaft within certain limits defined by stops, and there is consequently a disc, D, forming a counterbalance to the weight of the pulley, which prevents it from falling out of position during the disengagement of the pin at B.

FIG. 49.



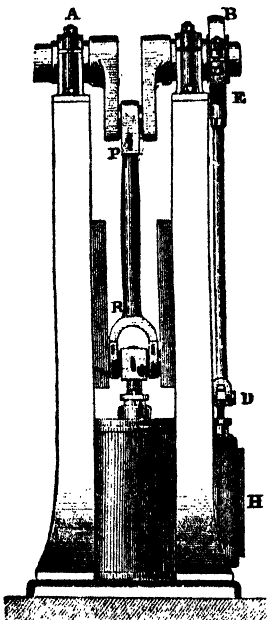
ART. 44.—It will be understood that the crank and connecting rod labour under the disadvantage of entailing a division of the shaft whenever it is required to place the crank anywhere except at one end, for the connecting rod is continually traversing over the centre of the shaft.

If, therefore, a crank is wanted in some intermediate portion of an axle or shaft, the axle must be *cranked* in the manner shown in fig. 50, or be divided, and the two cranks or arms will be con-

nected by a pin. These cranks and the pin are frequently forged in one solid mass upon the shaft, and shaped afterwards by the machinery of the workshop.

The annexed sketch is taken from a lecture diagram in Sir J. Anderson's collection, and represents a small vertical engine suitable for driving light machinery.

Fig. 50.



The steam cylinder is marked C, and H is the slide case, the piston rod being connected with the crank pin by the connecting rod PR. The slide valve is worked by an eccentric, shown at E, and the eccentric rod attached to the valve spindle is marked ED.

The great value of the eccentric arises from the circumstance that it enables us to derive the motion, which would be given by a crank, from any part of a shaft without the necessity of subdividing it. This is particularly noticeable in the mechanism here set forth, where the crank of small throw which is required for moving the steam slide-valve is furnished by the aid of an eccentric keyed upon the main shaft.

Art. 45.—It is hardly necessary to explain that when a circle revolves about an axis perpendicular to its plane, and a little out of the centre, it will be enveloped in all positions by a somewhat larger circle, the increase of the radius being equal to the eccentricity.

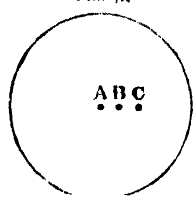
This fact has been applied to a useful purpose in the production of gun-stocks by machinery, as, for example, at the Small Arms Factory at Enfield. The practice has been to form a hardened steel block with cavities, shaped so as exactly to correspond with the cavities which it is intended to carve out of the gun-

stock. The clearing out of the recesses is effected by revolving drills, making some 6,000 revolutions per minute, which are carried over the work, and are guided on the copying principle by a dumb tracer, which travels over every portion of the steel block. The tracer and the rotating drill exactly correspond in size. For more delicate portions of the work smaller bits with corresponding tracers are employed, and thus the wood is carved out with the details of form which are to be found in the pattern.

But now comes a difficulty. The tracer and pattern are of hardened steel, and each tracer has its corresponding bit or drill, which is an exact revolving counterpart thereof, so long as its cutting edges are not worn away. But when the drills wear by sharpening, and become smaller, the copy will be defective.

In order to compensate for this source of error, the conical hole in the end of the running spindle into which the drills are fitted is made slightly eccentric. The drill is also eccentric on its shank to the same amount.

Fig. 51.



It is therefore possible to bring the axis of the drill itself exactly in the same line as the axis of the running spindle, in which case the drill will revolve in a cylinder of its own size. Or the axis of the drill may be set on one side of the axis of the running spindle, in which case the drill will carve out a cylinder larger than itself.

The end of the spindle and drill are graduated round the circumference, and thus the amount of eccentricity, and the consequent enlargement of the drill spindle, can be adjusted with the utmost nicety.

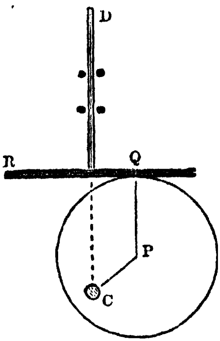
In the drawing A represents the centre of the running spindle, B is the centre of the conical hole made therein, C is the centre of the drill spindle. When the drill is at work C describes a circle round A. But C can be shifted into any position round B as a centre, and therefore C can be brought either to coincide with A, or can be placed at any distance from A, lying between the limits zero and AC. Hence the result which has been stated.

Art. 46.—There is yet another method of arranging the eccen-

tric circle, which gives us the combination of a *crank* with an *infinite link.*

We here intend, as explained in a former article, that the reciprocating piece shall be driven by a crank and connecting rod in such a manner that the connecting rod shall always remain parallel to itself, a result which could only happen in theory if the connecting rod were indefinitely lengthened.

FIG. 52.



Suppose the roller at Q to be replaced by a cross bar QR, standing at right angles to DC. As the circle revolves, it will cause the bar to reciprocate, CP will remain constant, and PQ will always be at right angles to RQ, and will therefore remain parallel to CD in all positions.

But this is the motion of a crank with an infinite link.

If C were in the circumference of the circle, the motion would be just the same, except that now the crank would be the radius of the eccentric circle. We shall presently notice a useful illustration of this particular case. (Art. 48.)

Take the following as an example of the movement under discussion.

In Sir J. Anderson's machine for compressing elongated rifle bullets, which was in use during the Crimean war, there are punches fixed at the two ends of a strong massive rod, to which a reciprocating motion in a horizontal line is imparted, and a piece of lead is compressed into the required form at each end alternately.

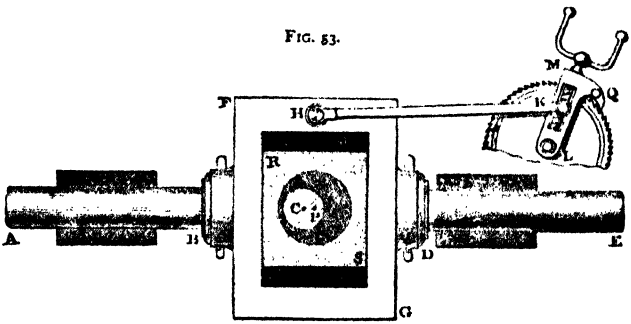
The object of one part of the mechanism is to cause this rod to reciprocate, and the movement is obtained as follows.

A small circle centred at C represents in section a shaft caused to rotate by the power of an engine. Upon this shaft a short cylindrical block is forged so as to form part of it, and is afterwards accurately turned into the form of a circular cylinder, whose axis passes through P upon one side of the original axis.

A rectangular brass block, RS, is bored out to fit the larger cylinder and slides in the rectangular frame FG, to which the

cylindrical pieces AB, DE, which carry the punches, are attached. The whole is put together in the manner shown, and it requires very little effort to understand that the rotation of the eccentric cylinder round an axis through C will impart to RS simultaneous

FIG. 53.

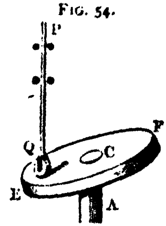


movements in a horizontal and vertical direction, whereof one, viz., that in a direction perpendicular to AE, will be inoperative, and the other will be communicated directly to FG, and so to the rods carrying the punches. Thus AB and ED will be made to oscillate in the guides indicated in the sketch. In this case, then, the eccentric circle, whose centre is at P, is caused to rotate about the point C, and gives motion to the sides of the frame FG, just as it moved the bar QR in the last article, and hence the resulting motion is that of a crank, CP, with an infinite link.

The contrivance of the ratchet wheel at the right hand will be explained in a subsequent chapter.

ART. 47.—The crank with an infinite link also appears under the guise of a *swash-plate.*

FIG. 54.



Here a circular plate, EF (fig. 54), is set obliquely upon an axis, AC, and by its rotation causes a sliding bar, PQ, whose direction is parallel to AC, to oscillate continually with an up and down movement, the friction between the end of the bar and the plate being relieved by a small roller.

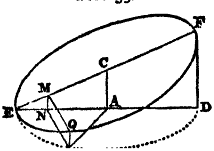
We must now try and ascertain what is the law which governs this motion, and we observe that since PQ

remains always parallel to AC, the actual path of Q, as projected upon an imaginary plane through the lowest position of Q, and perpendicular to AC, will be a circle.

If this be so, it follows that the path of Q upon the plate itself will not be a circle, but an oval curve, and, as a matter of geometry, we can prove that the line CQ will vary in length as Q rises or falls during the rotation of the plate, in the precise degree necessary for the description of the curve known as an ellipse.

In fig. 55 let EQF represent the actual path of Q on the plate, and let the circle ERD be the projection of this path upon a plane perpendicular to the axis AC.

FIG. 55.



Draw QM perpendicular to EF, QR perpendicular to the plane ERD, and RN perpendicular to ED, which is the diameter of the circle ERD.

Join MN, and suppose the plate to rotate through an angle $EAR = A$, and thus to carry the roller at Q through a vertical space equal to RQ.

Then
$$RQ = MN = AC \times \frac{EN}{EA}$$
$$= AC\left(\frac{EA - AN}{EA}\right)$$
$$= AC\left(1 - \frac{AN}{AR}\right)$$
$$= AC\,(1 - \cos A);$$

or the motion is that of a crank AC with an infinite link.

This is a curious result. Hitherto, in the motion of a crank with an infinite link, the reciprocation has always taken place in a plane perpendicular to the axis of rotation, but here we get the very same movement in a plane which contains the axis instead of being perpendicular to it.

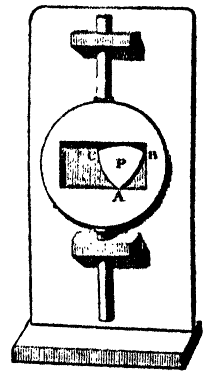
ART. 48.—It is sometimes required that the reciprocating motion shall be intermittent, or have intervals of rest.

This motion may be provided for by placing a loop at the point where the eccentric bar engages the pin. It is evident that the pin will only move when one end of the loop takes it up; but in

doing this a blow is struck, which it may be well to avoid, and hence an intermittent motion has been obtained in a much better manner by a movement adapted for working the slide-valves of a steam-engine.

We can readily see that if any portion of the plate in Art. 42 be shaped in the form of a circle round C, such portion will have no power of moving the sliding bar.

FIG. 56.

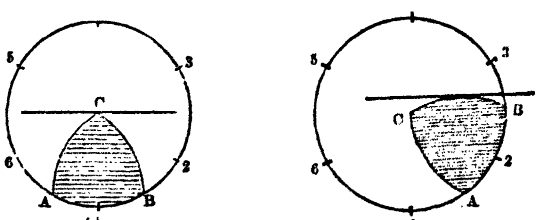


Let the pin P assume the form of a circular equilateral triangle, CAB, formed by three circular arcs, whose centres are in A, B, and C respectively, and let it be embraced by a rectangular frame attached to a sliding rod.

As CAB revolves round the point C, the portion CB will raise the plate; the point B will next come into action, and will raise the plate still higher; the upper edge of the groove will then continue for a time upon the curved surface AB, which is a circular arc described about C as a centre, and here the motion will cease; the plate will next begin to fall, will descend as it rose, an interval of rest will succeed, and thus we shall produce an intermittent movement, which may be analysed as follows:—

Suppose the circle described by B to be divided into six equal parts, at the points numbered 1, 2, 3, 4, 5, 6.

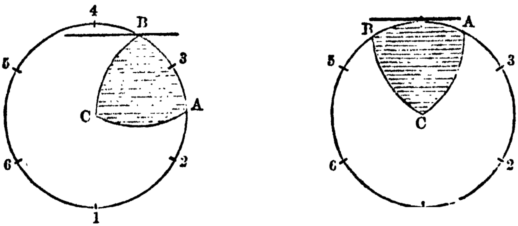
FIG. 57.



As B moves from 1 to 2, the frame remains at rest; from 2 to

3 the arc CB drives the frame, the centre of motion of the eccentric circle being now a point in its circumference, and the horizontal bar is driven as it would be by a crank CB with an infinite

FIG. 58.



link (see Art. 46); from 3 to 4 the point B drives, and the motion is again that of a crank, CB, with an infinite link (see Art. 46); *i.e.*, the motion from 3 to 4 is the same as that from 2 to 3, except that it is decreasing in velocity instead of increasing.

From 4 to 5 there is rest, then an increase of motion from 5 to 6, and finally a decrease to zero as B passes through the arc 6 to 1 and completes an entire revolution.

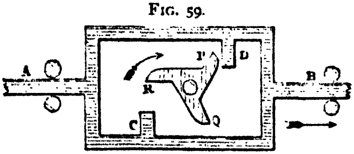
ART. 49.—Circular may be converted into reciprocating motion by the aid of *escapements.*

An *escapement* consists of a wheel fitted with teeth which are made to act upon two distinct pieces or *pallets* attached to a reciprocating frame, and it is arranged that when one tooth escapes, or ceases to drive a pallet, the other shall commence its action.

One of the most simple forms is the following :—

A sliding frame, AB, is furnished with two projecting pieces at C and D, and within it is centred a wheel possessing three teeth, P, Q, and R, which tends always to turn in the direction indicated by the arrow.

FIG. 59.



The upper tooth, P, is represented as pressing upon the projection D, and driving the frame to the right hand: when the tooth

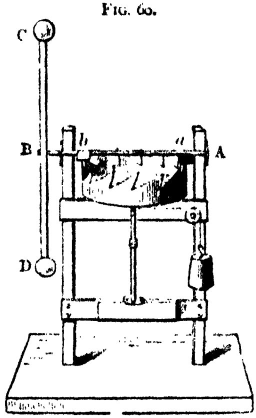
P escapes, the action of Q commences upon the other side of the frame, and the projection C is driven to the left hand. Thus the rotation of the wheel causes a reciprocating movement in the sliding piece, AB.

It is clear that the wheel must have 1, 3, 5, or some odd number of teeth upon its circumference.

ART. 50.—The *Crown Wheel* escapement was invented for the earliest clock of which we possess any record.

The form of the wheel is that of a circular band, with large saw-shaped teeth cut upon one edge; the vibrating axis, AB, carries two flat pieces of steel, *a*, *b*, called pallets, which project from the axis in directions at right angles to each other, and engage alternately with teeth upon the opposite sides of the wheel. In order to ensure that each pallet shall be struck in succession, either the wheel must have an odd number of teeth, or the axis of the pallets must be set a little out of the central line. Suppose the wheel to turn in the direction towards which the teeth incline, and let one of its teeth encounter the pallet *b* and push it out of the way; as soon as *b* escapes, a tooth on the opposite side meets the pallet *a* and tends to bring the axis AB back again: thus a reciprocating action is set up, which will be very rapid unless AB is provided with a heavy arm, CD, at right angles to itself. Such an arm possesses *inertia*, so that its motion cannot be suddenly checked and reversed, and a recoil action is set up in the wheel, which materially subtracts from the utility of the contrivance. For it will be seen that the vibration of CD cannot be made to cease suddenly, and that the wheel must of necessity give way and recoil at the first instant of each engagement between a tooth and its corresponding pallet.

FIG. 60.



The more heavily CD is loaded at a distance from the axis the more slowly will the escapement work, and the greater will be the amount of the recoil.

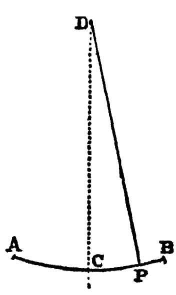
Here we have an invention which has done good service to mankind. It was used in the first clock which was ever made, and dealt out time through the *step-by-step* movement of the wheel with pointed teeth. This wheel, urged on by a weight, and hampered always by the vibrating bar, whose pallets were perpetually getting into the way of its teeth, moved round with a slow, intermittent, and step-by-step movement, checked and advancing alternately, but solving for mankind, in a clumsy though tolerably accurate manner, the great problem of the mechanical measurement of time, and giving birth, by the idea suggested, to those marvellous pieces of mechanism which have finally resulted in the modern astronomical clock and the chronometer.

Being, however, in some particulars, a defective or imperfect contrivance, it has gradually sunk from one level to another; it has disappeared from clocks and from watches also, and is now seldom to be met with except in the homely contrivance of the kitchen-jack for roasting meat.

ART. 51.—We purpose, before going further, to examine a little more particularly the mechanism of an escapement, so as to gain some idea of the refinements of its construction when applied to the best made clocks or watches, and we will review very rapidly the elementary facts which are to be found in the books on mechanics.

An imaginary or *simple pendulum* is a conception of mathematicians, and is defined as being a single particle of matter, P, suspended by a string, DP, without weight.

FIG. 61.



This particle may swing to and fro in a *vertical plane* under the action of the pull of the earth, and the *oscillation* of the pendulum is the whole movement which it makes in one direction before it begins to return, viz., ACB.

The *time of a small oscillation* is the period of this movement, and is given by the formula:—

$$t = \pi \sqrt{\frac{l}{g}},$$

where t = time of a swing in seconds, l = length of the string in feet, $g = 32{\cdot}2$ feet.

Make $t = 1$, and we have the length of a pendulum oscillating seconds, which is a little less than a mètre, being equal to 39·14 inches.

The discovery of the so-called pendulum law was made by Galileo, who noticed that a lamp swinging by a chain in the metropolitan church at Pisa made each movement in the same time, although gradually oscillating through smaller arcs while coming to rest.

For ordinary purposes the time of a swing may be supposed to depend only on the length of the pendulum, and not at all upon the arc through which it swings. It is practically very nearly the same for arcs up to 2 or 3 degrees on each side of the vertical, and it may be shown by calculation that the error introduced by assuming the law to be strictly true, in the case of a pendulum moving through an arc of 5 degrees from the vertical position, would only amount to $\frac{1}{2000}$th part of the time of a swing.

A seconds' pendulum in a well-made clock swings through about 2 degrees on each side of the vertical.

The question now arises, how is the law of the swing of this imaginary pendulum to be applied in the regulation of clocks, and wherein does a solid heavy pendulum, which we must of necessity employ, resemble or differ from an ideal pendulum?

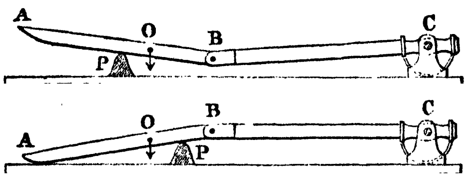
Conceive a straight uniform bar of iron to be suspended close to one end upon small triangular wedges of hardened steel, technically called knife edges, which rest upon perfectly horizontal agate or steel planes, and then to be set swinging. It is evident that each particle of the bar will endeavour to observe the pendulum law stated above, and will tend to swing in different times. But all the particles must swing together, and the result is that a sort of compromise takes place between the different tendencies, and the whole bar swings as if its material were all collected into one dense point at a certain distance from the knife edge, which is known as the *centre of oscillation.* Thus the solid pendulum swings as an ideal pendulum would do whose length was equal to the distance between the centres of suspension and oscillation.

This is the theory of the *rigid pendulum*, which has been investigated by Huyghens, and by others after him, and has led to many interesting experiments in applied mechanics.

ART. 52.—A method of showing roughly to a class the position of the centre of oscillation is the following :—

The drawing represents a wooden sword, having a pin at C passed through its handle and resting in two small upright supports, in such a manner that the sword can freely turn in a vertical plane about the point C.

FIG. 62.



The sword is divided into two parts at B in the proportion shown, and there is a tightening screw at B, which may be set to make it somewhat difficult to bend the sword out of shape. The sword is then allowed to hang freely as a pendulum from C, and a ball suspended by a silk thread at C is swung by the side of it. When the sword and the bullet swing together the position of the centre of the bullet is marked at O upon the sword.

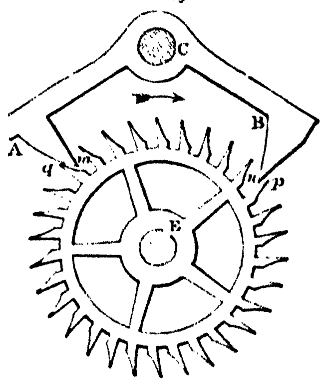
The experiment can now be completed. An anvil, P, is placed under the sword as in the figure, the sword is raised, and is allowed to fall and strike upon P. It will bend as shown. If CP be greater than CO it will bend with its end upwards, whereas if CP be less than CO it will bend downwards, just as if CA were a thin wire carrying a weight, O, upon it. The whole weight of the sword is for the purpose of the blow collected in the point O, and if CP be equal to CO the sword will not bend at all.

ART. 53.—The application of the pendulum in the manner in which it is now employed in clockwork forms an important branch of mechanism, and a great advance was made by Dr. Hooke, the contemporary of Newton, who devised the so-called *Anchor Escapement.*

Looking at the contrivance without regard to its connection with a pendulum, we find in it a wheel with pointed teeth, which is centred at E, and tends always to turn in the direction indicated by the arrow.

A portion of this wheel is embraced by an anchor, A C B, centred at C, the extreme ends of which are formed into pallets, A *m* and B *n*: these pallets may be flat or slightly convex, but they are subject to the condition that the perpendicular to A *m* shall pass above C, and the perpendicular to B *n* shall pass between C and E. The point of a tooth is represented as having escaped from the pallet B *n* after driving the anchor to the right hand; and the point *q*, by pressing against A *m*, is supposed to have already pushed the anchor a little to the left hand, and thus the wheel can only proceed by causing a vibratory motion in the anchor, A C B.

FIG. 63.



If the escape wheel engaged only a light metal anchor, such as that shown in the drawing, the rapidity of the vibration set up, as above described, would be very great; but in a clock the object is to provide that the wheel shall advance by slow and regular steps, and the anchor is therefore controlled by the inertia of a comparatively heavy swinging pendulum.

There is one uniform method of connecting the anchor and the pendulum which can be seen in any clock. The pendulum, consisting often of a compound metal rod with a heavy bob, is swung by a piece of flat steel spring, and vibrates in a vertical plane a little behind that in which the anchor oscillates. To the centre of the anchor is attached a light vertical rod, having the end bent into a horizontal position, and terminating in a fork which embraces the pendulum rod. It follows that the anchor and the pendulum swing together as one piece, although each has a separate point of suspension.

The same recoil is experienced upon each swing of the pendulum as that which we noticed in the last article, and the contrivance is commonly known as the *Recoil Escapement*.

ART. 54.—The exact character of the action which takes place between the pendulum of a clock and the scape wheel has been the subject of a long and interesting mathematical investigation, and before mentioning the results which have been arrived at, we may state in general terms the nature of the problem.

The going part of a clock consists of a train of wheels tending to move under the action of a weight or spring: if the last wheel of the train were left to itself, it would spin round with great velocity, and we should fail in obtaining any measure of time.

The escapement is one part of a contrivance for regulating the velocity of the train of wheels, but the escapement alone is not sufficient: we require further a vibrating body possessing *inertia*, the motion of which cannot be suddenly stopped or reversed.

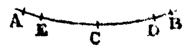
Such a body is found in the pendulum, and a very intricate mutual action exists between the pendulum and the scape wheel. The function of the pendulum is to regulate and determine the periods and amount of onward motion in the scape wheel, whereas the office of the wheel is to impart such an impulse to the pendulum at each period of this onward movement as may serve to maintain its swing unimpaired, and may cause it to move with the same mathematical precision which would characterise the vibrations of a body swinging *in vacuo*, and uninfluenced by any disturbing causes.

It is remarkable that in the case of the ideal pendulum, where there is no artificial resistance and no friction, the movement is in theory perpetual. In the case of the rigid pendulum the friction at the point of suspension and the resistance of the air would gradually diminish the arc of swing, and the movement would slowly subside and die away, although it might be many hours before it became quite imperceptible. The mechanism of a clock must therefore so act upon the pendulum as to maintain its swing unimpaired by these resistances, and it should be borne in mind that the swing of the pendulum is identical with that of the anchor in the last-mentioned escapement, whence it follows that any impulse or check given to the anchor is felt at once as an impulse or check upon the pendulum.

Refer now to the recoil escapement described in Art. 53, and conceive that the escape wheel is urged onwards in the direction

of the arrow by the force of the clock train, so as to press its teeth slightly against the pallets of the anchor, the pendulum being hung from its point of suspension by a thin strip of steel, and vibrating with the anchor in the manner already stated.

Fig. 64.



Let the arc A E C D B be taken to represent the arc of swing of the centre of the bob of the pendulum.

As the pendulum moves from B to E the point *q* of the escape wheel rests upon the oblique surface A *m* of the pallet, and presses the pendulum onward until the point of the tooth escapes at the end of the pallet. For an instant the escape wheel is free, and tries to fly round, but a tooth is caught at once upon the opposite side by the oblique edge B *n*, and the escape wheel then presses against the pendulum and tends to stop it, until finally the pendulum comes to rest at the point A, and commences the return swing.

What now has been the action? From B to E the force of the train as existing in the escape wheel has been acting with the pendulum and has performed its proper office in assisting to maintain the swing; whereas from E to A this force has acted *against* the pendulum.

So also on the return swing, the escape wheel will act *with* the pendulum from A to D, and *against* it from D to B.

The action then is alternately *with* and *against* the pendulum, and it might be supposed that the injurious effect of a pressure against the pendulum would be entirely corrected by the maintaining force in the other part of the swing; but this is not the case. The pendulum no longer moves with what we may call its natural swing, as a free pendulum would oscillate, and any variation in the maintaining force will disturb the rate of the clock.

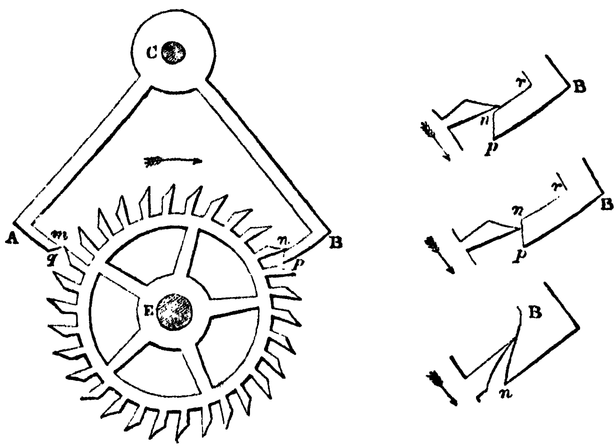
The matter has been carefully analysed by mathematicians, and they have shown that the principle of this escapement is radically bad, because it is impossible to remedy entirely the harm which is done by continually interfering with the swing of the pendulum.

There occurs also the useless expenditure of energy. It is almost superfluous to remark that no mechanical arrangement will ever bear a close scrutiny when it is so constructed as to throw away work.

ART. 55.—The *dead-beat escapement* was invented by Graham, and at once removes this primary objection. It is, however, most worthy of note that the change in construction which abolishes the defects due to the recoil, and gives the astronomer an almost perfect clock, separates the combination entirely from its original conception, viz., that of an apparatus for converting circular into reciprocating motion. No such conversion can be effected by Graham's escapement.

The improvement is made clear by the sketch, and the student will observe that the pallet A has its lower edge in the form of a

FIG. 65.



circular arc, A*q*, whose centre is C, and again that the upper portion of the pallet B is also a circular arc struck about the same centre. The oblique surfaces *qm*, *np* complete the pallets. Take the case shown in the diagram, which is enlarged so as to make the action more apparent. As long as the tooth is resting on the circular portion *nr* of the pallet, the pendulum is free to move, and the escape wheel is locked. Hence in the portion EA, and back again through AE, there is no action against the pendulum except the very minute friction which takes place between the tooth of the escape wheel and the surface of the pallet. Through

a space ECD the point of the escape wheel is pressing against the oblique edge *np* and is urging the pendulum forward.

Then at D the tooth upon the opposite side falls upon A*q*, and the escape wheel is locked; from D to B, and back again to D, there is the same friction which acted through EA or AE; whereas from D to E the point of a tooth presses upon *qm* and urges the pendulum onward; at E another tooth is locked upon the pallet B*n*, and thus the action is reproduced in the order in which it has been described.

It follows that any action against the pendulum is eliminated, or, more correctly, is rendered as nearly as possible harmless, and the difference between the 'recoil' and the 'dead beat' will be understood upon contrasting the three enlarged diagrams, which sufficiently explain themselves, the lower sketch referring to the recoil escapement.

The term 'dead beat' has been applied because the seconds' hand which is fitted to the escape wheel stops so completely when the tooth falls upon the circular portion *nr*. There is none of that recoil or subsequent trembling which occurs when a tooth falls upon B*n* and is driven back.

The actual construction of the dead-beat escapement having been explained, it only remains for us to state two of the principal conclusions which follow from a theoretical inquiry into the motion of the pendulum.

1. All action against the pendulum should be avoided, and if some such action be inevitable, it should at any rate be reduced to the smallest amount that is practicable.

2. The maintaining force should act as directly as possible, and the impulse should be given through an arc which is bisected by the middle point of the swing.

That is, the arc of impulse DCE should be bisected at the lowest point C.

This latter condition cannot be exactly fulfilled, because the point of the escape wheel must fall a little beyond the inclined slope of the pallet in order that it may be locked with certainty.

ART. 56.—In a printing telegraph instrument the recoil escapement has been employed to control the rapidity of motion in a train of wheels, and the number of vibrations of the anchor are

appreciated by listening to the musical note which it imparts to a vibrating spring.

The anchor ACB (fig. 66) is centred at C, and vibrates rapidly as the scape wheel E revolves; a strip of metal, F, carries on the oscillation to a steel spring which gives the note, and the velocity of the train can be regulated by an adjustable weight attached to the spring.

Again, the same escapement forms part of the mechanism of an *Alarum clock*, where a hammer is attached by a bar to the anchor, and blows are struck upon the bell of the clock in rapid succession as the scape wheel runs round.

FIG. 66.

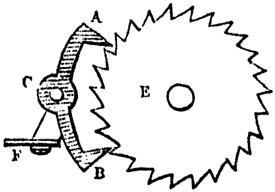
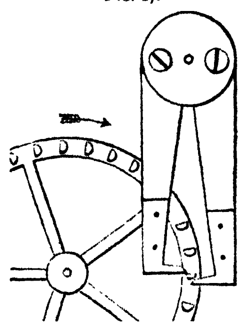


FIG. 67.



ART. 57.—The teeth of the wheel in an anchor escapement are sometimes replaced by pins, in which case the form of the anchor may be so altered that the action shall take place upon one side of the wheel, as shown in fig. 67.

ART. 58.—Circular may be converted into reciprocating motion by the aid of *cams*.

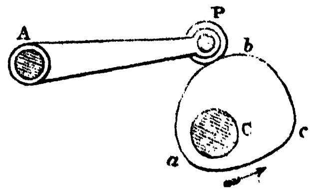
The term 'cam' is applied to a curved plate or groove which communicates motion to another piece by the action of its curved edge.

Such a plate is shown in fig. 68, and, as an illustration, we shall suppose that the portions *ab*, *ca* are any given curves, and that *bc* is a portion of a circle described about the centre of motion.

It is easy to understand that as the cam rotates in the direc-

tion of the arrow, the roller P at the end of the lever AP will be raised gradually by the curved portion *ab*, will be held at rest while *bc* passes underneath it, and, finally, will be allowed to fall by the action of *ca*.

Fig. 68.

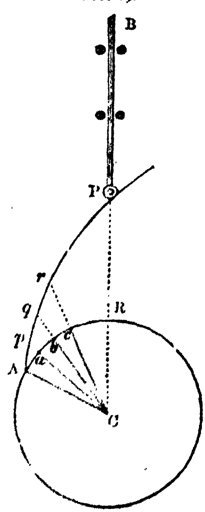


In this way a cam may be made to impart any required motion, and may reproduce in machinery those delicate and rapid movements which would otherwise demand the highest effort of skill from a practised workman.

Art. 59.—The circular motion being uniform, the reciprocating piece may also move uniformly, or its velocity may be varied at pleasure.

1. Suppose that the reciprocating piece is a sliding bar, whose direction passes *through the centre of motion* of the cam-plate. Take C as this centre, let BP represent the sliding bar, and let A be the commencement of the curve of the cam-plate.

Fig. 69.



The curve AP may be set out in the following manner.

With centre C and radius CA describe a circle, and let BP produced meet its circumference in the point R.

Divide AR into a number of equal arcs A*a*, *ab*, *bc*, &c.

Join C*a*, C*b*, C*c*, &c., and produce them to *p*, *q*, *r*, &c., making *ap*, *bq*, *cr*, &c., respectively equal to the desired movements of BP in the corresponding positions of the cam-plate. The curve A*pqr* . . . P will represent the curve required.

This curve will often present in practice a very irregular shape, but in the particular case where the motion of PB is required to be uniform, it assumes a regular and well-known form.

Let CA $= a$, CP $= r$, PCA $= \theta$, and let BP move in such a

manner that the linear velocity of P shall be constantly m times that of the point A, in other words, let RP $= m$. RA.

$$\text{Now RP} = r - a, \text{ and RA} = a\theta.$$

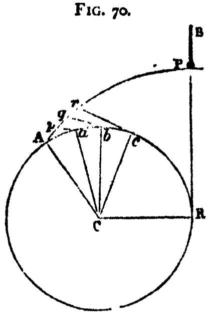
$$\therefore r - a = m \,.\, a\theta,$$

which is the equation to the spiral of Archimedes.

2. We will next examine the case where the centre of motion of the cam-plate lies upon one side of the direction of the sliding bar, and we shall find that the method of setting out the curve changes accordingly.

Suppose that the direction of BP passes upon one side of the centre of motion C, draw CR perpendicular to BP produced, describe a circle of radius CR, and conceive the motion to begin when A coincides with R.

FIG. 70.

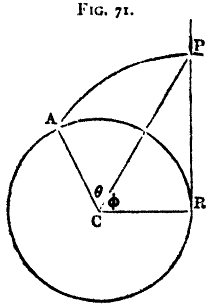


As a matter of theory such an extreme case is possible, and we will imagine it to exist in order to obtain the equation which represents the complete curve. Practically, the cam would be more effective in straining the bar than in moving it when the point P was near to the point R.

Divide AR into the equal intervals A a, ab, bc, &c., but now draw ap, bq, cr, &c., tangents to the circle, and equal in length respectively to the desired movements of BP during the corresponding periods of motion of the cam-plate.

The curve Apqr ... P will be that required, and the analytical representation of it is the following :—

FIG. 71.



Let CP$=r$, CA$=a$, ACP$=\theta$, PCR$=\phi$. The curve AP is now of such a character that the linear velocity of A shall be constantly m times that of P, or, in other words, RA$=m$. RP.

$$\text{But RA} = a(\theta + \phi), \text{ RP} = a \tan\phi,$$

$$\therefore a\,(\theta+\phi) = ma \,.\, \tan\phi.$$

$$\text{Now } \cos\phi = \frac{a}{r}, \text{ and } \tan\phi = \sqrt{\frac{1 - \cos^2\phi}{\cos^2\phi}}$$

$$\therefore \tan\phi = \sqrt{\frac{r^2}{a^2} - 1},$$

whence $\theta + \cos^{-1}\frac{a}{r} = m\sqrt{\frac{r^2}{a^2} - 1}$.

Cor. Let $m=1$, or RA=RP, which would happen if AP were a stretched string unwound from the circle. The curve traced out by the end P of the string becomes in this case a well-known curve called the involute of the circle, and our equation takes the form

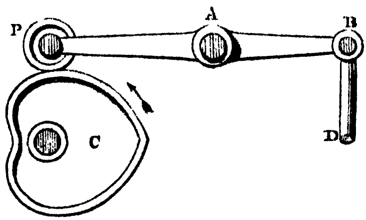
$$\theta + \cos^{-1}\frac{a}{r} = \sqrt{\frac{r^2}{a^2} - 1},$$

which is the equation to the involute of a circle.

Art. 60.—The heart wheel has been much used in machinery, and is formed by the union of two similar and equal cams of the character discussed in the first part of Art. 59.

A curved plate, C, shaped like a heart, actuates a roller, P, which is placed at the end of a sliding bar, or which may be attached to a lever PAB, centred at some point A, and connected by a rod BD to the reciprocating piece. The peculiar form of the cam allows it to perform complete revolutions, and to cause an alternate ascent or descent of the roller P with a velocity which may be made quite uniform.

Fig. 72.



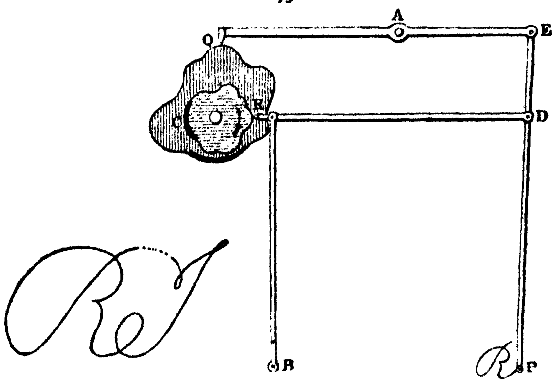
Since a cam of this kind will only drive in one direction, the follower must be pressed against the curve by the reverse action of a weight or spring.

Art. 61.—In order to illustrate in a lecture the power of cams to produce any required movement, the late Professor Cowper arranged a model which would write the letters R I, selected probably as a compliment to the Royal Institution.

The principle of this combination of cams will be readily understood if we remember that the successive movements of a point in directions parallel to two intersecting lines will suffice to enable the point to take up any position in the plane of the lines.

The bars shown in the drawing have fixed centres at A and B, and it is apparent that if we were to remove the cam and fasten the joint R to the plane, we should be able to give P a vertical movement by the swing of the arms AE and RD. In the same way, if we fastened E to the plane and liberated R, the arm EP could swing about E, and P would then describe a small circular arc which would closely approach to a horizontal line.

FIG. 73.



Connect now the bars with the cam as in the sketch, and press the pointers at Q and R against the curves of the respective cams. Let these cams revolve slowly about a centre, C (marked as a round spot in the smaller cam-plate), in the direction shown by the arrow, and the required letters will be traced out by the pencil at P.

In the figure the letter R has just been completed, and the pencil is about to trace out the lower tail of the letter I.

The two darkened lines in the cams are arcs of circles about the centre of motion where nothing is being done, the pencil remaining at rest while the cam rotates through a small angle.

This example shows us that a combination of two cam-plates actuating a simple framework of levers will give the command of any movement in a plane perpendicular to the axis of rotation. We shall presently see how to obtain a motion parallel to the same axis, and thus we can secure any required movement in space.

ART. 62.—In like manner two simultaneous rectilinear movements in lines at right angles to each other are obtained in a well-known form of sewing machine by the operation of grooved cams upon the face of a plate.

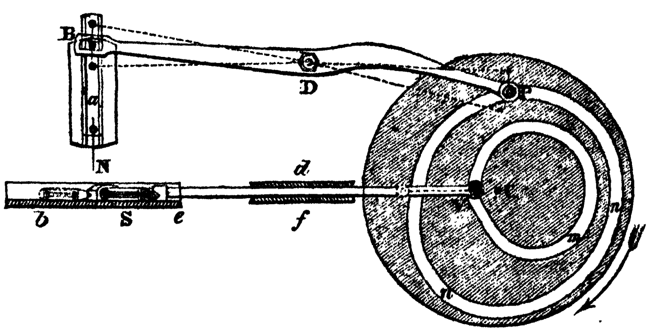
The sketch is taken from a lecture diagram.

1. The needle bar *a*, carrying the needle N, is constrained to move up and down in a vertical line between guides: it is driven by the lever BDP, which has a centre of motion at D, and is connected by a roller P with the groove marked *nn* in the cam-plate. The centre of motion of this plate is at C, and is marked by a strong dot.

If the groove were formed in a circle round C the needle would remain at rest, and it receives its required motion under the constraint of the cam. The dotted lines show the extreme positions of the lever during the motion.

2. The shuttle S moves to and fro in the path or 'race' *be*.

FIG. 74.



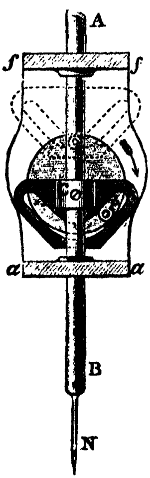
It is actuated by rod SV passing between guides *df* and terminating in a pin V which runs in the groove *m*. As the cam-plate rotates the pin V moves to and fro in a horizontal line, the extreme positions both of V and S being indicated by dotted lines in the diagram, and thus the needle and shuttle operate together in the manner required. The direction of rotation of the cam-plate is marked by an arrow.

ART. 63.—The movement of the needle in a sewing machine

is sometimes obtained from a peculiar form of cam, of the type of the heart wheel described in Art. 60.

The peculiarity consists in driving the cam by a pin P, placed on the face of the circular plate. In the ordinary heart wheel the pin or roller, P, moves up and down while the heart-shaped piece rotates upon its centre. Here the heart-shaped piece moves up and down while the pin rotates. The result is that we find only the apex of the heart instead of the complete outline. The contrivance is well worth studying as an example of the combination of movements.

FIG. 75.

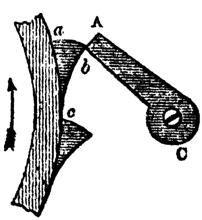


The drawing shows the needle bar AB, carrying a needle at N, and guided by openings in the fixed frames *aa* and *ff*.

The bar is attached to the darkly-tinted heart-shaped piece connected with it at the boss marked C, and the result is that the heart and the needle bar move as one thing. The pin P is attached to the circular plate, which rotates about its centre of figure, and it is clear that the needle bar is now nearly at rest in its lowest position, and will remain so until P gets round to the upper edge of the heart. The bar will then rise, and continue to rise while P is apparently descending the sloping side into the position shown by the dotted lines. The needle bar will then have reached its highest position, and will afterwards descend.

ART. 64.—In the striking part of a large clock the hammer may be raised by a cam, and may then be suffered to fall abruptly.

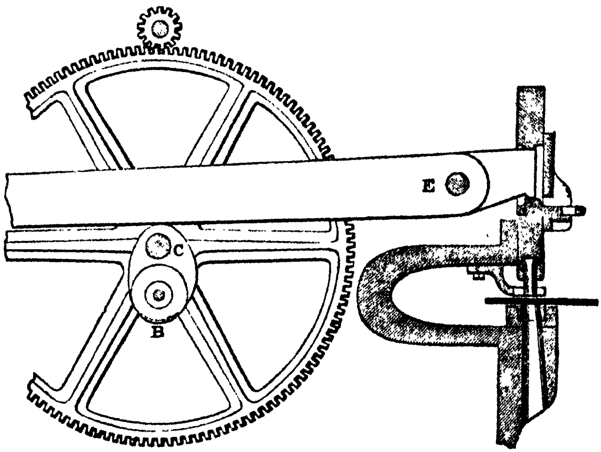
FIG. 76.



The figure represents the cam devised for the Westminster clock; the hammer rises and falls with the lever, AC, and the cam is so formed that its action commences at the extremity of the lever, and never departs sensibly from the same point; the cam, *ab*, is a circle whose centre is at the point of intersection of the tangents to the rim of the wheel at *a* and *c*.

Art. 65.—The lever punching machine is worked by a cam resembling that which we give as an example by Mr. Fletcher, of Manchester. The cam is here shown attached to the axis of the driving wheel, and the lever, which carries the punch in a slide connected with its shorter arm, is centred on the pin at E.

Fig. 77.



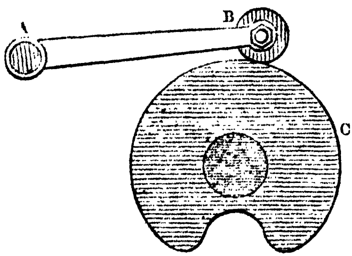
The curve of the cam is adapted to raising the longer arm of the lever bar in $\frac{1}{4}$ of a revolution of the driving shaft, it allows the lever to fall in the next $\frac{1}{4}$ of a revolution, and finally leaves the punch raised, as shown in the sketch, during the remaining $\frac{1}{2}$ of a revolution, thereby giving the workman an interval of time for adjusting the plate of iron before the next hole is punched.

That this action occurs will be quite evident upon inspecting the form of the cam, and it will also be seen that the cam is provided with a circular roller B, which determines the form of the driving surface while the work is being done, and which is merely an arrangement for lessening the friction just at the time when the greatest pressure is being exerted.

Art. 66.—Cams are employed when it is required to effect a movement with extreme precision. Thus in a now obsolete machine of Mr. Applegath for printing newspapers, the sheet of paper

used to start upon its journey to meet the type at a particular instant of time; an error of one-twelfth of a second would cause the impression to deviate half a foot from its correct position, and would throw two columns of letter-press off the sheet of paper. The accuracy with which the sheet was delivered was therefore very remarkable, and was insured by the assistance of the cam represented in the diagram (fig. 78)

FIG. 78.



As C revolves, the roller at B drops into the hollow of the plate, thereby determining the fall of the lever AB, and by it the fall also of another roller which starts the paper upon its course to the printing cylinder.

ART. 67.—Hitherto we have considered the cam to be a plane curve or groove, but there is no such restriction as to its form in practice. Let us examine the following very simple case:—

FIG. 79.

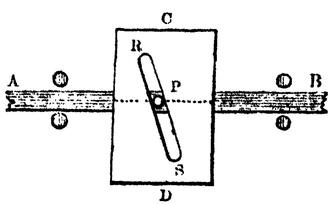
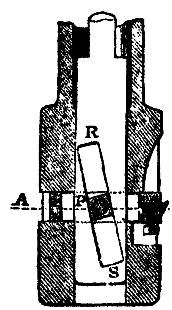


FIG. 80.



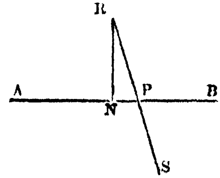
CD is a rectangle with a slit RS cut through it obliquely: a pin P fixed to the sliding bar AB works in the slit. If the rectangle CD be moved in the direction RS, it will impart no motion to the bar AB; but if it be moved in any other direction, the pin P will be pushed to the right or left, and a longitudinal movement will be communicated to the bar AB (fig. 79).

The contrivance here sketched is of frequent use in some form or other, and we may point out its application in the rifling bars used at Woolwich in the manufacture of rifled guns. In work of this kind, where the greatest accuracy is demanded, the bore of the gun acts as a guide to the head of the rifling bar, and the cutter does its work while the bar is being twisted and pulled out of the gun. It is essential, therefore, to keep the cutter within the head while the bar is being inserted preparatory to the removal of a strip of the metal, and to bring it out again at the end of the stroke.

In order to arrive at this result the bar is made hollow, and the tool-holder in the rifling head, shown in fig. 80, is made to move in and out laterally by means of a pin P working in an inclined slot, RS, in the internal feed rod. As the feed rod is pushed through a small definite space in either direction along the axis of the bar, the cutter will also move in or out in the direction of the dotted line AB.

In discussing this motion there are two cases to consider.

Fig. 81.



1. Suppose that CD is moved at right angles to AB.

Draw RN perpendicular to AB.

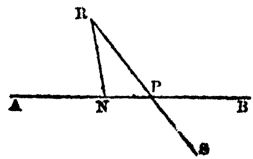
$$\text{Then } \frac{\text{travel of CD}}{\text{travel of AB}} = \frac{\text{RN}}{\text{PN}}$$

$$= \tan \text{RPN}.$$

2. Let CD move in a direction inclined at any given angle to the direction of the groove RS.

Draw RN in this direction, and we have

Fig. 82.



$$\frac{\text{travel of CD}}{\text{travel of AB}} = \frac{\text{RN}}{\text{NP}}$$

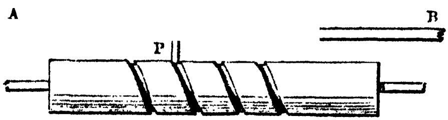
$$= \frac{\sin \text{RPN}}{\sin \text{NRP}}.$$

In other words, the *velocity ratio* of CD to AB is expressed by the fraction

and takes the form tan RPN when the angles at R and P make up a right angle.

ART. 68.—Next let CD be wrapped round a cylinder; it will form a screw-thread, and the revolution of the cylinder upon its axis will be equivalent to a motion of the rectangle at right angles to the bar, in the manner shown in the preceding article.

FIG. 83.



We shall have, therefore, by the arrangement in the figure, a continuous uniform rectilinear motion of the bar AB during the revolution of the cylinder upon which the screw-thread is traced.

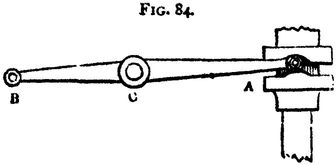
If the pitch of the screw be constant, the motion of PB will be uniform, and any change of velocity may be introduced by a proper variation in the direction of the screw-thread.

If the screw be changed into a circular ring, AB will not move at all.

It is, then, a matter of indifference whether the cam be a groove traced upon a flat plate or a spiral helix running round a cylinder. In the first case motion ensues when the groove departs from the circular form, and the distance from the centre varies; in the second case motion ensues the moment the groove deviates from the form of a ring, whose plane is perpendicular to the axis.

As an illustration of a cam of the latter character, we may refer to the diagram, which shows a form very much used where a small motion of a lever is required; the lever ACB is centred upon the point C, and will commence to move as soon as the pin at the end A reaches that portion of the ring which departs from the circular form.

FIG. 84.



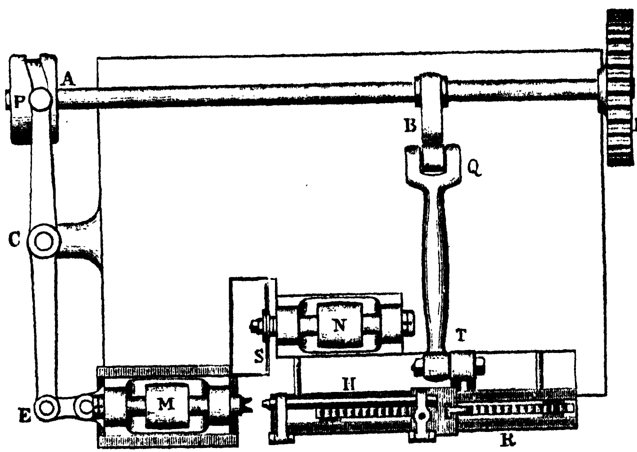
'c.—This kind of cam has the property of giving a motion parallel to the axis upon which it is shaped.

ART. 69.—As an example of two cam grooves in juxtaposition, one formed upon a flat plate and the other upon a cylinder, we may refer to a machine which was formerly used for shaping conical box-wood plugs of the kind required for expanding an elongated bullet into the grooves of a rifle.

Bullets of this class have a hollow recess at the base, into which a plug is fitted, and when the powder is fired the thin sides of the recess are forced into the grooves of the rifle by the action of the plug. At the present time these plugs are made of compressed clay, but formerly they were cut from a rod of box-wood by machinery.

The wood is first cut up into small square rods by means of circular saws, and one of the wooden rods is fixed in a saddle, as shown at H in fig. 85. Our drawing is taken from a lecture diagram by Sir J. Anderson.

FIG. 85.



The next operation is to bring a revolving cutter, which carves out the shape of the plug, upon the end of the rod H. This is effected by the lever PCE, centred at C, and having the end E jointed to a spindle which carries the cutter at its opposite end. A driving pulley M is rotated at a high velocity by means of a strap and pulley not shown in the drawing, and the longitudinal movement to and fro of the spindle, with its cutter and driving pulley, is effected by a grooved cam on the cylinder A. It will be apparent that this cam ought to be, and, in fact is, shaped precisely after the fashion of the cam in fig. 84.

As soon as the cutter had been forced down so as to form the end of the plug, it was withdrawn, and the final operation was to cut off the shaped piece. In the earlier machines a circular saw made a transverse movement, to cut off the plug, whereby the plugs and the chips were mixed together in the same heap, and Sir J. Anderson has stated that the labour for their separation cost more than for their manufacture. But soon it was arranged that the whole saddle HR carrying the plug rod should be shifted bodily back through a small space against a circular saw S, whereby the plugs might be dropped into a separate box.

The circular saw S is driven by a pulley N, and the transverse motion of HR is effected by the cam-plate B, which acts upon a roller at Q and connecting rod QT. The saddle HR is placed upon the top of a weighted rocking frame, and it is apparent that the cam B would give the transverse movement required.

D is a driving spur wheel which gives motion to the cam shaft. There is also a contrivance which we do not explain for pushing the rod H forward by the length of a plug at the end of each stroke.

ART. 70.—We subjoin a further example, devised many years ago, in which a reciprocating movement is imparted to a frisket frame in printing machinery, and it will be presently seen that the required result can be obtained in a much more simple manner.

The use of a cam-plate allows of an interval of rest at each end of the motion, and enables the printer to obtain an impression, and to place a fresh sheet of paper upon the form.

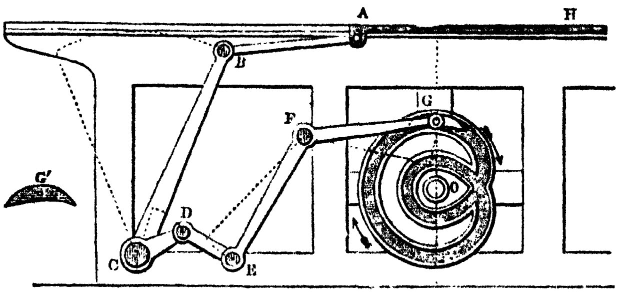
Here AH is the reciprocating frame attached to the combination of levers GFEDCB by the link AB (fig. 86).

At the end of the lever, FG, is a sliding pin which travels along the grooves in the flat plate centred at O, and determines, by its position, the angular motion of the levers about the fixed centres at F and C.

Where the groove is circular, which occurs in those portions which are to the left hand of the vertical dotted line, the levers remain at rest, and they change into the position shown by the dotted lines when the sliding pin passes from the outer to the inner channel. The pin is elongated in form, as shown at G′,

and is thus capable of passing across the intersections of the groove.

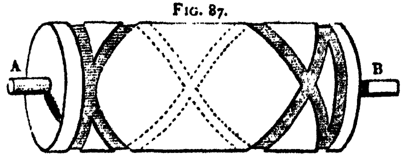
Fig. 86.



Precisely the same character of movement may be obtained by the aid of a helical groove traced upon a revolving drum. The intervals of rest occur when the groove assumes the form of a flat ring, whose plane is perpendicular to the axis of the drum.

A right and left-handed screw-thread is traced upon the worm barrel, AB, which revolves in one uniform direction a pin attached to the table of a printing machine follows the path of the groove upon the barrel, and its form is elongated so as to enable it to pass in the right direction at the points where the grooves intersect.

Fig. 87.



The interval of rest commences with the entry of the pin into the flat ring at either end of the barrel, and may be made to occupy the whole or any part of a revolution of AB, according as the grooves enter and leave the ring at the same or different points.

This construction dispenses with the complicated system of levers, which constitutes such a serious defect in the other arrangement.

Mr. Napier has patented an invention which causes the in-

terval of 'rest' to endure beyond the period of one revolution of the barrel.

At the entrance to the circular portion of the groove a movable switch is placed, and it is provided that the switch shall be capable of twisting a little in either direction upon its point of support, and also that the pin upon which the switch rests shall admit of a small longitudinal movement parallel to the axis of the barrel, the pin itself being urged constantly to the right hand by the action of a spring.

FIG. 83.

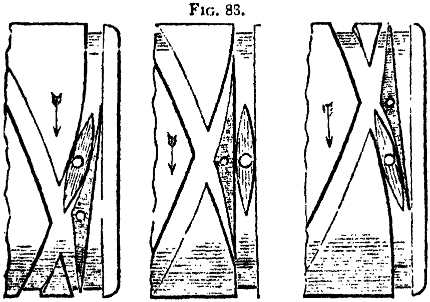


Fig. 1. Fig. 2. Fig. 3.

In fig. 1 the shuttle is seen entering the circular portion of the groove, and twisting the switch into a position which will allow the shuttle to meet it again, as in fig. 2, and to make a second journey round the circular ring.

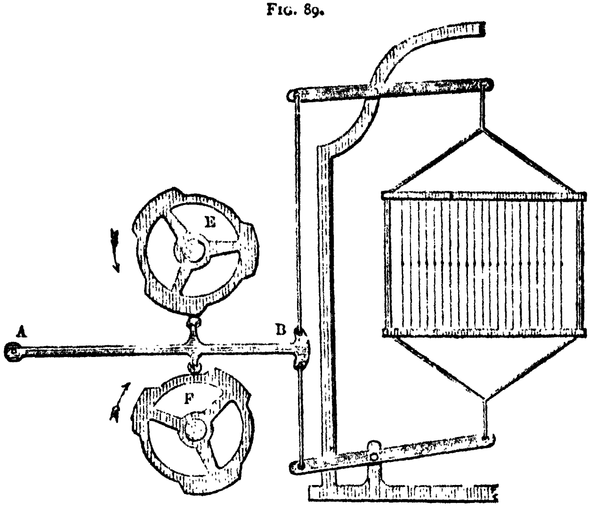
The spring which presses the point of support of the switch to the right hand will now cause it to twist by means of the reaction which the passing pin affords, and the consequence will be, that the switch will be left in the position shown in fig. 3, and will guide the shuttle into the helical portion of the groove. Thus the period of rest will be that due to about one and two-thirds of a revolution of the barrel.

ART. 71.—We remark, in conclusion, that when the mechanic causes the moving body to be influenced by a pin which exactly fits the groove along which it travels, it is obvious that the moving body will take the exact position determined by the pin; on the other hand, where the cam is merely a curved plate pushing a body before it, there is no certainty that this body will return unless it be brought back by a weight or spring. Hence it arises that double cams have sometimes been employed in machinery, and we take the next example from an early form of power-loom.

AB is the treadle, E and F are the cam-wheels or tappets, which revolve in the directions shown by the arrows, and in such

relative positions that the projections and hollows are always exactly opposite to each other. As the cams rotate, the treadle, AB, is alternately elevated and depressed, and the threads of the

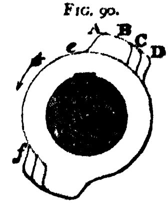
FIG. 89.



warp are opened so as to permit the throw of the shuttle during the operation of weaving the fabric.

ART. 72.—Cams are frequently employed for the purpose of opening or closing with rapidity the valves of a steam cylinder, or other valves concerned in what is termed the expansive working of steam, that is, the cutting off the supply of steam before the end of the stroke of the piston.

FIG. 90.



In a movement of this kind the cam is required to lift the valve rapidly from its position of rest, then to hold it up for a time while the steam is passing through, and next to allow it to drop into its seat and remain at rest.

The cam usually operates upon one end of a lever, the other end of which is connected with the valve, and it is apparent that it will suffice to surround the crank shaft of the engine by a plate or

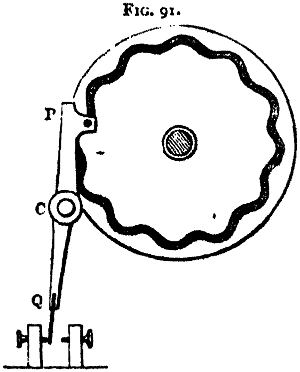
cylinder having a circular portion *ef*, on which the end of the valve lever rests when the valve is closed, and a raised portion, AB, also circular, upon which the end of the valve lever runs when the valve is to be opened. Thus there are two circular portions which determine the opening and closing of the valve, and an arbitrary sloping portion connects AB with *ef*, and determines the rapidity with which the changes take place.

For some purposes, as where steam is to be expanded in varying degrees, the raised portions are of different lengths, as AB, AC, AD, arranged in successive steps, one behind the other, whereby the valve may be held open for different periods.

Also, it is manifest that the cam may lie on the face of the plate instead of being part of its edge, and that in effect two portions of flat plates rotating about a common axis perpendicular to each, and raised one above the other, with a sloping surface connecting them, would be a mechanical equivalent for the cam described. Such a cam-plate was used by Sir W. Fairbairn.

ART. 73.—Where the cam-plate is required to effect more than one double oscillation of the sliding bar during each revolution, its edge must be formed into a corresponding number of waves.

FIG. 91.



There is an example in telegraph commutators, the interruptions of the current being caused by the vibrations of a lever, PCQ, centred at C, and whose angular position is determined by a pin travelling in the groove.

As the wheel revolves, it can impress any given number of double oscillations upon the lever.

ART. 74.—We have hitherto confined our attention to simple examples in the geometry of motion: we shall now extend our view of the subject, and shall consider the communication of motion when the driver is a toothed wheel or pulley rotating continuously upon a fixed centre or axis; and in order to generalise still further, we shall suppose the reciprocating motion to be either

rectilinear or circular. In this manner we shall be enabled to bring under one point of view a great variety of useful mechanical contrivances.

The student will be aware that in the transfer of force by machinery, the moving power is carried from one piece of shafting to another, throughout the whole length and breadth of the factory; it passes from point to point, enters each separate machine, and gives movement to all the several parts which may be prepared for its reception.

Now it must be remembered that the engine is never reversed, and that the power continues to flow onward in one uniform direction.

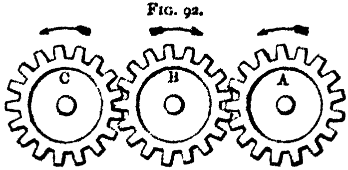
Take the case of a machine for planing iron: here the principal movement is that of a heavy table sliding forwards and backwards, and carrying the piece of metal which is the subject of the operation.

There are two methods of obtaining the desired result: the power may be poured, as it were, into the machine by a stream running always in one direction, and the reciprocation may be provided for by the construction of the internal parts, or the flow of the stream may be reversed by some intermediate arrangement external to the machine itself.

ART. 75.—The former method is that usually adopted, and we shall now examine those machines where the reciprocation depends upon the internal construction of the moving parts.

And, first, we shall discuss a very simple and useful reversing motion which is obtained by a combination of two or three spur wheels, and which depends upon an obvious fact.

FIG. 92.



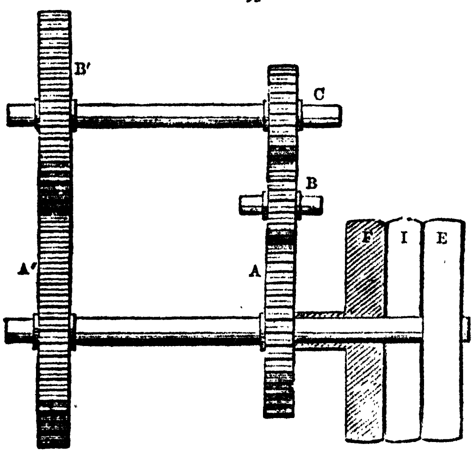
Let A,B,C represent three spur wheels in gear; it will be seen that A and B turn in opposite directions, while A and C turn in the same direction. If then we connect two parallel axes by a

combination of two and three spur wheels alternately, and properly arrange our driving pulleys, so that the power shall travel first through one combination and then through the other, we shall have a movement which has been adopted by Collier in his planing machines, and which has been subsequently much used by other makers.

The power is now derived from the shafting by means of a band passing over a drum on the main shaft and over one of the three pulleys, E, I, F, at the entrance into the machine.

Of these pulleys E is keyed to the shaft, I rides loose upon it, while F is attached to a pipe or hollow shaft, through which the shaft connecting E with A′ passes, and which terminates in the driving wheel A.

FIG. 93.



There is also a second shaft B′C, which carries the toothed wheels B′ and C.

B is an intermediate wheel riding upon a separate stud.

When the band drives the pulley E, it is clear that A′ and B′ turn in opposite directions; whereas the motion is reversed when the band is shifted to F, for in that case A and C turn in the same direction. When the driving band is placed upon I, the machine remains at rest.

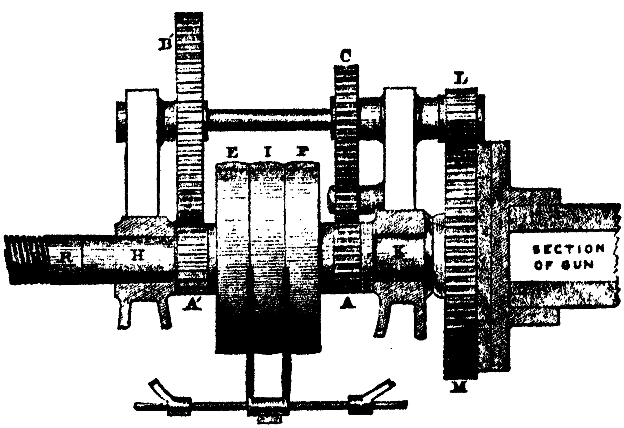
The rotation of B′C may be made much more rapid in one direction than in the other, and the construction is therefore particularly useful in machinery for cutting metals.

The slow movement occurs while the cutting tool is removing a slip of metal, and the return brings the table rapidly back into the position suitable for a new cut.

ART. 76.—This contrivance is in common use, and the drawing is from a machine arranged for cutting a screw-thread in the interior of the breech of an Armstrong gun.

In this case the driving pulleys are placed between the wheels A and A′, and are formed in such a manner that the pulley F and the wheel A make one piece, and ride loose upon the shaft HK, as do also, in their turn, the pulley E and the wheel A′: the wheel M is keyed to HK, so as to rotate with it, and is further attached by a coupling to the muzzle of the gun which is to be operated upon.

FIG 94



When the strap is upon E, the motion travels from A′ to B′, and so on to L and M, causing the gun and the shaft HK to rotate together slowly in one direction; whereas, upon shifting the strap to F, the motion passes from A to C through a small intermediate wheel, and thence to L and M, whereby the rotation of the gun is reversed, and a higher speed is introduced.

The object of the machine is to copy upon the interior of the breech of the gun a screw-thread which is formed upon the end R of the shaft HK.

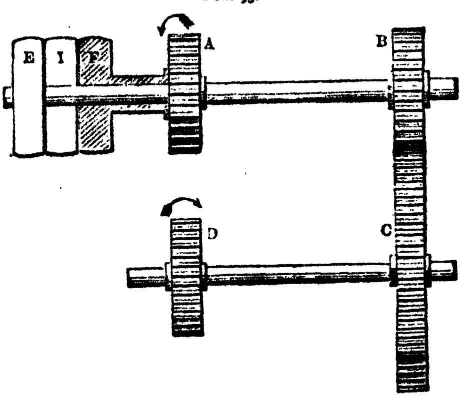
For this purpose the shaft HK is screwed, as shown, and a slide-rest carrying a cutter advances longitudinally along the gun, with a motion derived directly from a nut which travels along the screw-thread formed upon R. Since the cutter can only remove the metal while passing in one direction, there is a loss of time during the return motion, which it is the object of this combination to reduce as much as possible.

ART. 77.—The same combination, slightly modified, is adopted generally in planing machines, and is valuable by reason of the uniformity of the movement, the rate of advance of the table being perfectly constant.

It also possesses the important advantage of causing the table to traverse with a *quick return* movement when the cutter is not in action.

We give so much of the machine as will explain the method of reversing the motion of the table. When the strap is upon the pulley F, the wheel A turns in one direction. When the strap is

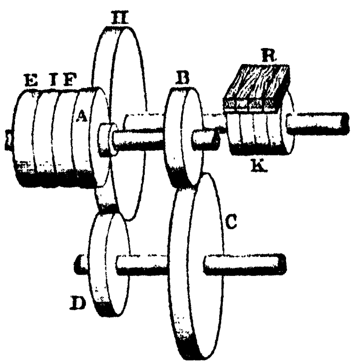
FIG. 95.



upon the pulley E, the motion passes to B, which turns with E, and thus the axis, CD, is made to revolve in the opposite direction with a reduced velocity.

The wheels A and D both engage with another wheel which actuates the table, and the reversal takes place when the moving power is transferred from the wheel A to D; but inasmuch as it would be difficult to give a clear representation of the movement without a sketch in perspective, an additional lecture diagram has been prepared wherein the toothed wheels are represented by circular discs with smooth edges.

FIG. 96.



Taking the two diagrams together, it will be apparent that the pulleys, E, I, and F, are arranged by the side of the first toothed wheel, A, whereby either F drives A, or E drives B, C, and D.

It has been stated that the wheels A and D both engage with another wheel which actuates the table, and the second drawing shows this additional wheel, H, engaging both with A and D on one side, and connected directly with a compound or stepped wheel, K, which is placed under the rack R.

The object being merely to indicate the position and arrangement of the working parts, the long rack which lies underneath the whole bed of the table is here marked as a short piece R rolling upon the disc K.

ART. 78.—We shall now examine another class of reversing motions, and shall commence in the most elementary manner.

Conceive a disc E, having a flat edge, to run between two parallel bars, AB and CD, arranged in a rectangular frame; and

conceive, further, that the frame can be raised or depressed so as to bring AB and CD alternately into contact with E (fig. 97).

If the disc rotates always in the direction of the arrows, it will move the frame to the left when brought into contact with CD, and to the right when brought into contact with AB.

We have therefore a reversing motion within the limits of the frame.

FIG. 97.

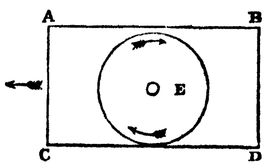
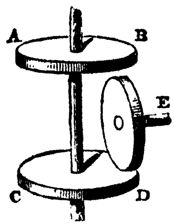


FIG. 98.



In order to make the motion continuous, it will only be necessary to alter the bars into circular strips or discs, as shown in fig. 98, and we shall reverse the motion of the vertical axis by bringing the upper or lower discs AB and CD alternately into close contact with the driver E.

In this way we obtain the first idea of a reversing motion, and it only remains for us to improve the general construction and arrangement of the working parts so as to make it practically useful. And we should observe that inasmuch as the rolling action of cones is more perfect than that of circular discs, for the reasons already explained in the introductory chapter, it will be better to substitute cones for the discs, in the manner shown in fig. 99, and the reversal will occur, just as before, when AB and CD are alternately brought into frictional contact with the driving cone E.

The geometrical condition of rolling will demand that the vertex of the driving cone E shall coincide with that of AB in one position of contact, and with that of CD in the other; hence the vertices of the two cones AB and CD must be separated through a small space equal to that through which the common axis is shifted.

If we desire to transmit force beyond the limit at which the cones would begin to slip upon each other, we must put teeth

upon the rolling surfaces, as in fig. 100, and we thus obtain a reversing motion which has been used in spinning machinery.

FIG. 99.

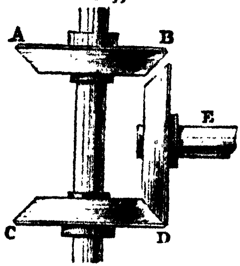
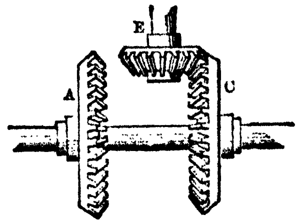


FIG. 100.

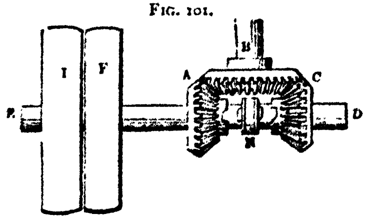


Here a bevel wheel, E, is placed between two wheels, A and C, which are keyed to the shaft whose motion is to be reversed, the interval between A and C being enlarged so that E can only be in gear with one of these wheels at the same time; the reversal is then effected by shifting the piece AC longitudinally, so as to allow E to engage with A and C alternately.

ART. 79.—A reversing motion which depends upon the shifting of wheels in and out of gear is not perfect as a piece of mechanism; we must try, therefore, to convert it into another, so arranged as to give the reversal by passing a driving clutch from one wheel to the other, the wheels concerned in the movement remaining continually in gear and fixed in position.

For this purpose we employ one working pulley F, keyed upon the shaft ED; by its side we place a second pulley I, which rides loose upon the shaft, and which carries the driving band when no work is being done.

FIG. 101.



The wheels A and C ride loose upon the shaft ED, and the intention is to impart the motion of the shaft ED, which is driven by steam power, to the wheels A and C alternately.

We now fit upon the shaft E a sliding clutch N, having projections which serve to lock it to

A or C as required, and we place also a projection, or feather, upon the inner part of the clutch which slides in a corresponding groove formed in the shaft, so that N must always turn with ED. It is clear, therefore, that if we allow the clutch N to engage with A we shall communicate to B a rotation in one direction, and that, further, we shall reverse the rotation of B if we connect C with N, for the student will see that in this combination A and C must always rotate in opposite directions, and that the rotation of B as derived from A must be different from that which B would derive from C.

This reversing motion may be commonly seen in steam cranes. The shaft ED is then driven directly by a steam-engine attached to the crane, and the sliding clutch may be locked to either bevel wheel by a friction cone, and is pushed to the right or left by means of a lever which grasps it without preventing its rotation.

There is another application in screwing machines where a rapid reversal is required. In this case the shaft ED is reversed by the action of the bevel wheels, instead of imparting its rotation to each of them in turn. The driving pulley F being attached by a pipe to the wheel A, the reversal is effected by shifting the clutch, and thereby locking the shaft ED to the wheels A and C alternately.

ART. 80.—We have next to examine the application of this reversing motion in planing machines, and shall describe the combination of three pulleys with three bevel wheels which has been adopted by Sir J. Whitworth.

In pursuing an inquiry into machinery of this character we may remark that the principle of machine copying, whereby a form contained in the apparatus itself is directly transferred to the material to be operated upon, is the distinguishing feature of all planing machines. The application of this principle is perfectly general, and, as a rule, wherever a process of shaping or moulding is well and cheaply performed by the aid of machinery, we find that some skilful and carefully arranged contrivance for transferring a definite form is contained within the machine.

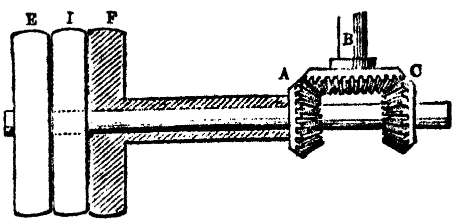
In the earliest form of planing machine, a method of carrying the cutter along parallel bars was adopted, and the present practice is to employ perfectly level and plane surfaces called Vs, which are

placed on either side of the machines, and are shaped exactly as their name indicates; their form gives a support to the table, prevents any lateral motion, and allows the oil required for lubrication to remain in a groove at the bottom, from whence it may be worked up by the action of the machine. The table has projecting and similar Vs which rest upon the former, and the object of the mechanism is primarily to cause the table to move in either direction along the grooves, and thus to copy upon a piece of iron supported thereon, and carefully bolted down, an exact plane surface which possesses the truth of the guiding planes.

Whether it may be better to move the table by a rack and pinion or by a screw is a subject upon which different opinions are held, and at all events the quick return movement which is given by a combination of spur wheels, as already described in Art. 77, is extremely valuable.

To recur to Sir J. Whitworth's arrangement, we find that he effects the required movement by rotating a screw which runs along the central line of the bed, and which imparts to the table a perfectly smooth traversing motion, equal of course in exactness, if not superior, to that which could be obtained by the best-constructed wheelwork.

FIG. 102.



There are now three pulleys, E, I, and F, whereof I is an idle pulley, and rides loose upon the shaft; E is keyed to a shaft terminating in the bevel wheel C, and F fits upon a pipe through which the shaft connecting E and C passes, and which terminates in the bevel wheel A.

B is a bevel wheel at the end of the shaft whose direction of rotation is to be reversed.

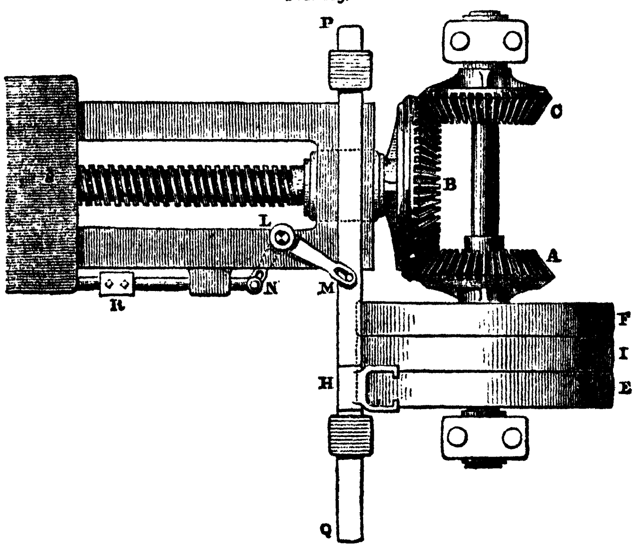
It is clear that the motion of the wheel B is reversed when the driving strap is shifted from E to F.

One objection to this movement consists in the fact that it does not permit the motion of B to be more rapid in one direction than in the other, and in order to economise the steam-power to the fullest extent, a method of rotating the tool-box was adopted by which means the cut was made while the table traversed in either direction. This reversal answers very well in planing ordinary flat surfaces.

It may, however, be so arranged as to obtain a quick return by making A and C of equal size, and by causing them to gear respectively with two *unequal* wheels upon the axis of B, or a combination of spur and bevel wheels may be employed.

ART. 81.—The contrivance just described is shown in fig. 103,

FIG. 103.



as applied in a machine for rifling guns, and the method adopted is precisely that so generally employed in planing machines.

The three pulleys and the three bevel wheels are connected to-

gether in the manner already indicated, and the bevel wheel B, by its rotation, causes a saddle S carrying the rifling bar to move along the screw in the direction of its length. A bell crank lever, MLN, controls the bar PQ, which carries a fork used to shift the strap, the arms of the lever lying in different horizontal planes, while a movable piece, R, fixed at any required point of the bar, NR, is caught by a projection on the saddle as it passes to the right hand, and thus the bell crank lever is actuated, and the strap is carried along from E to F.

A weight falls over when this is taking place, and gives the motion with sharpness and decision, so as to prevent the strap from resting upon I during its passage. On the return of the saddle to the other end of its path, a similar projection again catches a second piece upon the sliding bar NR, and the strap is thrown back from F to E.

This bell crank lever, as employed for shifting the strap, is worthy of notice; it consists of two arms, LN and LM, lying in different planes and standing out perpendicularly to an axis. It is a contrivance which affords a ready means of transferring a motion from one line, RN, to another, PMQ, which lies in a perpendicular direction at some little distance above it (Art. 36).

ART. 82.—Where the reciprocation is effected by a contrivance external to the machine, two driving bands may be employed: of these one is crossed, and the other is open, and it has been already pointed out that the followers will turn in opposite directions, although they derive their motion from a single drum, which, being driven directly by the engine, must rotate always in one direction.

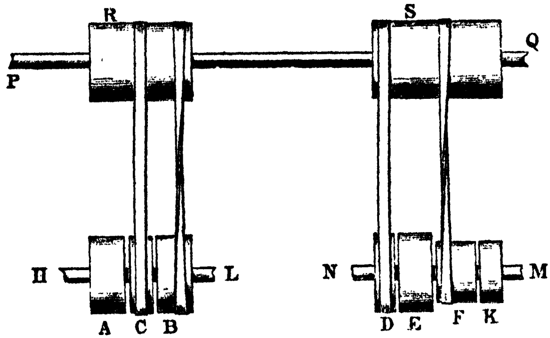
The form which the arrangement assumes in practice is shown in the sketch, fig. 104, where PQ is the driving shaft carrying the drums R and S.

Confining our attention at first to the left-hand diagram, we observe that one of the driving bands is represented as crossed, and that the rotation of the lower shaft HL is to be derived from each band alternately.

There are three pulleys, whereof A and B are each loose upon the shaft, and are about twice as broad as C, which is a working pulley.

The bands are shifted by two forks, and remain always at the same distance from each other. In the diagram the crossed strap

FIG. 104.



is upon the idle pulley, and the open strap is on the working pulley, the result being that the shafts PQ and HL rotate in the same direction. When the bands are shifted a little to the left both straps will lie on the respective idle pulleys, A and B, and the shaft HL will cease to rotate.

Whereas, upon shifting the straps still more to the left the crossed strap comes upon C, and the shaft HL begins again to rotate, but in the opposite direction to that in which it moved previously.

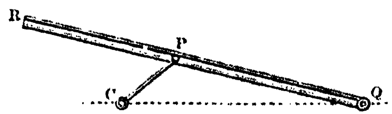
Here the rate of rotation is the same in either direction, but it may be varied by connecting the drum S with two pairs of pulleys, viz., D, E and F, K of unequal size, upon the lower shaft NM. The extreme pulleys D and K are the working pulleys, and the reversal is effected just as before, but NM rotates more rapidly in one direction than in the other.

ART. 83.—There is yet another most ancient contrivance for changing circular into reciprocating motion, which will repay the trouble of analysing it. It is deduced from the same triangle as that concerned in the motion of the crank and connecting rod, but the varying dimensions of the sides are arrived at in a different manner. One simple form is obtained when the points C and Q in the triangle CPQ become fixed centres of motion, the

crank CP being *less than* CQ, and the extremity P of the crank CP moving in a slot or groove running along the line QR.

The drawing shows an arm CP centred at C, and conveying motion to the grooved arm QR by means of a pin, P, which fits into the groove. As CP revolves with a uniform velocity, and in a direction opposite to the hands of a watch, it will cause QR to swing up and down to equal distances upon either side of the line QC, but with this peculiarity, that the upward swing will occupy less time than the downward swing.

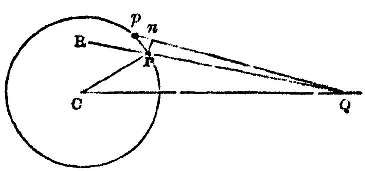
FIG. 105.



The motion of QR will be variable, its velocity changing at every instant, and we must endeavour, in the first instance, to discover an expression for its rate of motion as compared with that of CP. According to well-established rules, we estimate the relative rates of motion of two revolving pieces by comparing the sizes of the small angles described by either piece in a very minute interval of time reckoned from any given instant.

ART. 84.—Let now Pp represent the small arc described by P in a very minute interval of time, such as the $\frac{1}{1000}$th part of a second. Join Qp, Cp, and draw Pn perpendicular to Qp.

FIG. 106.



$$\text{Then } \frac{\text{angular vel. of QR}}{\text{angular vel. of CP}} = \frac{\angle \text{ PQ}p}{\angle \text{ PC}p}, \text{ in the limit,}$$

$$= \text{limit of } \frac{\text{P}n}{\text{QP}} \div \frac{\text{P}p}{\text{CP}}$$

$$= \text{limit of } \frac{\text{CP}}{\text{QP}} \times \frac{\text{P}n}{\text{P}p}.$$

But $Pn = Pp \cos pPn = Pp \cos RPC = Pp \cos (C+Q)$.

$$\therefore \frac{\text{angular vel. of QR}}{\text{angular vel. of CP}} = \frac{\text{CP} \cos (C+Q)}{\text{PQ}}.$$

We may test this formula in the usual way ; for instance, let $C + Q = 90°$, in which case QR touches the circle, then

$$\cos (C + Q) = \cos 90° = 0\,;$$
$$\therefore \text{angular vel. of QR} = 0,$$

or QR stops, as we know it must do.

Next, let $C = 0$, $Q = 0$, or let P be crossing the line CQ, then $\cos (C+Q) = \cos . 0 = 1$.

$$\therefore \frac{\text{angular vel. of QR}}{\text{angular vel. of CP}} = \frac{\text{CP}}{\text{QP}},$$

or the vel. of QR is as much less than that of CP as QP is greater than CP, a result which is evidently true.

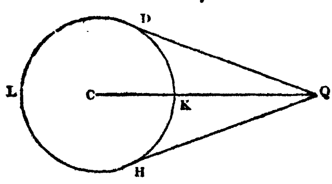
If it be required to find the position of QP when P is at any given point of its path, we have the equation

$$\tan \text{PQC} = \frac{\text{CP} \sin C}{\text{CQ} - \text{CP} \cos C},$$

whence the angle PQC is known in terms of C.

If we draw QD, QH, tangents to the circle described by P, it will be evident that the times of oscillation of the arm will be

FIG. 107.



unequal, and will be in the same proportion as the lengths of the arcs DLH, DKH.

The ratio of the angular velocities of QR and CP may also be obtained by analysis.

$$\text{Let CQP} = \phi,\ \text{PCQ} = \theta,\ \text{CP} = a,\ \text{CQ} = c,$$

$$\text{Then } \frac{a}{c} = \frac{\sin \phi}{\sin (\theta + \phi)},$$

$$\therefore 0=\sin(\theta+\phi)\cos\phi\frac{d\phi}{d\theta}-\sin\phi\cos(\theta+\phi)\left\{1+\frac{d\phi}{d\theta}\right\}$$

$$\therefore \sin\phi\cos(\theta+\phi)=\{\sin(\theta+\phi)\cos\phi-\cos(\theta+\phi)\sin\phi\}\frac{d\phi}{d\theta}$$

$$=\sin\theta.\frac{d\phi}{d\theta},$$

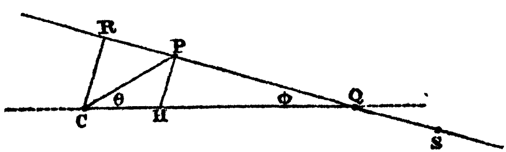
$$\therefore \frac{d\phi}{d\theta}=\frac{\sin\phi}{\sin\theta}\cos(\theta+\phi)=\frac{CP}{PQ}\cos(\theta+\phi).$$

ART. 85.—Another way of looking at the motion, which is technically known as a *slit-bar motion*, is to consider that PQ is a line of indefinite length jointed at P to the crank CP, and constrained to pass through an opening at the fixed point Q. As before, the condition that CP is less than CQ must maintain, or the line PQ will no longer oscillate.

This again is the movement in an oscillating engine, where the steam cylinder is swung upon trunnions, and the crank CP is connected by a piston rod to a piston moving up and down in the cylinder.

Whichever of the above forms may be selected as an illustration, the movement is precisely the same.

FIG. 108.



Taking CPQ as the triangle, draw CR perpendicular to QP produced, and draw PH parallel to CR.

$$\text{Then }\frac{CP\cos(\theta+\phi)}{PQ}=\frac{CP}{PQ}\times\frac{RP}{CP}=\frac{RP}{PQ}=\frac{CH}{HQ}$$

$$\text{or, }\frac{\text{angular vel. of QR}}{\text{angular vel. of CP}}=\frac{CH}{HQ}.$$

Referring to the oscillating cylinder, let V be the linear velocity of the crank pin P, and v the velocity of the piston, which moves in a cylinder swinging on trunnions at Q, and which, for simplicity, we will indicate by the point S in the drawing.

Then the velocity of P resolved along PQ is equal to the velocity of S.

$$\therefore \text{V} \sin \text{RPC} = v,$$

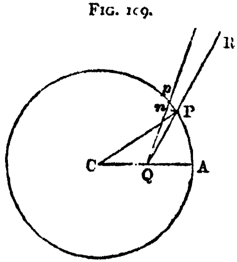
$$\text{or, } \frac{v}{\text{V}} = \frac{\text{CR}}{\text{CP}},$$

which gives the velocity ratio of the piston to the crank pin in an oscillating engine.

ART. 86.—Hitherto we have supposed CP to be less than CQ, and the result has been that QR swings about the point Q in unequal times; but we will now arrange that CP shall be greater than CQ, in which case QR will sweep completely round with a circular but *variable* motion. We shall, in fact, have solved the problem of making a crank revolve in such a manner that one half of its revolution shall occupy less time than the other half.

Now this is a very important result, and is of great value in machinery, because if the crank be made to perform its two half revolutions in unequal times, it follows that any piece connected with it by a link may be caused to advance slowly and return more rapidly; a movement which, as we have already pointed out, is peculiarly useful in machines for cutting metals.

FIG. 109.



Constructing as before, let C be the centre of the circle described by P. Then the equation $\tan \text{PQA} = \dfrac{\text{CP} \sin \text{C}}{\text{CP} \cos \text{C} - \text{CQ}}$ gives the position of QP when that of the crank is assigned.

$$\text{Also } \frac{\text{angular vel. of QR}}{\text{angular vel. of CP}} = \frac{\text{P}n}{\text{PQ}} \div \frac{\text{P}p}{\text{CP}} = \frac{\text{P}p \cos \text{CPQ}}{\text{PQ}} \times \frac{\text{CP}}{\text{P}p}$$

Cor. 1. If CQ be small, the angle CPQ will be small also, and we shall have $\cos \text{CPQ} = 1$ nearly; in which case the angular vel. of QR varies as $\frac{1}{\text{QP}}$, while that of CP remains constant.

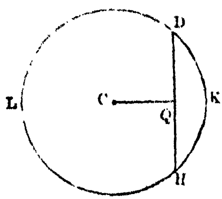
Cor. 2. When CQP is a right angle, we have

$$\cos CPQ = \frac{PQ}{CP}, \text{ or } \frac{CP \cos CPQ}{PQ} = 1,$$

that is, the angular vel. of QR = the angular vel. of CP.

This happens twice during a revolution, and gives the line of division of the inequalities of the motion of QR. Hence, if we draw DQH perpendicular to CQ, and cutting the circle described by P in the points D and H, the times of each half-revolution of QR will be in the proportion of the arcs DKH and DLH.

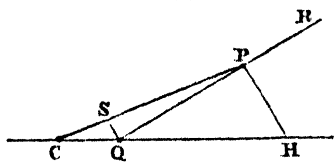
FIG. 110.



Cor. 3. The angular velocity ratio between QP and CP may be set out in a simple geometrical form just as in the previous case.

Draw PH perpendicular to QP, and QS parallel to PH.

FIG. 111.



$$\text{Then } \frac{\text{angular vel. of QP}}{\text{angular vel. of CP}} = \frac{CP}{PQ} \cdot \cos CPQ$$
$$= \frac{CP}{PQ} \times \frac{PQ}{PS}$$
$$= \frac{CP}{PS}$$
$$= \frac{CH}{QH}.$$

ART. 87.—If BR be made to carry a link, RQ, as in the case of the crank and connecting rod, the linear motion of Q will be the same in amount as if BR revolved uniformly, but the periods of each reciprocation will in general be different. (Fig. 112.)

The difference in the times of oscillation will depend upon the direction of the line in which Q moves.

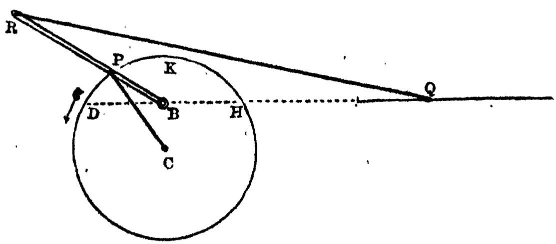
The best position for that line is in a direction perpendicular to CB. We have shown that the times of oscillation are always as the arcs DKH and DLH, and it is also evident that the inequality

between these arcs is greatest when DH is perpendicular to CB, and diminishes to zero when DH passes through CB.

We have now an arrangement very suitable for effecting a quick return of the cutter in a shaping machine.

Let one end of a connecting rod be made to oscillate in a line perpendicular to CB, or nearly so, and let the crank BR be driven by an arm, CP, which revolves uniformly in the direction of the arrow, we at once perceive that Q will advance slowly and return quickly, the periods of advance and return being as the arcs DLH and DKH.

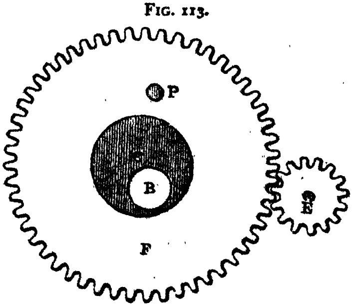
FIG. 112.



ART. 88.—Such a direct construction is not very convenient for the transmission of force, and it has been so modified by Sir J. Whitworth in his *Shaping Machine*, that the principle remains unchanged, while the details of the moving parts have undergone some transformation.

This machine is analogous to a planing machine, but there is no movable table; the piece of metal to be shaped is fixed, and the cutter travels over it. The object is to economise time, and to bring the cutter rapidly back again after it has done its work.

FIG. 113.



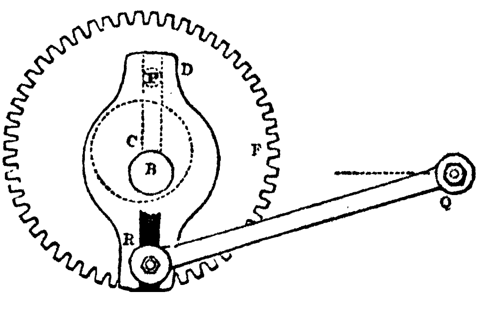
The arm CP is here obtained *indirectly* by fixing a pin, P, upon the face of a plate, F, which rides loose upon a shaft, C, and is driven

As the wheel F revolves upon the shaft represented by the shaded circle, the pin moves round with it, and remains at a constant distance from its centre.

A hole, B, is bored in the shaft, C, and serves as a centre of motion for a crank piece, DR, shown in fig. 114. The connecting rod, RQ, is attached to one side of this crank piece, and the pin, P, works in a groove upon the other side. Thus the rotation of the crank causes the end Q to oscillate backwards and forwards, and to return more rapidly than it advances.

The length of the stroke made by Q must be regulated by the character of the work done, and is made greater or less by shifting R farther from or nearer to B. This adjustment does not affect the

FIG. 114.



inequality in the relation between the periods of advance and return which the machine is intended to produce.

ART. 89.—As a further illustration of this slit-bar motion, we give a sketch of a curvilinear shaping machine used at the Crewe Locomotive Works.

There have been instances of unequal wear of the tyres in the leading wheels of locomotive engines, which have been traced to the circumstance of the wheel itself being a little out of balance; that is to say, the centre of gravity of the wheel did not exactly coincide with its centre of figure.

In one case a wheel was found upon trial to be 9 lbs. out of balance.

Now we learn in mechanics that a weight of W lbs. describing a circle of radius (r) with a velocity of (v) feet per second, will,

during its whole motion, exert continually a pull upon the centre in the direction of a line joining the body and the centre, which will be measured in pounds by the expression

$$\frac{Wv^2}{32{\cdot}2 \times r}.$$

Suppose a wheel 3 ft. 6 in. diameter to run at a velocity of 50 miles an hour. In this case v will be equal to $\frac{220}{3}$, and r will be $\frac{7}{4}$,

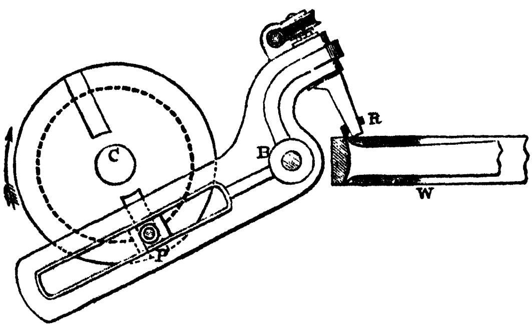
$$\text{and} \quad \frac{Wv^2}{32{\cdot}2 \times r} = \frac{1 \times 220 \times 220}{9 \times 32{\cdot}2 \times \frac{7}{4}} \quad (\text{taking } W = 1)$$

$$= \frac{8 \times 12100}{7 \times 9 \times 16{\cdot}1}$$

$$= \frac{96800}{1014{\cdot}3} = 95{\cdot}4.$$

whence the pull of only one pound weight at a distance of $3\frac{1}{2}$ ft. from the centre will amount to rather more than 95 lbs., and a weight of 9 lbs. would produce a pressure upon the bearing of rather more than $7\frac{1}{2}$ cwts.; and then, in the time of a half-revolution, viz., about $\frac{1}{13}$th part of a second, the same pressure in the opposite direction.

Fig. 115.



It is, of course, only at high speeds that the defects due to want of balance become serious, and this numerical result shows very plainly the necessity of great care in the construction of wheels which are required to run at a high velocity.

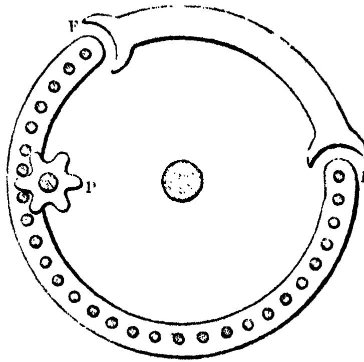
The machine intended to shape the curved inner face of the rim of locomotive wheels has the quick return movement which we have just discussed.

The point B is the centre of motion of the lever bar, and coincides with the centre of the circular portion forming the inner surface of the rim of the wheel W. The tail end of the lever has a long slot in which the crank pin P works: this pin is attached to the driving disc centred at C, and the length of the stroke can be adjusted by shifting P in the direction of the radius CP.

ART. 90.—*Mangle wheels* form a separate class of contrivances for the conversion of circular into reciprocating motion.

A mangle wheel is usually a flat plate or disc furnished with pins projecting from its face; these pins do not fill up an entire circle upon the wheel, but an interval is left, as shown at F and E.

FIG. 116.

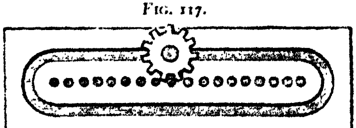


A pinion, P, engages with the pins, and is supported in such a manner as to allow of its shifting from the inside to the outside, or conversely, by running round the pins at the openings F and E.

The pinion, P, always turns in the same direction, and the direction of rotation of the mangle wheel is the same as that of P when the pinion is inside the circular arc, and in the opposite direction when the pinion passes to the outside.

The mangle wheel may be converted into a *mangle rack* by placing the pins or teeth in a straight line. Here the pinion must be so suspended as to allow of its shifting from the upper to the under side of the rack.

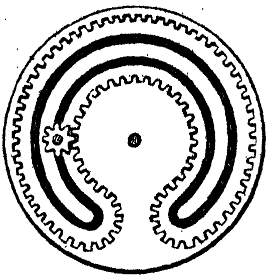
FIG. 117.



As to the velocity ratio between the wheel and pinion, it will

be shown hereafter that the inner and outer pitch circles coincide in the case of a pin wheel, and therefore that the relative rotation of the mangle wheel to the pinion is the same in both directions.

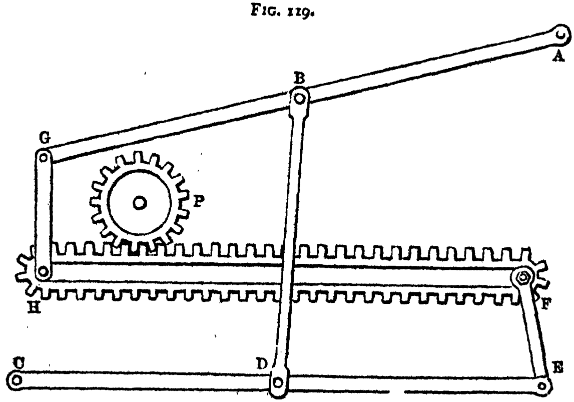
FIG. 118.



If the pins be replaced by a curved ring furnished with teeth, the mangle wheel will move more rapidly when the pinion is upon the inside circumference, and by giving certain arbitrary forms to this annulus, the velocities of advance and return may be modified at pleasure. Contrivances such as this are seldom met with at the present time.

ART. 91.—Sometimes the pinion is fixed, and the rack shifts laterally. An excellent form of this arrangement was introduced by Mr. Cowper, and serves to give a reciprocating movement to the table in his printing machine.

FIG. 119.



The rack HF is attached to the system of bars in the manner exhibited in the diagram. A and C are centres of motion, and are the points where the bars are attached to the table. AG and CE are bisected in B and D, and are joined by the rod BD; the rack HF is attached to the bars AG and CE by the connecting

links GH and FE, and it must be remembered that it is the intention to obtain this so-called mangle motion by the reverse process of fixing the pinion and causing it to drive a continuous rack which runs upon each side of it alternately.

The precise value of the contrivance consists in the arrangement of the bars, which will be understood upon referring to the section upon *Parallel Motion*, and it will be seen that when the pinion has pushed the rack to either end of its path, the bars will so act as to move together, and will shift this rack HF to the opposite side of the pinion, without allowing it to deviate from a direction coincident with that in which the table is moving.

This is the object of the contrivance, and, as we have said, the method by which the result is arrived at will be apparent when the subject of parallel motion has been examined.

Thus the table carrying the parallel bars and the rack oscillates backwards and forwards, while the pinion, which transmits the force, remains fixed in space.

When this machine was applied to the printing of newspapers, the table moved at the rate of 70 inches in a second, and its weight, including the form of type, would be about a ton and a half. When urged to its highest speed the machine would give 5,500 impressions in an hour, which is about the greatest number attainable under a construction of this kind; the true principle in rapid printing being that announced in the year 1790 by Mr. Nicholson, who proposed to place the type upon a cylinder having a continuous circular motion, and upon which another cylinder holding the paper should roll to obtain the impression. But although Mr. Nicholson enunciated the principle nearly a hundred years ago, and took out a patent for a mode of carrying it out, there is a wide difference between saying that a thing ought to be done, and showing the world how to do it in a practicable manner; hence it was not until late years that Mr. Applegath, and finally Mr. Hoe, were enabled so to arrange their cylinder printing machines upon the principle of continuous circular motion as to satisfy the wants of the daily papers, and to print some twelve or fourteen thousand sheets in an hour.

To recur to our shifting rack, it must be remarked that by reason of the great weight of the table, and the rapidity with which

it moves, it would be quite unsafe to leave the rack and pinion in the present unassisted condition; a guide roller therefore determines the position of the pinion relatively to the rack, while the rack itself shifts laterally between guides.

But since, theoretically, the rods would cause HF to move always in a direction parallel to itself, they practically enforce the desired movement in the path of the guides, with as little loss of power as possible.

ART. 92.—If it be required that the reciprocation shall be intermittent, *i.e.*, that there shall be intervals of rest between each oscillation, we may employ a segmental wheel and a double rack, as shown in fig. 120.

The teeth upon the pinion engage alternately with those upon either side of a sliding frame, and the motion is of the character required. The intervals of rest are equal, and are separated by *equal* periods of time.

A pin upon the wheel and a guide upon the rack will ensure the due engagement of the teeth.

FIG. 120.

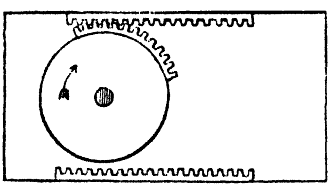
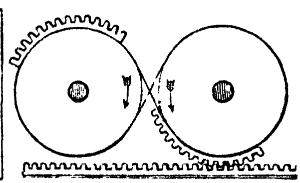


FIG. 121.



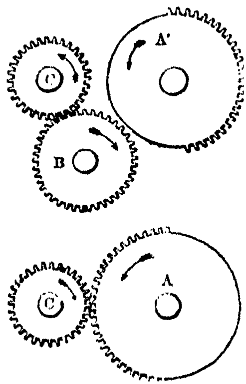
A mechanical equivalent to the above is found in the use of two segmental wheels and a single rack (fig. 121).

These segments must be equal, but they may be placed in different relative positions upon the discs to which they are attached; and, as a consequence, the intervals of rest may be separated by *unequal* periods of time.

These segmental wheels have been employed in the earlier days of mechanism, and there was a well-known instance in Mr. Cowper's printing machine, where a segment of a wheel engaged with a small sector at each revolution, and so fed on the sheets of paper by the push given while the segments were in action.

Sir J. Whitworth has proposed the subjoined arrangement for the reversal in a machine for cutting screws: we take it as a further example of the use of these wheels, which, however, should always be avoided if possible. There is only one driving pulley, and two segmental wheels are keyed upon the driving shaft. They are close together in the machine, and for the sake of the explanation we have *placed one above* the other. The object is to effect the reversal of a shaft C: the segmental wheels A and A′ have teeth formed round one half of each circumference, and the toothed segments are in situations opposite to each other, as in fig. 122.

FIG. 122.



When the action of A ceases, that of A′ begins, and we have the wheels A and C, or the wheels A′, B, and C alternately in action, *i.e.*, we have a reciprocation of C. This is a direct example of the case given in Art. 75.

CHAPTER III.

ON LINKWORK.

ART. 93.—In the present treatise the term 'linkwork' is applied to combinations of jointed bars movable in one plane, the joints being pins, whose axes are respectively perpendicular to the plane in which the bars move. The crank and connecting rod is an example of linkwork, but the present chapter deals principally with combinations of three or more links.

As a fundamental case, and one which will repay the trouble of examining it, we take two cranks or levers centred at a distance from each other, movable in one plane, and connected at their extremities by a jointed bar.

FIG. 123.

Such a combination is represented by CPQB, where C and B are fixed centres of rotation formed by two parallel cylindric pairs, the arms CP, BQ being cranks or levers movable about axes through C and B, while PQ is technically known as a link or coupling rod, and is attached to the respective cranks by pins.

There are, in fact, four parallel cylindric pairs, and three movable bars.

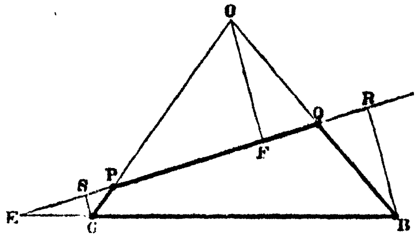
Prop. When two unequal arms or cranks are connected by a link, as in fig. 123, the angular velocities of the arms are to each other inversely as the segments into which the link divides the line of centres.

1. This may be proved by reference to an instantaneous axis.

Upon causing the combination to rack a little, it will be found

that P begins to describe a circle round C, while Q begins to describe a circle round B, whence it becomes evident that the instantaneous centre, about which PQ is rotating at any instant, lies in the point O, where CP and BQ meet when produced.

FIG. 124.



Produce PQ to meet BC in E, and draw CS, OF, BR respectively perpendicular to PQ. Let PCE$=\theta$, QBC$=\phi$, and when the figure racks a little, let CP, BQ describe the small angles $d\theta$ and $d\phi$. At the same time P and Q will each describe the same small angle α about the centre O.

$$\therefore \mathrm{CP}\times d\theta=\mathrm{OP}\times\alpha,$$

$$\mathrm{BQ}\times d\phi=\mathrm{OQ}\times\alpha,$$

$$\therefore \frac{\mathrm{CP}}{\mathrm{OP}}.\ d\theta=\frac{\mathrm{BQ}}{\mathrm{OQ}}.\ d\phi$$

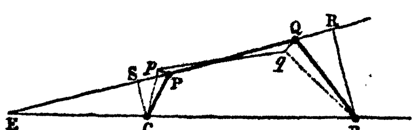
$$\therefore \frac{d\theta}{d\phi}=\frac{\mathrm{BQ}}{\mathrm{OQ}}\times\frac{\mathrm{OP}}{\mathrm{CP}}=\frac{\mathrm{BR}}{\mathrm{OF}}\times\frac{\mathrm{OF}}{\mathrm{CS}}=\frac{\mathrm{BR}}{\mathrm{CS}}=\frac{\mathrm{BE}}{\mathrm{CE}}.$$

2. The same may be proved by the resolution of velocities.

Let P and Q shift to p and q during the smallest conceivable interval at the beginning of the motion.

Then the resolved part of the motion of Q in the direction QP is ultimately equal

FIG. 125.



to Qq cos qQP,

$=$Qq sin BQR

$$=\mathrm{Q}q\times\frac{\mathrm{BR}}{\mathrm{BQ}}$$

$$=\mathrm{BR}\times\frac{\mathrm{Q}q}{\mathrm{BQ}}$$

$=$BR $\times$ angle QBq.

So also the resolved part of the motion of P in the direction QP$=$CS $\times$ angle PCp.

But in the first instant of the motion these resolved parts are equal to each other, because PQ remains for a brief space parallel to its first position.

$$\therefore \text{BR} \times \text{angle QB}q = \text{CS} \times \text{angle PC}p,$$

$$\therefore \frac{\text{angle QB}q}{\text{angle PC}p} = \frac{\text{CS}}{\text{BR}}.$$

But although the angular velocities of the arms BQ and CP change continuously, yet they will be at any instant in the same proportion as the limiting ratio of the angles described by these arms in a very minute interval of time, the relative motions of the arms not being supposed to change during that interval.

$$\text{Hence } \frac{\text{angular vel. of CP}}{\text{angular vel. of BQ}} = \frac{\text{angle PC}p}{\text{angle QB}q}$$

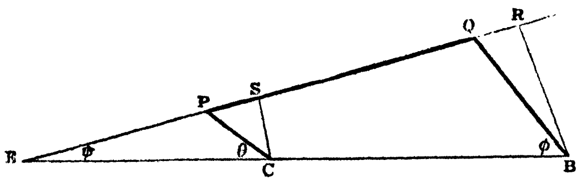
$$= \frac{\text{BR}}{\text{CS}} = \frac{\text{BE}}{\text{CE}}.$$

3. By analysis, the same may be proved.

$$\text{Let } \left.\begin{matrix}\text{CP}=a\\ \text{BQ}=b\end{matrix}\right\} \quad \left.\begin{matrix}\text{CS}=h\\ \text{BR}=k\end{matrix}\right\} \quad \left.\begin{matrix}\text{PQ}=d\\ \text{CB}=c\end{matrix}\right\}$$

θ and ϕ as before, and the angle CEP being equal to ψ.

FIG. 126.



Then $h = a \sin(\theta+\psi)$

$k = b \sin(\phi+\psi)$

$d = a \cos(\theta+\psi) + c \cos\psi - b \cos(\phi+\psi)$

$\therefore$ by differentiation we have

$$0 = -a \sin(\theta+\psi)(d\theta+d\psi) - c \sin\psi\, d\psi$$
$$+ b \sin(\phi+\psi)(d\phi+d\psi)$$

$$\therefore c \sin\psi\, d\psi = -h(d\theta+d\psi) + k(d\phi+d\psi)$$
$$= (k-h)\, d\psi - h\, d\theta + k\, d\phi.$$

$$\text{But} \quad k - h = c \sin\psi.$$

$$\therefore 0 = -h\, d\theta + k\, d\phi.$$

$$\therefore \frac{d\theta}{d\phi} = \frac{k}{h} = \frac{\text{BR}}{\text{CS}}$$

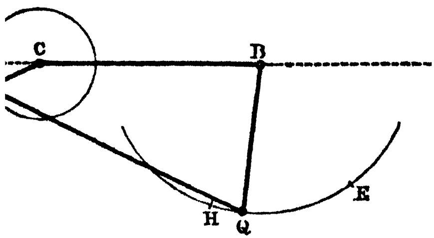
$$\therefore \frac{\text{angular vel. of CP}}{\text{angular vel. of BQ}} = \frac{BR}{CS} = \frac{BE}{CE},$$

which proves the proposition in its general form.

ART. 94.—Let it be required that one crank (viz. CP) shall sweep round in a circle, while the other (viz. BQ) oscillates to and fro through a given angle.

In the diagram, the point P describes a circle, while the point

FIG. 127.



Q oscillates to and fro in a circular arc, the points H and E marking its extreme positions.

$$\text{Then} \quad CH = PQ - CP,$$
$$CE = PQ + CP.$$

Also since CP makes complete revolutions, it is essential that CP and PQ should come into a straight line before BQ and PQ have the power to do so. Hence we must have

CB + BQ greater than CE,
and CB − BQ less than CH.

It will be readily seen, upon testing this statement, that if CB be taken equal to PQ, the crank CP will revolve, and BQ will oscillate, so long as CP is sensibly less than BQ.

The angle of swing of BQ increases also as CP becomes more nearly equal to BQ, and tends to reach two right angles as a limit.

Ex. Let $CP = 2$, $CB = PQ = 15$, $BQ = 8$, to prove that BQ will oscillate through 30° from a position perpendicular to CB.

When BQ is at the end of its swing on the right hand we have, since $CE = 17$,

$$\cos CBQ = \frac{225+64-289}{2\times 15\times 8} = 0.$$

$$\therefore\ CBQ = 90^\circ, \text{ or } BQ \text{ is vertical.}$$

When BQ is at the end of its swing on the left hand we have, since $CH=13$,

$$\cos CBQ = \frac{225+64-169}{2\times 15\times 8} = \frac{120}{240} = \frac{1}{2}.$$

$$\therefore\ CBQ = 60^\circ, \text{ which proves the statement.}$$

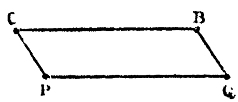
ART. 95.—It has been stated that an extreme case occurs when $CP = BQ$, and when the connecting link PQ is also equal to the distance between the centres, viz., CB.

Under these circumstances one crank, as CP, will make complete revolutions while the other, viz., BQ, oscillates through 180°.

But at the same time CPBQ may form a parallelogram, whose opposite sides and angles are equal, and if any provision be made for retaining PQ parallel to CB, the crank BQ will no longer oscillate but will perform complete revolutions.

In the diagram, CP, BQ represent two cranks connected by a link PQ, which is equal to CB, then it is apparent that CPQB forms a parallelogram so long as PQ remains parallel to itself, or that the parallelism of PQ is the condition which ensures the joint rotation of the cranks.

FIG. 128.

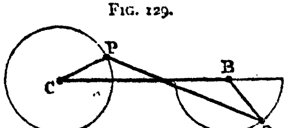


It is also apparent that when the driving crank comes upon the line of centres the joint CPQ will bend if there be any resistance to the motion in the follower, and BQ will then return, while CP alone continues its rotation.

This state of things is shown in fig. 129, where CP sweeps round in a circle, while BQ oscillates through 180°. The difference is that in the first case PQ remains parallel to itself, and that in the second case it oscillates through an angle $2a$, such that

$$\sin\frac{a}{2} = \frac{CP}{CB}.$$

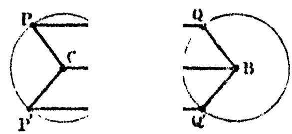
FIG. 129.



But the oscillation is put an end to by superposing a second

combination exactly similar to the first, as in fig. 130, where the bell crank lever PCP′ is connected with a second identical bell crank QBQ′, by means of the equal links PQ, P′Q′.

FIG. 130.



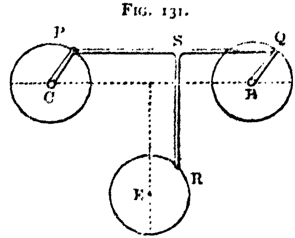
In this way, when PQ is passing through the dead points, P Q′ will hold it in a parallel position, and each connecting rod will prevent the other from taking that oblique position which is destructive of the required motion.

This is the principle of the coupling link between the two driving wheels in a locomotive engine. There are always two links, one on each side of the engine, and the cranks are of course at right angles.

The necessity of the second pair of cranks, with their link, is obvious upon a little consideration, and may be made very clear by constructing a small model of the arrangement; it is only necessary to make the links move in different planes so that they may be able to pass each other.

ART. 96.—We next observe that any point in PQ will describe a circle equal to either of the circles described by P or Q, so long as PQ remains parallel to itself, and hence that a third crank equal to either CP or BQ, and placed between them, would be driven by PQ, and would further prevent PQ from getting into an oblique position at the dead points, or would produce the same result as the second pair of cranks with the link in the locomotive engine.

FIG. 131.

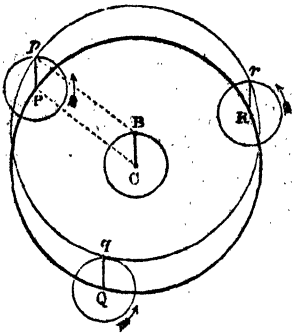


Again, the same would be true of any point R in a bar SR connected rigidly in any way with PQ, the point R would describe a circle equal to either of the primary circles so long as PQ remained parallel to CB.

Also, if a crank were supplied at ER, the three cranks would go round together, and PQ would remain parallel to itself.

Conceive now that three equal cranks, P*p*, Q*q*, R*r*, are centred at equal distances along a circle PQR, as shown in fig. 132, and let a second circle *pqr*, equal to PQR, be jointed to the cranks at the points, *p*, *q*, *r*.

FIG. 132.



If the circle *pqr* be shifted so that the cranks are allowed to rotate, each of them will describe a circle, the respective cranks will always remain parallel to each other, and the circle *pqr* will move in such a manner that any line drawn upon it remains always parallel to itself.

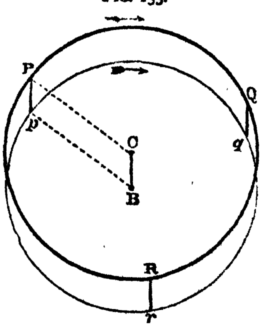
Hence the circle *pqr* may be employed as a driver to rotate all three cranks at the same time, and while doing so, it will itself sweep round without the slightest movement of rotation upon its own centre.

It has what is sometimes called a motion of circumduction.

ART. 97.—A very small alteration in the construction will give a combination which has been useful in rope-making machinery, and which was the first movement suggested for feathering the floats of paddle-wheel steamers.

Let the centres of the two circles PQR and *pqr* be made fixed centres of motion, and let P*p*, Q*q*, R*r*, remain as before.

FIG. 133.



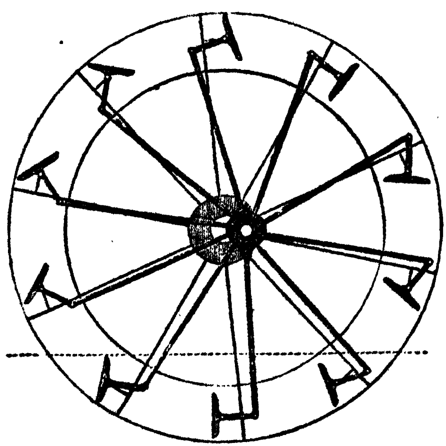
A power of rotation will now be given to both the circles, and P*p*, Q*q*, R*r* will be the connecting links which always remain parallel to CB, the line of centres. That is to say, the rotation of the circle PQR, about the centre C, will cause an equal rotation in the circle *pqr*, about its centre B, and P*p*, Q*q*, R*r*, must remain parallel to CB. It is the same combination that we started with, under a different aspect, by reason that the proportionate size of the pieces has been

changed. The two circles have been enlarged and brought together, so that their circumferences overlap, and CP, Bp are the parallel cranks.

Regarding the contrivance as a method of feathering the floats in paddle wheels, we find that, in the year 1813, Mr. Buchanan patented a form of paddle wheel in which one circle, as PQR, carried the floats, and another circle, pqr, rotating with the former, held these floats always in a vertical position, and so made them enter the water edgeways, instead of striking it obliquely with the flat surface, as is the case in an ordinary paddle wheel.

ART. 98.—This wheel of Buchanan has not been used for very sufficient reasons. It is not a good arrangement for the floats to enter the water in an exactly vertical line, because the motion of the vessel must compound with that of the floats, and the supposed vertical path will not be one in reality, any more than it would be in the case of a stone dropped from the same vessel into the water. The stone appears to fall in a vertical line, but is really projected forwards.

FIG. 134.



According to this view the float should enter the water at an angle such that its line of direction will pass through the highest

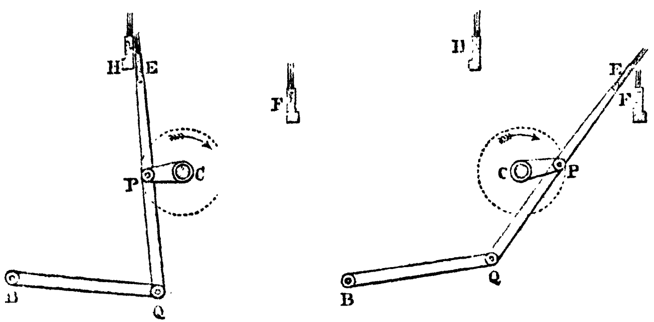
point of the wheel, this being the direction of the resultant of the two *equal velocities* impressed upon a point in the wheel, the one being that due to the vessel, the other being due to the rotation of the wheel.

This result may follow very closely from the construction in the drawing, which represents Morgan's wheel, where the floats are connected by rods with a ring that rotates round a fixed centre in the paddle-box. The floats are attached to small cranks and pivoted upon centres, one of them (the lowest in the drawing) being driven by a rigid bar which springs from a solid ring. Each float passes the lowest point in a vertical position, and is somewhat inclined when entering or leaving the water.

ART. 99.—In the conversion of circular into reciprocating motion by two cranks and a connecting link, it is a condition of the movement that the connecting link shall swing through a given angle.

This fact has been usefully applied in wool-combing machinery, and in 1852 Messrs. Lister and Ambler patented (No. 13,950) an improved arrangement for transferring wool from one carrying comb to another in the act of working the same.

FIG. 135.



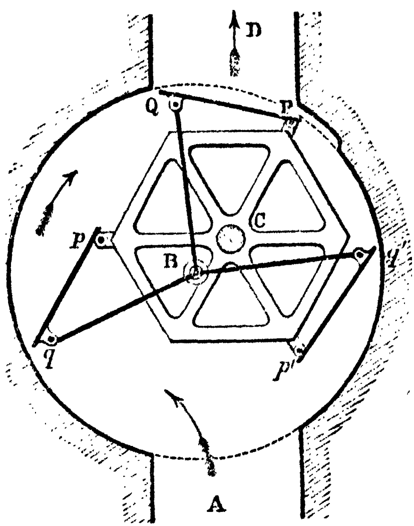
The diagram represents the combination CPBQ, the connecting link QP being prolonged to E, and carrying a comb at its extremity. H and F are two fixed combs, and the object in view is to detach a lock of wool from H by the comb E, and to transfer it to F. The proportions of the parts are so chosen that CP per-

forms complete revolutions, and the positions of the passing comb are shown. In the left-hand figure E is in the act of rising through the wool at H, and detaching it, while the other sketch shows E as about to deposit the tuft on the distant comb F.

ART. 100.—The changes in the position of PQ may also be applied to a useful purpose when both the arms CP and BQ make complete revolutions.

A remarkable instance occurs in a mechanical ventilating machine proposed some years ago by M. Lemielle as a substitute for furnace ventilation in mines.

FIG. 136.



The apparatus consists of a circular chamber of masonry communicating by an air passage A with the shaft of the mine on one side, and having a discharge opening D on the other side. Within the chamber is placed a revolving drum centred on an axis at C, and having shutters or vanes, as PQ, connected by rods, as BQ, with a second fixed axis at B.

It is apparent that CPBQ forms our well-known combination,

and, with the proportions in the drawing, it is also clear that CP, BQ will both make complete revolutions.

In doing so, PQ opens out and closes up again, sweeping out before it the air in the open portion of the chamber, and driving the mass before it from A to D.

The several positions of PQ are shown at *pq*, *p'q'*, and it will be seen that PQ begins to open out on the left-hand side, and gets to its extreme position at or near to *pq*. In practice there are at least three vanes employed, as shown.

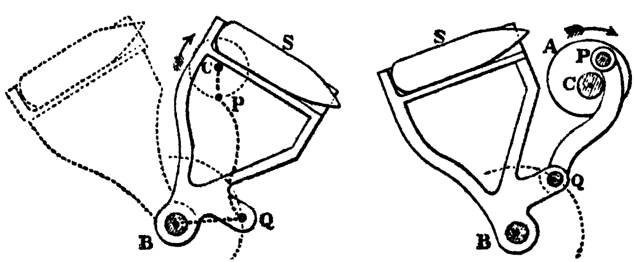
A 'Lemielle ventilator' has been constructed and set to work where the chamber is 14 ft. in diameter and 7 ft. deep, with a fan making some 37 revolutions per minute, and discharging in that time 25,000 cubic feet of air.

ART. 101.—In sewing machines the 'shuttle race' or path of the shuttle is commonly a straight line, but the shuttle is sometimes caused to oscillate to and fro in a circular arc.

The drawing shows an arrangement for this purpose, founded upon the above combination of two cranks and a connecting link.

Here the shuttle S is carried in a frame which oscillates on a fixed centre at B. The driver is a pin P placed on the face of a cam-plate whose centre is at C, and connected by a link PQ to the stud Q attached to the frame carrying the shuttle. The plate carrying P is not circular, and its edge A is used as a cam driver for another part of the operation of the machine.

FIG. 137.



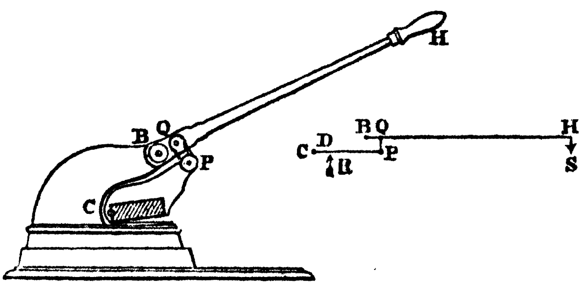
It is apparent that we have assigned such dimensions to the working parts, viz., CP, BQ, and PQ, as will produce the result that Q will oscillate while P performs complete revolutions.

Accordingly, the left-hand diagram shows two extreme positions of S and of the frame. It will be seen that P describes a circle of radius CP while Q moves through a portion of the dotted circular arc formed round the centre, B. The carrying backwards and forwards of the shuttle is thus arrived at very simply.

ART. 102.—The combination CP, BQ, and PQ appears in the form of a hand machine for cutting metals.

The diagram will make the construction quite apparent. The lever BH, having a handle at H and a fulcrum at B, is connected by the link PQ with a second piece CP carrying a knife-blade. This latter piece has a centre at C, and the cutting is performed by depressing the handle H.

FIG. 138.



As a question of mechanism it will be found that the angular motion of CP is much less in amount than that of BH, and the principle of work comes in and affords an easy explanation of the power of the machine.

A skeleton diagram will show the mechanical advantage,

Let S be the force applied at H,

R the resistance in cutting the metal as felt at D.

Then, if T be the thrust in QP, we have—

$$T \times BQ = S \times BH,$$
$$R \times CD = T \times CP.$$
$$\therefore R \times BQ \times CD = S \times BH \times CP$$
$$\therefore R = S \times \frac{BH \times CP}{BQ \times CD}.$$

Ex. Let $CD = BQ = 1$, $CP = 4$, $BH = 10$,

$$\text{Then } R = S \times \frac{10 \times 4}{1 \times 1}$$

$$= 40\,S,$$

which is a much better result than would be obtained by the use of a single lever CH carrying a knife.

ART. 103.—The combination of two unequal arms with a connecting link forms the celebrated Stanhope levers which have been so generally employed in printing presses worked by hand.

From the invention of the Art of Printing in the year 1450 till the year 1798 no material improvement was made in the printing press. The earliest representation of a press occurs as a device in books printed by Ascensius. There is scarcely any difference between it and a modern press, and it is truly a matter of astonishment that so long a period as 350 years should have rolled on without some improvement being made in so important a machine.

The wooden press consists of two upright pieces of timber joined by transverse pieces at the top and near the bottom ; a screw furnished with a lever works into the top piece, and by its descent forces down a block of mahogany, called the 'platten,' and thus presses the sheet of paper upon the type, which is laid upon a smooth slab of stone embedded in a box underneath. In the year 1798 Lord Stanhope constructed the press of iron instead of wood, and at once transferred the machine from the hands of the carpenter to those of the engineer ; he further added a beautiful combination of levers for giving motion to the screw, causing thereby the platten to descend with decreasing rapidity and consequently increasing force, until it reached the type, when a very great power was obtained.

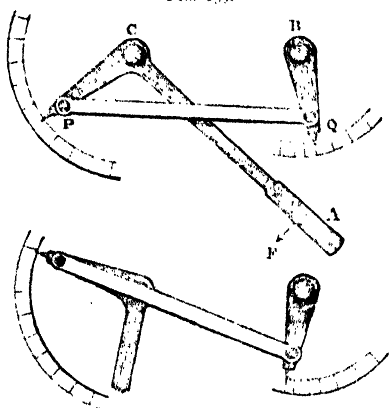
These levers consist of a combination of two arms or cranks, CP, BQ, connected by a link, PQ, in such a manner that the connecting link shall come into a position perpendicular to one of the arms at the instant that it is passing over the centre of motion of the other arm.

In order that this may happen, it is evident that the various pieces must satisfy the relation.

$$PQ - CP = \sqrt{CB^2 - BQ^2}.$$

For the convenience of the workman who is employed upon the press, a handle, CA, is attached to the crank, CP, and moves as part of it, but the introduction of this handle does not affect the principle of the movement, which, regarded as a question in mechanics, depends simply on the combination of CP, BQ, and PQ.

Fig. 139.



If, now, a force, F, be applied at the end of the handle, AC, so as to turn the crank, CP, uniformly in the direction indicated, the arm, BQ, will, under the conditions already stated, move with a continually decreasing velocity until it comes to rest, and then any further motion of CP will cause BQ to return.

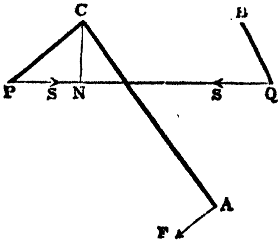
The lower diagram shows the levers in this extreme position, and the graduated scales at P and Q indicate the relative angular movements of CP and BQ.

Now the motion, interpreted with relation to the transmission of force, implies that the resistance at Q necessary to balance the moving power which turns the crank is increasing rapidly as the rotation of BQ decreases, and that there is no limit theoretically to the pressure which will be felt as a pull at Q by reason of the force F. In practice this extreme pressure is exerted through so very small a space that the theoretical advantages are scarcely realised, but the arrangement is exceedingly useful as applied in the printing press.

The lever BQ is there employed to turn the screw which acts upon the platten; the workman gives a pull to the handle AC, and by doing so causes the platten to descend with a motion which is at first considerable, and afterwards rapidly dies away. Thus the limited amount of power which is being exerted comes out with greatly magnified effect in impressing the paper upon the type.

ART. 104.—In order that this contrivance may be better understood, take the annexed sketch to represent it, and draw CN perpendicular to PQ.

FIG. 140.



A force F acting at A in a direction perpendicular to CA would be balanced by a force S acting in PQ, such that

$$S \times CN = F \times CA,$$

$$\text{or} \quad S = \frac{F \times CA}{CN}.$$

This force S, necessary to balance F, would be supplied by the resistance to motion in the arm BQ, and would, in fact, be the pull felt at Q. Now, as the arms turn, the link PQ gets nearer and nearer to C, and CN becomes less and less until it has no appreciable magnitude, and the consequence is that S increases enormously in the last instant of the motion.

Ex. Let $F = 20$ lbs., $CA = 20$ inches, $CN = \frac{1}{10}$th of an inch, we have

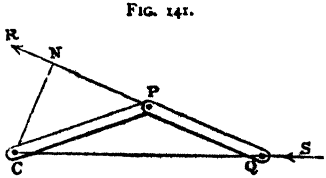
$$S = \frac{20 \times 20}{\frac{1}{10}} \text{ lbs.} = 4000 \text{ lbs.}$$

ART. 105.—In the Stanhope levers the work is completed just as PQ overlaps CP, and there is no advantage to be gained in carrying the motion further; but regarding the combination in its general form, it has been shown that the joint CPQ straightens into a line twice while CP is making a complete revolution.

When this happens we obtain a subordinate combination of levers, which is known as a *knuckle joint*, or *toggle joint*, and is commonly used in hand printing presses, as well as in machinery for punching and shearing iron.

The first form of the joint is shown in Fig. 141, where two arms, CP, PQ, generally equal in length, but which may be unequal, are jointed together, the point C is *fixed*, and the end Q exerts a pressure which may be carried on by a piece moving in the direction CQ.

FIG. 141.



The force F, which produces the result, is supposed to act upon the joint at P, and the reaction S, which is felt at Q, will be transmitted also to P in the direction QP.

Let the thrust, so set up in QP, be called R, and draw CN perpendicular to PR.

Then, by the principle of the lever, we have

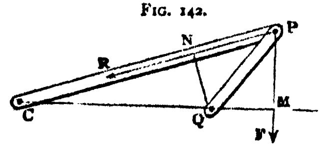
$$R \times CN = F \times \text{(perpendicular from C upon F)}.$$

But as CPQ straightens, CN diminishes, and ultimately becomes equal to zero while the product F × (perpendicular from C upon F) remains approximately constant. It follows, therefore, that the product of R × CN remains nearly constant while CN diminishes to nothing, but this can only happen if R increases in the same proportion that CN diminishes, that is, without limit. Hence the power of the combination.

It is upon this principle that the heavy chain of a suspension bridge cannot be stretched into a straight line; it would break long before it straightened.

In the same way, if the joint be doubled back, the point Q being *fixed*, and the force F being supposed to act in a line perpendicular to CQ, we shall have the pull upon C, which we may call R, felt as a pressure in the line CP.

FIG. 142.



If now QN be drawn perpendicular to the direction of R, as felt at P, the principle of the lever gives us the equation

$$R \times QN = F \times QM,$$

and R becomes infinite when QN vanishes, or when CP is passing over Q.

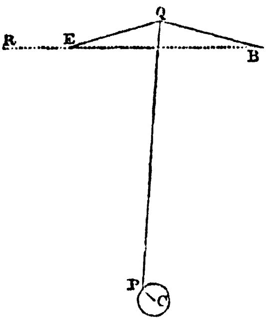
ART. 106.—If the toggle joint be regarded merely as a means of communicating a slow motion to the end of one of its pair of levers, it is not without practical utility.

But inasmuch as any force which may be transmitted through such a joint must be intensified at the period of slow motion, we shall generally find that some work is being done which demands an increase of pressure.

The joint is often attached to a revolving crank and connecting rod as shown in the diagram, where we have the combination

CPQB as in Art. 105, with the addition of a rod EQ jointed to Q, and guided so that the end E moves in the line BR.

FIG. 143.



In this case C should be in the vertical line passing through Q when the joint is straightened, and PQ should be just long enough to reach the lowest point of the circle at the same instant. It will follow that the joint can, under these circumstances, straighten itself only once during a complete revolution of CP, and the contrivance may then be applied so as to obtain a decreasing motion of the point E, and thereby to transmit a pressure which greatly and rapidly increases.

An example occurs in printing machinery, where a knuckle joint, actuated by a crank exactly as described above, is employed to depress the platten upon the impression table, and so to effect, in a large machine worked by steam power, the same thing which is done on a smaller scale by the pull of the lever of a hand press.

In the case last treated, the point Q never passed below the line BR, and thus the joint only straightened itself once during a revolution of CP; it is possible, however, to cause this straightening to occur twice in each revolution of the crank, and to effect this, it is only necessary to shift the point C a little nearer to BR, so that the joint may straighten when P is upon either side of the lowest or highest point of its circular path.

The sketch shows the knuckle joint as applied to a movement of this character in a power loom (fig. 144).

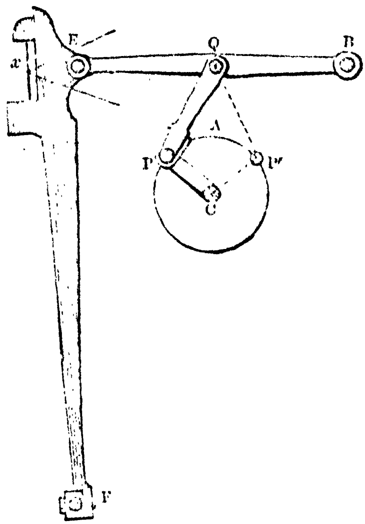
In this case the joint will straighten when P arrives at certain points on either side of the vertical line CA, as shown by the positions of CP, CP′, and thus we shall find that the point Q falls below BE, as well as rises above it, and that there will be two positions of P upon either side of A in which BQE becomes a straight line.

In weaving, the thread of the weft requires to be beaten up into its place after each throw of the shuttle; and in some cases, as in carpet weaving, *two beats* are wanted instead of one.

The arrangement which we are now discussing has been used

to actuate the movable swinging frame, or batten, which beats up the weft, and the result is that two blows are given in rapid succession.

FIG. 144.



In the figure referred to, EF is the batten, movable about F as a centre, and it is clear that when the crank takes the positions CP, CP′, the joint BQE will straighten, and, as a consequence, the batten will be pushed as far as it can go to the left hand, or a beat-up of the weft will take place.

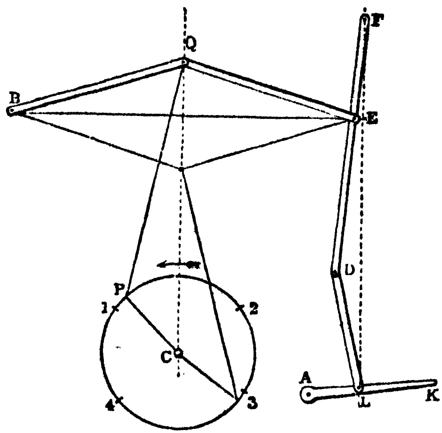
We thus solve the problem of causing a reciprocating piece to make *two* oscillations for each complete revolution of an arm with which it is connected.

ART. 107.—This principle of obtaining *two* vibrations of a bar for each revolution of the driving-crank may be extended still further, and we will alter the construction so as to obtain *four* vibrations instead of two.

The arrangement of the knuckle joint BQE and the crank CP remains as before, but the arm QE is now connected with a second knuckle joint FDL, and the piece AK, which is to receive *four*

vibrations, is centred at A, and is attached at the point L to the link DL.

FIG. 145.

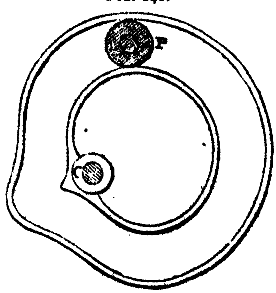


It is clear that the joint FDL will straighten itself four times in each revolution, viz., when the crank CP is in the positions marked 1, 2, 3, 4, upon the circle, and thus AK will make four complete vibrations for each revolution of the crank.

In other words there are two positions of the joint BQE in which FDL straightens, and each of these positions of BQE is obtained by two distinct positions of CP.

By recurring to the earlier part of the chapter, we shall understand that a cam-plate movable about C, and shaped as in Fig. 146, may be employed to drive the batten, and may replace the above combination, being, in point of fact, a mechanical equivalent for it.

FIG. 146.



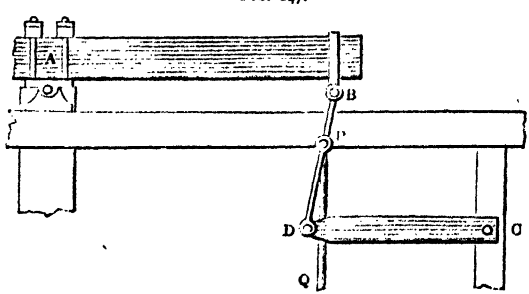
The roller P is then connected with levers attached to the batten, and the beat-up occurs when P passes through the hollows upon each side of the projection at C.

ART. 108.—The *Parallel Motion* used in steam-engines was the invention of James Watt, and was thus described by himself in the specification of a patent granted in the year 1784 :—

'My second new improvement on the steam-engine consists in methods of directing the piston rods, the pump rods, and other parts of these engines, so as to move in perpendicular or other straight or right lines, without using the great chains and arches commonly fixed to the working beams of the engine for that purpose, and so as to enable the engine to act on the working beams or great levers both by pushing and by drawing, or both, in the ascent and descent of their pistons. I execute this on three principles. . . . The third principle, on which I derive a perpendicular or right-lined motion from a circular or angular motion, consists in forming certain combinations of levers moving upon centres, wherein the deviations from straight lines of the moving end of some of these levers are compensated by similar deviations, but in opposite directions, of one end of other levers.'

The annexed sketch is copied from the original drawing deposited in the Patent Office.

FIG. 147.



AB is the working beam of the engine, PQ the piston rod or pump rod attached at P to the rod BD, which connects AB and another bar, CD, movable about a centre at C.

'When the working beam is put in motion the point B describes an arc on the centre A, and the point D describes an arc on the centre C, and the convexities of these arcs, lying in opposite directions, compensate for each other's variation from a straight

line, so that the point P, at the top of the piston rod, or pump rod, which lies between these convexities, ascends and descends in a perpendicular or straight line.'

ART. 109.—This invention being an example of our combination of two cranks and a connecting link, we proceed to discuss it in a careful manner, and to examine its peculiar features.

The lines AB and CD in the diagram represent two rods movable about centres at A and C, and connected by a link, BD. If BD be moved into every position which it can assume, the path of any point P in BD will be a sort of figure of eight, of which the portions which cross each other are nearly straight lines.

At the beginning of the motion let the rods be so placed that the angles at B and D shall be right angles.

FIG. 148.

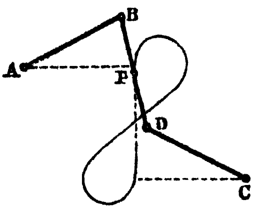
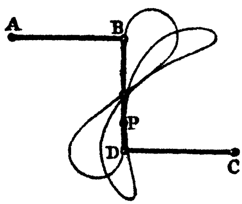


FIG. 149



We shall now endeavour to discover that point in BD which most nearly describes a straight line, and in doing so, we first remark that BD begins to shift in the direction of its length, and therefore that the straight line in question must coincide with BD.

The exact position of the so-called *parallel point*, that is, the point P in Watt's diagram, is determined very simply by analysis, and we shall give the investigation immediately. But we can readily predict where it must be found.

As stated by Watt, the points B and D describe circular arcs about the centres A and C, the convexities of these arcs lying in opposite directions, and if AB and CD be equal, the parallel point P must be so placed that its tendency to describe a curve with a convexity approaching to that of the path of B is exactly neutralised by its tendency to describe another curve with a like convexity in the opposite direction due to its connection with CD.

Hence P must lie in the middle of BD, and being solicited by two equal and opposite tendencies, it will follow the intermediate course, which is a straight line. If, however, AB and CD are unequal, the path of the point P will be affected by the increased convexity due to its connection with the shorter arm CD, and in order to escape from this effect it will be necessary to move P away from D, and to bring it nearer to the arm AB, whose extremity traces out a curve of less convexity.

It may be expected, since we are dealing with circular arcs, that the point P should now approach B in a proportion identical with that given by comparing AB with CD, or that we should have

$$\frac{BP}{PD} = \frac{CD}{AB}, \text{ as in fig. 150.}$$

It is very easy to construct a small model, and to verify in this way the principle of Watt's parallel motion.

If the arms AB, CD, be equal, but the describing point P does not bisect BD, but is brought near to the end D, as in fig. 149, the regular looped curve will become distorted and will incline towards C, in the manner shown in the diagram.

ART. 110.—Refer now to fig. 150, and suppose the rods to be moved from the position ABDC into another position A*bd*C.

FIG. 150.

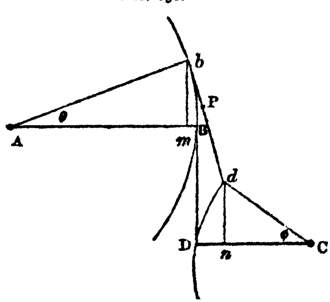
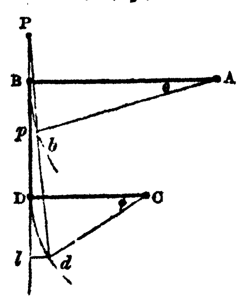


FIG. 151.



Draw *bm*, *dn* perpendicular to AB and CD respectively, and let P be the point whose position is to be determined.

$$\text{Let } AB = r, \; bP = x, \; BAb = \theta,$$
$$CD = s, \; Pd = y, \; DCd = \phi.$$

We shall suppose in what follows that the motion of AB and CD is restricted within narrow limits, and shall deal approximately with our equations, by putting

$$\sin\frac{\theta}{2}=\frac{\theta}{2}, \text{ and } \sin\frac{\phi}{2}=\frac{\phi}{2},$$

$$\text{then } \frac{x}{y}=\frac{b\text{P}}{d\text{P}}=\frac{\text{B}m}{\text{D}n}$$

$$=\frac{r\,(1-\cos\theta)}{s\,(1-\cos\varphi)}$$

$$=\frac{r}{s}\cdot\frac{2\sin^2\frac{\theta}{2}}{2\sin^2\frac{\varphi}{2}}$$

$$=\frac{r}{s}\frac{\theta^2}{\varphi^2} \text{ nearly.}$$

But the link only turns through a very small angle, which may be considered to be nothing as a first approximation, in which case the vertical motion of B is equal to that of D,

$$\therefore\ bm=dn, \text{ or } r\sin\theta=s\sin\phi,$$

$$\text{whence } r\,\theta=s\,\varphi \text{ nearly.}$$

$$\therefore\ \frac{x}{y}=\frac{r}{s}\times\frac{s^2}{r^2}=\frac{s}{r},$$

$$\text{or } \frac{b\text{P}}{\text{P}d}=\frac{\text{CD}}{\text{AB}},$$

i.e., the point P divides BD into two parts which are inversely as the lengths of the nearest radius rods.

In the case considered, which is that which occurs in practice, the parallel point lies in the connecting link, but if the rods be arranged on the same side of the link, as shown in fig. 151, the required point will lie in BD produced, and on the side of the longer rod.

Suppose now the rods to be moved into the position AbdC, and draw bp, dl perpendicular to BD and BD produced respectively.

$$\text{Then } \frac{b\text{P}}{d\text{P}}=\frac{bp}{dl}=\frac{r\,(1-\cos\theta)}{s\,(1-\cos\varphi)}=\frac{r\,\theta^2}{s\,\varphi^2}.$$

Also $r\,\theta = s\,\phi$, by parity of reasoning,

$$\therefore \frac{bP}{dP} = \frac{r}{s} \times \frac{s^2}{r^2} = \frac{s}{r},$$

$$\text{that is, } \frac{bP}{dP} = \frac{Cd}{Ab},$$

and the point P obeys the same general law whether it be found in the link itself or in the prolongation of the line of its direction.

ART. 111.—We have supposed that $\sin\theta = \theta$ and $\sin\phi = \phi$ in the previous investigation, and have examined the motion of that point in the connecting link which most nearly describes a straight line. We shall now inquire how much P really deviates from the rectilinear path at any given period of its motion.

In practice, the beam of an engine seldom swings through an angle of more than 20° on each side of the horizontal line, and within that limit the error consequent upon our assumption that the sine of an angle is equal to its circular measure would not be considerable; for we find, upon referring to the tables, that the circular measures of angles of 1, 5, 10, 15, 20 degrees, and the natural sines of the same angles are the following:—

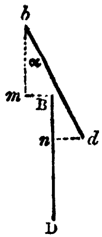
Angle	Circular Meas.	Natural Sine	Difference
1°	·0174533	·0174524	·0000009
5°	·0872665	·0871557	·0001108
10°	·1745329	·1736482	·0008847
15°	·2617994	·2588190	·0029804
20°	·3490659	·3420201	·0070458

In an engine of the usual construction AB is equal to CD, and we shall simplify our results by making this supposition.

Let BD move into the position *bd*, and turn through an angle α; it is very apparent that within the limits to which the rods move in practice, α will be much less than θ or ϕ, so that we may regard α as a close approximation to the actual value of $\sin\alpha$ even when we do not adopt the same supposition with regard to θ and ϕ. The object of the investigation will be to determine ϕ in terms

of θ, and we shall see that the deviation sought for depends upon the difference of the cosines of ϕ and θ.

FIG. 152.



As before, observing that $s = r$, we have

$$\mathrm{B}m = r\,(1 - \cos\theta),$$
$$m\,b = r\sin\theta,$$
$$d\,n = r\,(1 - \cos\phi),$$
$$\mathrm{D}n = r\sin\phi.$$

Let $\mathrm{BD} = l$, then $l + mb = l\cos a + \mathrm{D}n$,

$$\text{or } l + r\sin\theta = l\cos a + r\sin\phi,$$

$$\therefore r\sin\phi = r\sin\theta + l\,(1 - \cos a), \qquad (1).$$

Now a being the angle through which BD is twisted, and being moreover very small, we shall have

$$la = \mathrm{B}m + dn, \text{ very nearly,}$$
$$= r\,(1 - \cos\theta) + r\,(1 - \cos\phi)$$
$$= 2\,r\,(1 - \cos\theta), \text{ since } \phi \text{ is nearly equal to } \theta.$$

$$\therefore a = \frac{2\,r}{l}\,(1 - \cos\theta) \text{ very approximately.}$$

By substituting in equation (1) we can calculate ϕ with considerable accuracy, and then the deviation of P from the vertical

$$= \frac{dn - \mathrm{B}m}{2} = \frac{r}{2}\,(\cos\theta - \cos\phi),$$

and can therefore be ascertained.

Ex. Let $\theta = \frac{\pi}{9}$, and assume $r = s = 50$ in., $l = 30$ in.

$$\therefore a = \frac{2\,r}{l}\,(\cdot 0603074) = \frac{10}{3}\,(\cdot 0603074) = \cdot 2010247,$$

or a represents the angle $11^\circ\ 31'$.

Substituting in equation (1) we have

$$\sin\phi = \sin 20^\circ + \frac{3}{5}\,(1 - \cos 11^\circ\ 31')$$
$$= \cdot 3420201 + \frac{3}{5}\,(\cdot 0201333)$$
$$= \cdot 3541001.$$

$\therefore \phi$ represents an angle of $20^\circ\ 44'$ nearly.

Hence the deviation of P from the vertical

$$= \frac{50}{2}\,(\cos 20^\circ - \cos 20^\circ\ 44) = 25\,(\cdot 0044544)$$
$$= \tfrac{1}{10}\text{th of an inch approximately.}$$

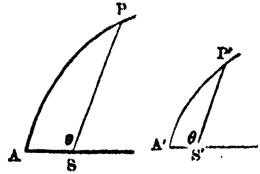
It may be shown that this amount of deviation is again capable of reduction if we cause the centres of motion, A and C, to approach each other by shifting them horizontally through small spaces.

ART. 112.—The point B, whose motion has been examined, is usually found at the end of the air-pump rod. We have now to obtain a second point, also describing a straight line, and suitable for attachment to the end of the piston rod.

We require, in the first instance, to know when two curves are similar, and in a Cambridge treatise on Newton's 'Principia' the test of similarity is stated in the following terms:—

Two curves are said to be similar when there can be drawn in them two distances from two points similarly situated, such that, if any two other distances be drawn equally inclined to the former, the four are proportional.

FIG. 153.



Ex. Thus all parabolas are similar curves, and all ellipses with the same eccentricity are similar curves.

Let A, A′, be the vertices, S, S′, the foci of two parabolas.

Then SA, S′A′, are two lines drawn from two points similarly situated, viz., the foci of the curves.

Let SP, S′P′ be radii inclined at the same $\angle\theta$ to SA, S′A′ respectively.

$$\text{Then SP}=\frac{2\text{SA}}{1+\cos\theta},$$

$$\text{S}'\text{P}'=\frac{2\text{S}'\text{A}'}{1+\cos\theta}.$$

$$\therefore \frac{\text{SP}}{\text{S}'\text{P}'}=\frac{\text{SA}}{\text{S}'\text{A}'},$$

whence the curves are similar, and there is no exception to this rule.

Those who are conversant with the properties of a parabola know very well that it represents, with great exactness, the path of a stone thrown obliquely into the air, and gives the theoretical form of the path of a projectile when unaffected by the resistance of the air.

The similarity of all such curves to one another is by no means evident upon cursory observation, but it is at once established by this simple reasoning.

In the case of ellipses, we proceed in a similar manner, and now S and S′ represent the foci of two ellipses of eccentricity e and e' respectively.

$$\text{Here } SP = \frac{SA\ (1+e)}{1+e\cos\theta}$$

$$\therefore\ S'P' = \frac{S'A'\ (1+e}{1+e'\cos\theta}.$$

Let now $e=e'$, the eccentricities being identical,

$$\text{then } \frac{SP}{S'P'} = \frac{SA}{S'A'},$$

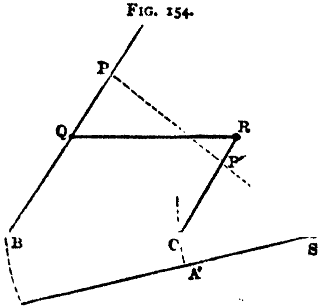
or the curves are similar only under the condition stated.

ART. 113.—Without any further enquiry into the nature of the curves which satisfy the condition of similarity, we will pass on to examine an extremely useful instrument called a *Pantograph*, which is formed as a jointed parallelogram with two adjacent sides prolonged to convenient lengths, and is used to enlarge or reduce drawings according to scale.

This parallelogram was incorporated by Watt into the invention of the parallel motion, and gave it that completeness which it has at the present time. We have now to show that the pantograph is an apparatus for tracing out similar curves.

In the diagram, let BQRC represent a parallelogram whose sides are jointed at all the angles, and having the two adjacent sides BC, BQ, lengthened as shown. Take a point S, somewhere in BC produced, as a centre of motion, place a pencil at any point P′ in the side RC, produce SP′ to meet BQ, or its prolongation in P, place another pencil at P, when it will be found that by moving about the frame over a sheet of paper, and at the same time allowing the joints free play, it will be possible to describe any

FIG. 154.



two curves that we please, and these curves will be similar to each other.

Our definition tells us that the two pencils will trace out similar curves if we can show that SP always bears to SP′ the same ratio that two other *fixed* lines radiating from S, and to which SP and SP′ are equally inclined, also bear to each other.

Conceive, now, that SCB originally occupied the position SA′A, and draw the line SA′A as a fixed line upon the paper, then we shall always have

$$\frac{SP}{SP'}=\frac{SB}{SC}=\frac{SA}{SA'}, \text{ which is a constant ratio,}$$

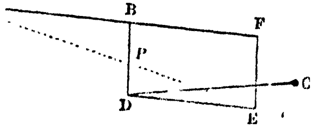
and the angle ASP is equal to the angle A′SP′, therefore the points P and P will trace out similar curves so long as SP′P remains a straight line.

This is, therefore, the only condition which we have to observe in using the instrument.

ART. 114.—We are now in a position to complete the *Parallel Motion of a Beam Engine*, for if one of the points describe a straight line, the other must do the same.

To the system, ABCD, we superadd the parallelogram BFED; ABF being the working beam of the engine.

FIG. 155.



The usual construction is to make the arms AB and CD equal to each other, in which case P, which is the point to which the air-pump rod is fixed, will be in the centre of BD; whereas the second point E, which we are about to find, lies in AP produced, and is the point of attachment of the end of the piston rod.

In order to find the side BF in this parallelogram BFED, assume that $AB=r$, $CD=s$, $BF=x$.

Then $\frac{x}{r}=\frac{DP}{PB}$ by similar triangles ABP, DPE.

Also $\frac{DP}{PB}=\frac{AB}{CD}=\frac{r}{s}$, by property of the parallel motion,

$$\therefore \frac{x}{r}=\frac{r}{s}, \quad \text{or } x=\frac{r^2}{s},$$

which equation determines the proportion between BF, AB, and CD, in order that the second point sought for may lie at the vertex, E, of the parallelogram BFED.

Thus we see that the complete arrangement consists of two distinct portions incorporated together.

1. The combination of AB, CD, and BD, which compels some point P to describe a straight line, the position of this point depending upon the relation between AB and CD.

2. The Pantograph ABFED, which has some point in FE, here for simplicity selected at E, that must necessarily describe a straight line parallel to the path of P.

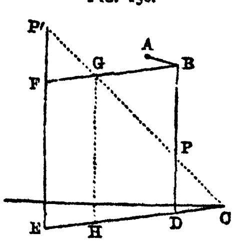
And, as we have stated, the points of attachment of the ends of the air-pump and piston rods to the main beam of the engine are thus provided for.

ART. 115.—Where a beam engine is used in a steam vessel the beam must be kept as low down as possible, and the motion is altered as in the figure, but it is precisely the same in principle.

Beginning with ABDC, a system of two arms and a connecting link, we obtain the parallel point P; we then construct the pantograph CDBFE so as to arrive at the point P′, whose path is similar to that of P.

Here CE represents the beam of the engine, and P′ is the point to which the end of the piston rod is attached. Draw GH parallel

FIG. 156.



to FE, and let $\begin{matrix} AB=r, & DE=c, \\ CD=s, & BD=l, \end{matrix}$ $FP'=x$,

$$\text{Then } \frac{HD}{CD}=\frac{GB}{CD}=\frac{BP}{PD}.$$

$$\text{But } \frac{BP}{PD}=\frac{CD}{AB}=\frac{s}{r}.$$

$$\therefore \frac{HD}{s}=\frac{s}{r}, \text{ or } HD=\frac{s^2}{r}.$$

$$\text{Also } \frac{FP'}{FE}=\frac{FP'}{HG}=\frac{EH}{CH}$$

$$\therefore \frac{x}{l}=\frac{\mathrm{DE}-\mathrm{HD}}{\mathrm{HD}+\mathrm{CD}}=\frac{c-\frac{s^2}{r}}{\frac{s^2}{r}+s}=\frac{rc-s^2}{s(r+s)}$$

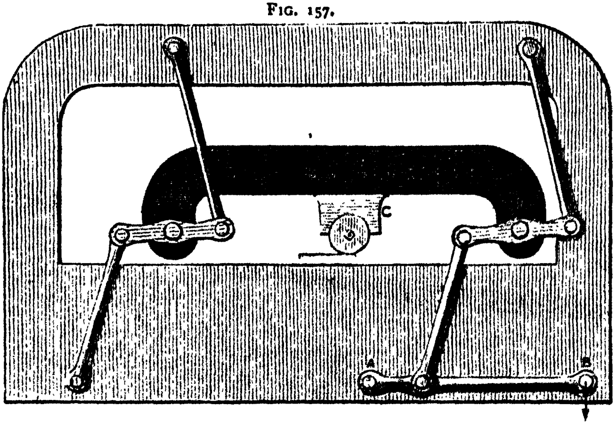
$$\therefore x=\frac{l}{s}\times\frac{rc-s^2}{r+s},$$

whence the position of the point P′ is ascertained.

ART. 116.—A *Parallel Motion* may also be useful in machinery.

In the old process of multiplying engraved steel plates at the Bank of England, which was practised before the art of electrotyping was understood, it was necessary to roll a hardened steel roller upon a flat plate of soft steel with a very heavy pressure, and so to engrave the plate. The difficulty of maintaining this pressure during the motion of the roller upon the surface was overcome by the aid of the parallel motion shown in the drawing.

FIG. 157.



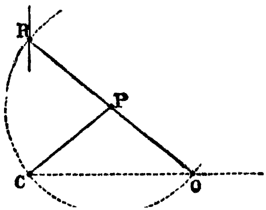
The system of jointed bars allowed the heavy frame C to traverse laterally, while the necessary pressure was obtained by a pull upon the end B of the lever AB, which lever was movable round A as a centre of motion, and was further connected at B with some source of power.

ART. 117.—A straight line motion which is founded on the proposition in Euclid that the angle in a semicircle is a right

angle has been suggested by Mr. Scott Russell. It is derived from the ordinary crank and connecting rod.

Let the rod RQ be bisected in P, and jointed at that point to another rod CP, which is equal in length to PQ. Suppose the point C to be fixed as a centre of motion, and the end Q to be constrained to move to and fro in the line CQ, then R will move up and down in a straight line pointing also to C.

FIG. 158.



Since CP = PQ = RP, the point P will be the centre of a circle passing through C, Q, and R.

Also RPQ is a straight line, and must therefore be the diameter of the circle, whence the angle RCQ is a right angle, or the point R must always be situated in a straight line through C perpendicular to CQ.

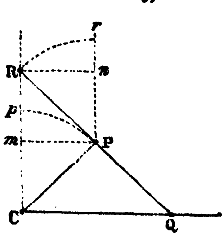
That is, the path of R is a fixed straight line pointing to C.

The motion fails after CP has rotated through two right angles from the upper vertical position. The point Q then gets back to C and remains there.

The motion is rather one for *copying* a straight line than for generating it, for the truth of the straight line described by R will be neither greater nor less than that of the guide CQ which directs its motion.

ART. 118.—The movement may be analysed on the doctrine of harmonics. Taking the rods in the position shown, it is apparent that CP has moved from C*p* and that P has completed a harmonic motion *m*P in a horizontal direction towards the right hand. At the same time PR has turned round P towards the left hand through a circular arc *r*R which is exactly the same as the corresponding path of P. The harmonic motion of R is therefore *n*R in a horizontal direction towards the left hand.

FIG. 159.



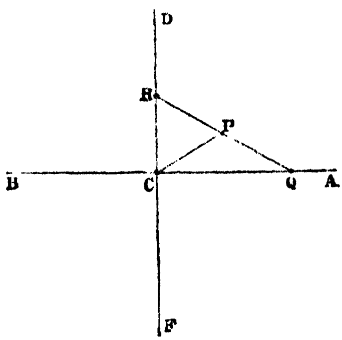
But R receives the motion of P in addition to its own proper

movement round P as a centre, and the horizontal components of these movements are equal and opposite. Hence R describes a vertical straight line.

Hereafter it will be shown that the backward rotation of RP at the same rate as the forward rotation of CP may be provided for by wheelwork, and that a straight line motion may be obtained without any guide along CQ.

ART. 119.—Since the path of P is a circle round C it follows that if AB, DF, represent grooves on a plane surface, and the rod RQ has pins at R and Q working in the grooves, the circular motion of CP will cause R and Q to oscillate to and fro in AB and DF through spaces equal to 4CP. And the converse is also true.

FIG. 16a.



This suggests an instructive model.

For if CP and PQ represent an ordinary crank or connecting rod, CP being *equal* to PQ, it will be found that the rotation of CP brings Q up to C and then the motion fails. The rods CP and PQ merely continue to maintain a circular motion round C.

Whereas if QP be produced to R, such that RP = PQ, and a pin at R works in grooves lying in DF but not continued quite down to the centre C, the rotation of CP will cause Q to move from a distance 2CP on the right, up to C, and then to pass to a distance 2CP on the left of C, thus having a throw equal to *four times* the length of the crank.

ART. 120.—Another form of parallel motion was devised for marine engines before the principle of direct action was so generally adopted. It was fitted to the engines of the 'Gorgon' by Mr. Seaward, and has since been applied in a modified form to small stationary engines, which are convenient in the workshop, and are known as *Grasshopper* engines; but except so far as the

latter application is concerned, it has not been regarded with particular favour.

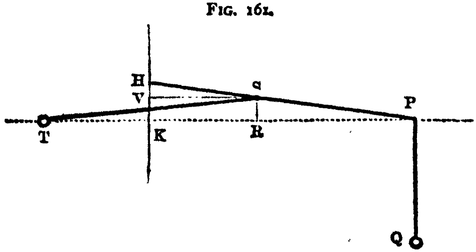
It is, however, remarkable as illustrating a mechanical principle for reducing the friction upon an axis, by causing the driving pressure and the resistance to be overcome to act upon the same side of the centre of motion; for here the connecting and piston rods are both attached to the rocking beam upon the same side of its axis. In this respect it has an advantage, for in ordinary beam engines the pressure upon the fulcrum of the beam is the sum of the power and the resistance, whereas here it is the difference of these forces, and the friction is proportionally diminished.

It is derived from Scott Russell's motion by replacing the guide by a portion of a circular arc of comparatively large radius. For the purpose of explanation we refer to the diagram, where the line HSP corresponds to the line RPQ in the last article, the point P moving very approximately in a horizontal line by reason of its connection with PQ, which has a centre of motion at Q, and the point H being that which most nearly describes a straight line. The system of rods is then TS, HSP, PQ, the points T and Q being centres of motion.

Draw SR and HK perpendicular to TP, and SV perpendicular to HK,

$$\text{and let } \left.\begin{matrix} TS = a \\ SH = b \end{matrix}\right\} \; SP = c, \; \left.\begin{matrix} STR = \theta \\ SPR = \phi \end{matrix}\right\}$$

FIG. 161.



$$\text{Then } TR = a\cos\theta = a\left(1 - 2\sin^2\frac{\theta}{2}\right)$$

$$= a\left(1 - \frac{\theta^2}{2}\right) \text{ nearly,}$$

$$SV = b \cos \phi = b \left(1 - \frac{\phi^2}{2}\right) \text{ nearly,}$$

$$\therefore \ TK = a - b - \frac{a\theta^2}{2} + \frac{b\phi^2}{2}.$$

But the point H describes the straight line HK,

$$\therefore TK = a - b,$$

$$\text{whence we have } \frac{b\phi^2}{2} - \frac{a\theta^2}{2} = 0,$$

$$\text{or } a\theta^2 = b\phi^2.$$

$$\text{But } \frac{a}{c} = \frac{\sin\phi}{\sin\theta} = \frac{\phi}{\theta} \text{ nearly,}$$

$$\therefore \frac{a}{b} = \frac{a^2}{c^2}$$

$$\therefore c^2 = ab,$$

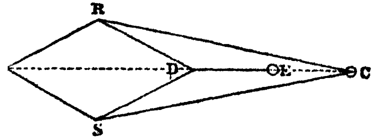
or SP is a mean proportional between TS and SH, a condition to be fulfilled by the rods giving the parallel motion.

In a grasshopper engine PH is the working beam, and PQ is a vibrating pillar at one end, the piston rod is attached at H, and the connecting rod is jointed to some convenient point in the beam.

ART. 121.—The important discovery of the method of drawing a straight line by a combination of bars jointed together, and some of which are movable upon fixed centres, was first made public in the year 1864 by M. Peaucellier, an officer of Engineers in the French army.

The combination consists of seven bars, as in Fig. 162, whereof CR = CS, C being a fixed centre of motion; ED = $\frac{1}{2}$ DC, the

FIG. 162.



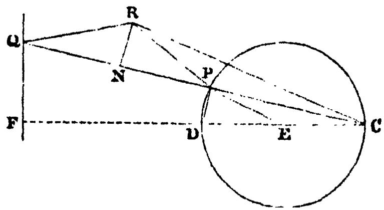
point E being a fixed centre of motion; and FR = RD = DS = SF, the whole of the respective bars being so jointed at their ends as to permit perfect freedom of motion in the plane of the paper.

If the system be moved within the limits possible by its con-

struction the diamond-shaped figure FRDS will open out or close up, the points R and S will describe circles about C, and the point D will describe a circle, which, if completed, would pass through the point C, but F will describe an exact straight line.

To prove this we refer to Fig. 163, where the bars have been moved upwards, one half of the combination being left out for greater simplicity in the diagram.

FIG. 163.



Join CP and produce it to Q, and draw RN perpendicular to PQ.

Let $CR = c$, $PR = b$, $PN = NQ = x$, $NR = y$.

$$\text{Then } c^2 = y^2 + (x + CP)^2,$$
$$b^2 = y^2 + x^2.$$
$$\therefore c^2 - b^2 = CP^2 + 2x \times CP$$
$$= CP\,(CP + 2x)$$
$$= CP \times CQ.$$

When the rods are in the normal position, as shown in fig. 162, let Q be at the point F. Join FQ and PD.

Then by parity of reasoning we have

$$CD \times CF = c^2 - b^2 = CP \times CQ.$$
$$\therefore CD : CP :: CQ : CF.$$

Hence in the triangles CQF, CDP there is one angle common to each, viz., the angle QCF, and the sides about this common angle are proportional : therefore the triangles are equiangular and the angle CPD = angle QFC (Euclid, book vi. prop. 6).

But CPD is the angle in a semicircle, and is therefore a right angle; hence QF is perpendicular to CF, and Q lies always in a straight line through F, which is at right angles to CF.

Hence the point Q moves in a straight line.

It is universally admitted among scientific men that this is a discovery of the highest value as a contribution to the science of geometry, and the student will do well to examine it carefully in detail. For this purpose he should take the combination when unfettered by the bar DE, and should establish by trial the relation between the sides, viz. :—

$$CD \times CF = CR^2 - RD^2.$$

It is usual to call the figure FRDS a *cell*, the points D and F being the *poles* of the cell, and the arm ED being introduced simply to control the motion.

Thus, if the pole D describes a circle of any given radius round some point in the direction of DC, the pole F must of necessity describe another circle whose centre lies also in the same line—it may be that these fragments of circles have their convexities in the same or in opposite directions. All that is a matter for trial or study. And, again, the relative sizes of the circles will vary, so that it becomes possible to draw an arc of a circle of almost any required radius.

The case in the text is where the two circular arcs, which always coexist when ED is centred at some point E in DC, or that line produced, are so related that the radius of one of them becomes infinite.

ART. 122.—In modern engines, the principle of the expansive working of steam is extensively carried out, and there are often two steam cylinders in the place of one, viz., a high-pressure cylinder of small dimensions, and a larger, or low-pressure cylinder by the side of it.

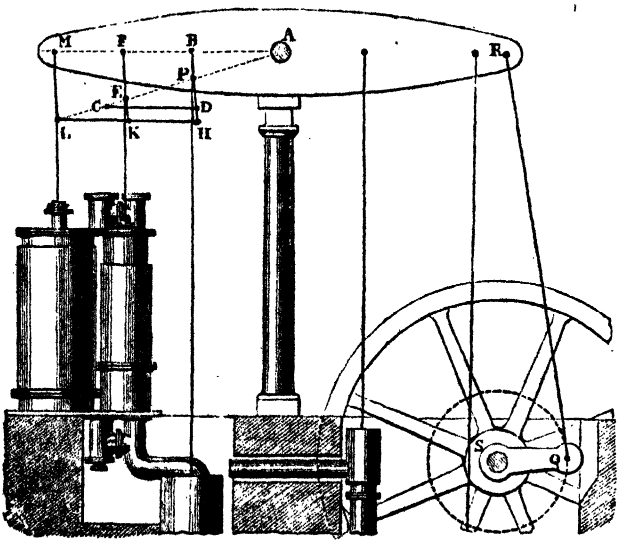
The steam is first admitted into the smaller cylinder, passes from thence into the larger one, and finally escapes into the condenser, as in an ordinary condensing engine.

The use of these double cylinders, with a swinging beam, necessitates a more complex form of parallel motion, but it is quite easy to understand the construction if we remember the principles already investigated.

The diagram, fig. 164, shows the arrangement of the Pumping Engine at the Lambeth Water Works, where four engines of 150 H.P. are placed side by side, arranged in two pairs, each pair

working into one shaft, with cranks at right angles and a fly-wheel between them. The stroke of the crank is equal to that of the larger cylinder, but the smaller cylinder, which receives steam direct from the boiler, has a shorter stroke, and its effective capacity is about one-fourth that of the large or low-pressure cylinder. The peculiarity of the engines consists in the use of a crank and fly-wheel for controlling the motion of the pistons. The main pumps are connected with the beam near the end R.

FIG. 164.



The first part of the parallel motion for connecting the two piston rods with the beam of the engine consists of the portion CDBA, C being a fixed centre of motion, and A being the axis of the beam. In this portion the two arms, CD, AB, and the link BD give the parallel point P.

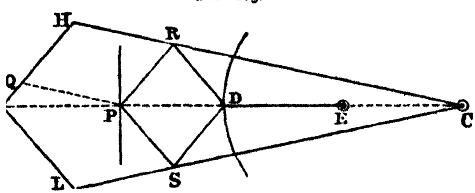
If we now join AP, and produce it to meet the sides of the superadded parallelograms in the points E and L, we shall obtain two other points whose motion is similar to that of P. To these

points the ends of the piston rods must be attached, and the arrangement is complete. The addition, therefore, of an intermediate bar, FEK, parallel to the side ML of the ordinary parallelogram, gives us what we require.

ART. 123.—So likewise, in the parallel motion of Peaucellier, the superposition of the pantograph will give two or more points describing straight lines.

Taking the ordinary combination, produce the lines CR, CS to the points H, L, making CH=CL, and add two bars, HT, LT,

FIG. 165.



of such length that they are respectively parallel to RP and SP. This parallelism must necessarily continue throughout the motion, and it follows that if PQ were drawn parallel to HR we should have an actual pantograph connecting P and T, just as Watt made it for a beam engine. Hence P and T must describe similar paths, which in this case are straight lines.

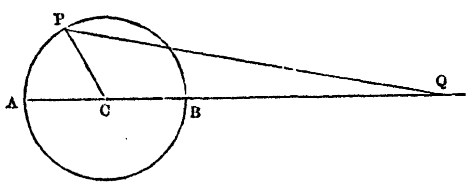
CHAPTER IV.

ON THE CONVERSION OF RECIPROCATING INTO CIRCULAR MOTION.

ART. 124.—It has been shown that the motion of a point in a circle results from the combination of two reciprocating movements—technically called simple harmonic motions—which take place in straight lines at right angles to each other, and that no single harmonic or other straight line motion can produce it. Also, that circular may be converted into reciprocating motion by the suppression of one of the above movements.

The reconversion of reciprocating into circular motion is not a problem of the same kind, as we now require the creation of a

FIG. 166.



movement, instead of its suppression. Such a creation is impossible in a strict mathematical sense, but is practically attainable by mechanical construction.

We recur to the contrivance of the crank, CP, and connecting rod, PQ, as one of the most obvious methods of solving the problem; it being clear that the travel of Q in a line pointing to C will cause the rotation of CP, and will compel P to move in a circular arc.

But unless CP possesses inertia, or is attached to some heavy body as a fly-wheel, which, when once set in motion, cannot sud-

denly come to rest, there will be two points where the power exerted at Q will fail to continue the motion, and these points are evidently at A and B, where CPQ straightens into a right line.

It is usual to call A and B the *dead points* in the motion, and P must be made to pass through them without deriving any aid from Q.

In the existence of these *dead points* we note the failure of the contrivance as a piece of pure mechanism, and theory would tell us beforehand that it must fail, because we cannot create motion by mechanism any more than we can create force: we may modify or interchange without limit the movements which exist among the parts of a system, but there our power ends. And, accordingly, the continuous rotation of CP can only be provided for by storing up in the arm itself, or in some body, such as a fly-wheel, connected with it, the energy which is necessary to overcome any external resistance during that portion of the movement where the driver ceases to act.

So, therefore, in applying the crank and connecting rod to beam-engines, the piston rod is attached by Watt's parallel motion to one end of a heavy iron beam, and the rotation of the fly-wheel is derived by the aid of a connecting rod or spear uniting the other end of the beam with a crank which turns the fly-wheel.

The application to direct-acting engines has been already noticed, and the student will now understand that the mechanical working would be incomplete unless the crank were attached to a fly-wheel or other heavy revolving body balanced upon its centre, which would carry P through those portions of its path near to the dead points, and would also act as a reservoir, into which the work done by the steam might be poured, as it were, unequally, and from which it might be drawn off uniformly, so as to cause the engine to move smoothly and evenly.

In marine engines, where the fly-wheel is not admissible, and where the engine must admit of being readily started in any position, two separate and independent pistons give motion to the crank shaft. In this case the two cranks are placed at right angles to each other, so that when one crank is in a bad position the other is in a good one. The same plan holds in the construction of locomotive engines.

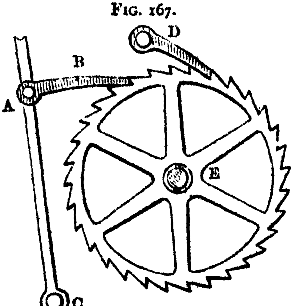
ART. 125.—In the instances considered, the circular motion derived from the reciprocating piece is *continuous*; it now remains for us to examine a class of contrivances for producing the like result where the circular motion is *intermittent.*

The circular motion being that of a wheel turning upon its axis, it may be arranged that one-half of the reciprocating movement shall be suppressed, and that the other half shall always push the wheel in the same direction. This is the principle of the *ratchet wheel.*

Or it may be arranged that the reciprocating piece shall be of the form shown in those escapements which produce a recoil, and the pallets will then act upon opposite sides of the wheel, and will drive it always in one uniform direction. Here we find ourselves again in the region of pure mechanism. These two classes of contrivances constitute the only methods by which the problem can be solved without external assistance.

ART. 126.—A wheel provided with pins or teeth of a suitable form, and which receives an intermittent circular motion from some vibrating piece, is called a *ratchet wheel.*

FIG. 167.



In the drawing E represents the ratchet wheel furnished with teeth shaped like those of a saw, and AB, the driver, is a click or paul, jointed at one end, A, to a movable arm AC, which has a vibrating motion upon C as a centre.

As AC moves to the right hand, the click, B, pushes the wheel before it through a certain space. Upon the return of AC, the click, B, slides over the points of the teeth, and is ready again to push the wheel through the same space as before, being in all cases pressed against the teeth by its weight or by a spring.

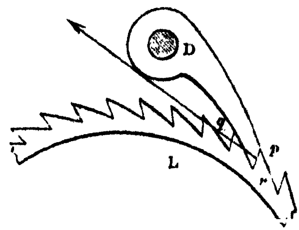
A detent, D, prevents the wheel from receding while B is moving over the teeth, for it is, of course, a condition in this movement that the ratchet wheel itself shall either tend always to fly back, or shall remain held in its place by the friction of the pieces

with which it is connected. In this way the reciprocating movement of AB is rendered inoperative in one direction, and the circular motion results from the suppression of one half of the reciprocating movement of the arm.

The wheel, E, and the vibrating arm, AC, are often centred upon the same axis.

ART. 127.—As regards the action between the teeth and the detent, we observe that the wheel must tend to hold the detent down by the pressure which it exerts, and that it will do so as long as the line of pressure on the surface *pr* falls below the centre D.

FIG. 168.



If the angle *qrp* were opened out much more, the perpendicular upon *pr* might rise above D, and the detent would then fail to hold the wheel.

Further, the click has to return by slipping over the points of the teeth. The condition for this result is that the perpendicular to the surface *qr* shall fall between D and the centre of the wheel.

Where very little force is required to hold the wheel, and the exact position is of consequence, as in counting machinery, the teeth may be pins, and the detent may be a roller pressed against them by a spring.

The usual form of the teeth is that given in fig. 167, and the result is that the wheel can only be driven in one direction; but

FIG. 169.

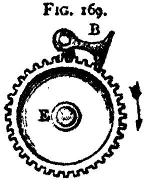
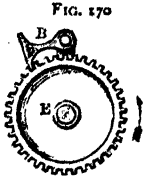


FIG. 170



in machinery for cutting metals it is frequently desirable to drive the wheel indifferently in either direction, in which case a different construction is adopted. The ratchet wheel has radial teeth, and

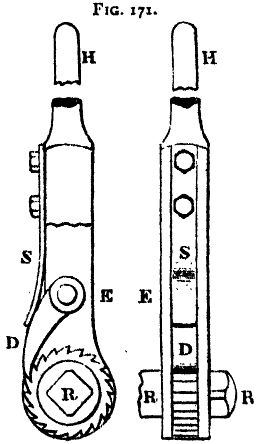
the click, B, can take the two positions shown in the diagrams, and can drive the wheel in opposite directions.

Here the click has a triangular piece fitted upon its axis, any side of which can be held quite firmly by a flat stop attached to a spring. There are, therefore, three positions of rest for the click, whereof two are shown in the figure, and the third would be found when the click was thrown up in the direction of the arm produced. We have by this simple contrivance a ready means, not only of driving the wheel in both directions, but also of throwing the click out of gear when required.

ART. 128.—A common method of applying a ratchet wheel in ordinary mechanism is to be found in the drawing of a screw-jack, as given at the end of the first chapter. (See fig. 38.)

The object of the ratchet there sketched is to drive the screw which traverses the framework of the jack, and it will be readily understood that the friction on the screw thread is so great that there is no tendency for the wheel to run back, and that no detent is necessary.

FIG. 171.



In fig. 171, the lever handle, EH, which actuates the driving paul, has a centre of motion coincident with that of the ratchet wheel, and the paul D is held against the teeth by a spring S. On moving the handle to the right the paul slips over the teeth, and on moving it to the left the spring presses the paul upon the teeth, and the ratchet is advanced in the usual way.

The sketches show the lever-handle and ratchet both in front and side elevation.

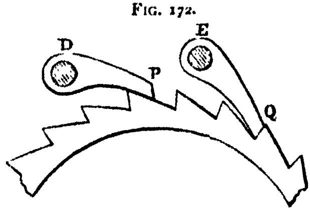
This is the same contrivance as an ordinary ratchet-brace used in drilling by hand.

ART. 129.—Everyone must have seen the application of the ratchet wheel to capstans and windlasses, where it is introduced in order to prevent the recoil of the barrel, the same purpose for which it is applied in clocks and watches.

It was a very early improvement to provide two pauls of different lengths, termed by the sailors *paul and half paul*, and thereby to hold up the barrel at shorter intervals during the winding on of the rope; in fact, a ratchet wheel of eight teeth thus became practically equivalent to one of sixteen teeth, and the men were better protected from any injury which might be caused by the recoil of their handspikes.

The principle of this contrivance is very intelligible, and is shown in fig. 172, where the two pauls DP, EQ, differ in length by half the space of a tooth.

FIG. 172.



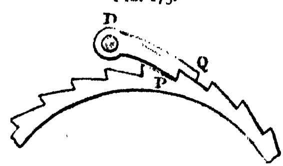
As the wheel advances by intervals of half a tooth, each paul falls alternately, and the same effect is produced as if the number of teeth were doubled, and there was one paul.

In the same way three pauls might be used, each differing in length by one-third of the space of a tooth, and so the subdivision might be extended.

In practice we may be required to move a ratchet wheel through certain exact spaces, differing by small intervals. Where such is the case it is better not to attempt a minute subdivision of the teeth, as they become liable to break and wear away and the action is uncertain, but recourse should be had to a method of placing three or four clicks upon the driving arm.

In such a case the pauls or clicks will be increased in number, and will act as drivers instead of detents, being arranged in order of magnitude as regards length. They may be placed upon separate driving arms, but there is no advantage in doing so, and it is usual to place them all upon one pin at the end of the driving bar.

FIG. 173.



In fig. 173, two clicks, DP, DQ, are shown hung upon a pin D, which is supposed to be at the end of the arm which drives the wheel. The clicks differ in length by half the space of a tooth, and they will manifestly engage the wheel alternately, and will move it

as if there were twice as many teeth driven by one click. And so for *three* or any greater number of clicks.

On referring back to fig. 53, the student will find an example of the use of a ratchet wheel. A link, HK, connects the reciprocating frame, FG, with an arm, LM, carrying a click at Q; thus the oscillations of the frame are received by the arm, and the wheel is advanced a certain number of teeth upon each motion to the right. The number of teeth taken up can be regulated by adjusting the distance of K from L by means of a screw; the nearer K is brought to L, the greater will be the advance of the ratchet wheel at each stroke.

The object of the arrangement is to feed on the wire of lead from which the material for each bullet is cut, and by placing *three* clicks at Q instead of one, according to the method just examined, the amount of advance for bullets of different sizes may be regulated with considerable nicety.

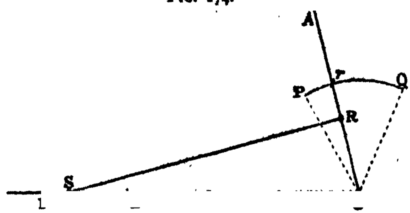
As, in some form or other, the principle of this construction is of great practical value, we will examine it more in detail. It will be found that one chief use of the ratchet wheel occurs in providing the feed motions in machinery for cutting or shaping metals, and the general plan adopted is to draw off, as it were, some definite amount of motion at proper intervals during the operation, and so to impart an unchangeable movement of vibration through a definite angle to a bar which comes as the first piece on the way to the ratchet wheel. We start by giving to such a bar some fixed and unvarying amount of vibration, and our object will be to draw off from this motion just so much or so little as we may require for the purposes of the work, and so to advance the ratchet by 1, 2, 3, or any convenient number of teeth.

If we can now arrange to do this, we have obtained the first part of a feed motion, viz., the power of advancing the ratchet at each stroke by an integral number of teeth; but we may want to go further, and advance the wheel with greater nicety, such as through $2\frac{1}{3}$ of a tooth each time, and the student will understand that for the integral part of the advance some definite contrivance, such as those we are about to discuss, will be wanted, and that for the fractional part of the advance this system of multiple clicks will be perfectly sufficient.

ART. 130.—In considering this problem of advancing the ratchet through any required number of teeth, the very obvious principle already suggested will become apparent upon inspecting the diagram.

Suppose the arm AC to represent a vibrating bar which swings with a definite amount of angular motion through the space PCQ. In a planing machine this bar would be pushed over each time that the table came to the end of its travel. If AC be connected to another point S by the link RS, we can impart to the point S a reciprocating motion in the line CS, which may be represented by the space FE under the conditions shown in the sketch. It is now clear that if R be moved towards C, this travel, FE, will diminish to nothing, while AC continues all the time to swing through the same angle PCQ, whereas FE will be increased in a like degree when R is moved away from the centre C in the direction R*r*.

FIG. 174.



Thus the rate of advance of the ratchet wheel may be regulated.

ART. 131.—We have seen that when a bar centred at one end is continually moved to and fro through a given angle, a feed motion can be derived from it, and it will be found in the workshop that all kinds of rough but effective contrivances are adopted for obtaining such a motion.

We subjoin an elaborate arrangement taken from a rifling machine in the Arsenal at Woolwich.

The student should refer back to Art. 67, where he will find an account of the head of a rifling bar, and will note an arrangement for pushing out the cutters when the head of the rifling bar reaches the breech of the gun, and for withdrawing the cutters while the head is being advanced into the bore. In other words, he will note that the cut is made while the head is being drawn out.

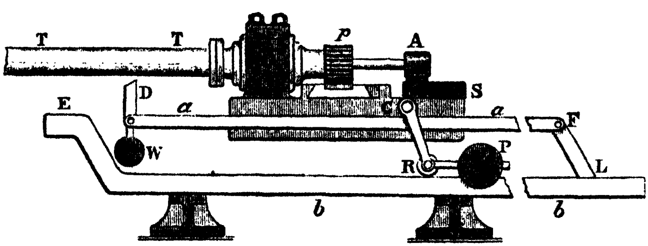
It will also be seen in fig. 80, that when a piece marked RS

is pulled back, the cutters enter the work, and when RS is pushed forward, the cutters are sheathed within the head. It follows that some contrivance must be adopted for moving RS longitudinally through a small space at each end of a stroke.

In our present lecture diagram the piece to which we have referred is not shown, for it lies at some distance to the left beyond the range of the drawing, and forms the head of the rifling bar TT. In the previous diagram the end of the bar TT, which is a hollow pipe, is shown in section, and the student must conceive that a slender rod passes from the head of the rifling bar along the whole length of TT, and terminates in the piece marked A in the annexed sketch.

The whole operation, with which we are now concerned, consists in moving A longitudinally to and fro through a small space. If that can be done at the right times, the feed motion is complete.

Fig. 175.



In the drawing there is a lever CR, having a fulcrum at C, and this lever terminates in a short arm concealed within the saddle S, which moves A when CR is turned through a definite angle. The movement of CR is determined by causing the end R to rest either upon *bb* or *aa*. When R is on *bb* the piece A is pulled out, and the cutters are at work; whereas, when R is on *aa* the piece A is pushed in, and the cutters are sheathed.

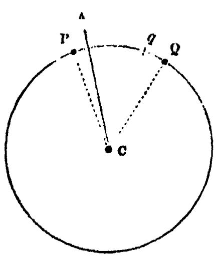
Conceive that the cut is being made, that R is travelling to the right along *bb*, and that the roller which is loaded by a weight P reaches FL. The piece FL is a bar jointed at F, and rises out of the way to allow R to pass. As soon as it has done so, and FL

has dropped into its place, the cut is complete, the mechanism of the machine is reversed, the rifling head begins to enter the gun, the saddle carrying CR moves to the left, the roller R travels up the incline, and the lever CR is raised so that R rests upon *aa*, at which time the piece A is pushed forwards, and the cutters are sheathed.

The roller R travels along *aa* until it reaches the weighted drawbridge D, which it overpowers, and so travels on to the part marked E. When it gets there, the rifling head has reached the breech end of the gun, and the mechanism is reversed. As the saddle returns the drawbridge has risen, and R runs down the slope on to its former path *bb*. Thus R travels to and fro along *bb* and *aa* alternately, and the necessary feed motion is obtained.

ART. 132.—A feed motion, extremely ingenious in principle, has been applied in Sir J. Whitworth's planing machines, and is worth a careful examination. In fig. 176, let CA represent a vibrating bar centred at C, upon which point there is also centred a circle carrying two pins, P and Q.

FIG. 176.



We will suppose that the circle vibrates, independently of the arm, through an angle exactly represented by PCQ, and that the object of the contrivance is to impart to the bar, CA, the whole or any portion of this vibration.

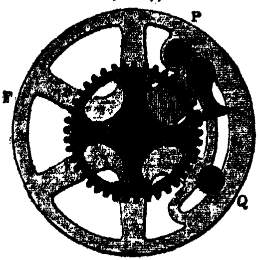
It is easy enough to impart the whole vibration. We have only to fasten P and Q close to CA on each side of it, and the bar and the circle will swing as one piece.

Again, if we wish the bar to remain at rest, we may separate P and Q as much as possible, and when the bar has been pushed as far as it can go by one of the pins, it will remain at rest, for the second pin can only just come up to it on the opposite side.

Conceive now that the pin Q is shifted to *q*, the arm CA will be pushed to Q by the pin P when moving to the right, but will only return as far as Q can carry it, *i.e.*, to *q*, and the vibration will take place through an angle QC*q* instead of an angle QCP, and in this way, by separating P and Q, the motion of AC may be reduced till it ceases altogether.

So, therefore, we obtain precisely the same result as in a previous case, and can advance the ratchet wheel through any integral number of teeth up to a limit fixed by the amount of the vibration of the arm.

FIG. 177.



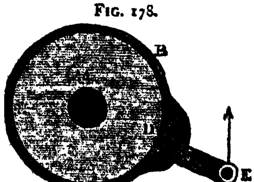
The practical arrangement is shown in fig. 177, where a ratchet wheel, an arm carrying a click, and another wheel provided with a circular slot, are placed in the order stated upon the same axis, and can all move independently of each other. There are two movable pins in the circular slot, which are capable of being fixed in any position by nuts at the back of the wheel, and which embrace the arm carrying the click, but do not reach the click itself.

The ratchet wheel is connected with a screw which advances the cutter across the table, and the object is to impart definite but varying amounts of rotation to the screw after each cut has been taken.

The wheel F receives a fixed amount of vibration from the table, and will impart the whole thereof to the click if P and Q be made to embrace the arm closely upon each side; or it will impart any less amount, gradually diminishing to zero, as P and Q are separated to greater intervals along the groove, and thus the feed of the cutter may be regulated.

ART. 133.—A mechanical equivalent for the teeth and click may be found in what is termed a ***nipping lever***, constructed upon the following principle. Conceive that a loose ring B surrounds a disc A, and that upon a projecting part of the ring there is a short lever, DE, centred. This lever is movable about a fulcrum at F near to the wheel, and terminates at one end in a concave cheek, D, fitting the rim of the disc. On applying a force at E the lever will nip or bite upon the disc, and the friction set up may be enough to cause them to move together as if they

FIG. 178.



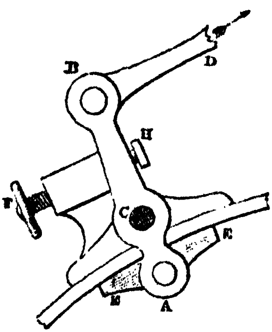
were one piece. This friction does, in fact, increase almost indefinitely, for the harder you pull at E the greater will be the pressure at D, and since the friction increases with the pressure, being always proportional to it in a fixed ratio, the resistance of friction will be developed just as it is wanted.

Upon reversing the pressure at E, the nipping lever will be released and the ring will slide a short space upon the disc; thus the action of a ratchet wheel is imitated.

ART. 134.—The ratchet wheel has been much used in obtaining an advance of the piece of timber at each stroke of the saw in sawing machines. A substitute has been found in an adaptation of this nipping lever, and is commonly known as the *silent feed.*

An arm AB, centred at C, rides upon a saddle which rests upon the outer rim of a wheel: a piece, EE, is attached to one end of the arm, and admits of being pressed firmly against the inside of the rim of the wheel which carries the saddle.

FIG. 179.



It is clear that when the end, B, of the arm, ACB, is pulled to the right hand, the rim of the wheel will be grasped or nipped firmly between the saddle and the piece EE, and that the pull in BD will move the saddle and wheel together, as if they were made in one piece. When B is pushed back, a stop prevents BCA from turning more than is sufficient to loosen the hold of EE, and the saddle slides upon the rim through a small space.

In this way the action of a ratchet wheel is arrived at, and, by properly regulating the amount of motion communicated by the link BD, we obtain an equivalent for a ratchet wheel, with an indefinite number of teeth.

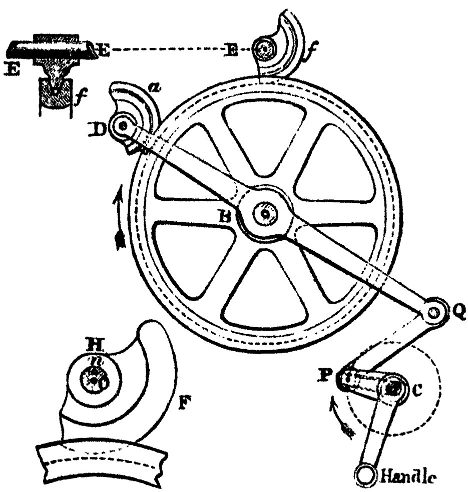
It is this circumstance which renders the contrivance so useful. The amount of feed of the timber can be regulated with the utmost nicety; the opposite end of the link BD is movable by a screw, according to the principle laid down in Art. 130, and can be set at

any distance from the centre of the arm upon which it rides, and thus the contrivance is as perfect as can well be imagined.

A screw, F, may be employed to bring up a stop, H, towards the arm, ACB, and so to prevent the arm from twisting into the position which gives rise to the grip of EE. The saddle will then slide in both directions without imparting any motion to the wheel, a result which is obtained in an ordinary ratchet wheel by throwing the click off the teeth.

ART. 135.—A like result may be obtained in a more simple manner. The annexed drawing of Worssam's feed motion has been taken from a model belonging to the School of Mines.

FIG. 180.



The wheel B is the feeding wheel, and the object is to impress upon it a step-by-step motion changing in every possible amount between two fixed limits.

A handle turns the crank CP, whereby P performs complete revolutions, and the student will recognise the well-known combination of CP with a second crank BQ and the connecting rod PQ, the result, of course, being that while P describes a circle round C, the arm BQ oscillates through a given angle. There is a con-

trivance for adjusting the end P of the connecting rod PQ at different distances from C, and indeed for bringing P to coincide with C. The consequence of this arrangement is that BQ will continually swing through a less and less angle, diminishing to zero, as P is brought nearer coincidence with C.

At the end D of the arm QB produced there is a circular piece which is shown on an enlarged scale in the detached sketch, and is there marked F. The object of drawing this piece a second time is to make it quite clear that the circular rim F is eccentric to the spindle on which it rides. In the drawing the centre of the spindle is marked O, and the centre of the circular rim F is marked *n*.

A second detached sketch shows the rim of the wheel and the piece F in section, from which it appears that F is a circular wedge fitting into a corresponding angular recess which runs round the whole rim of the wheel. The object of making F eccentric is now manifest, for as F rotates on its spindle towards the left hand, the wedge is driven more tightly into the recess and the surfaces grip together with great holding power. As soon, therefore, as BD turns through a definite angle in the direction of the arrow, the piece F holds to the wheel just as if it were a driving paul engaging with a ratchet tooth, and of necessity the wheel moves forward. On reversing the motion of BD the grip is released, and F slides back over the rim without any opposition. In like manner the upper piece corresponding to F acts as a detent, and allows the wheel to advance in the direction of the arrow but stops any return. By this simple construction we have a driving paul and a detent fitted to a wheel having any required step-by-step movement as determined by the angular motion imparted to BD.

ART. 136.—Where the ratchet wheel moves at *each* vibration of the driver, and not during every alternate movement, an *escapement*, or something approximating thereto, must be employed. The action now takes place alternately upon opposite sides of the wheel.

Upon looking back to the elementary form of escapement described in Art. 49, it is quite apparent that if the frame AB be moved to the left, the pallet D will push P before it, sufficiently far to bring R in front of C, and then, upon the return of the

frame, the pallet C will push R before it, and thus the scape wheel will rotate always in one direction.

We remark that this direction is the *opposite* to that in which the wheel revolves when driving the escapement.

The same thing is true, generally, of all *Recoil Escapements*, and upon examining them it will be found that the scape wheel may be driven backwards by the pallets.

ART. 137.—A like action results where two clicks are hung upon a vibrating bar, and one of them terminates in a hook.

The bar ECD vibrates on C as a centre, and the pieces QD, PD, hang at the extremity D. (Fig. 181.)

When P pushes on the wheel, the arms PD and DQ open, and the hook at Q slips over a tooth: whereas, upon reversing the motion of the driving lever, the hook drags the wheel with it, and P slips over a tooth, and thus the wheel advances upon each vibration of the moving arm.

This contrivance is due to Lagarousse.

FIG. 181.

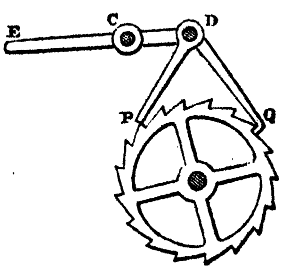
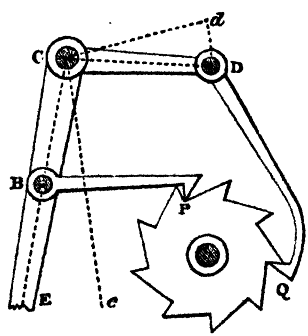


FIG. 182.

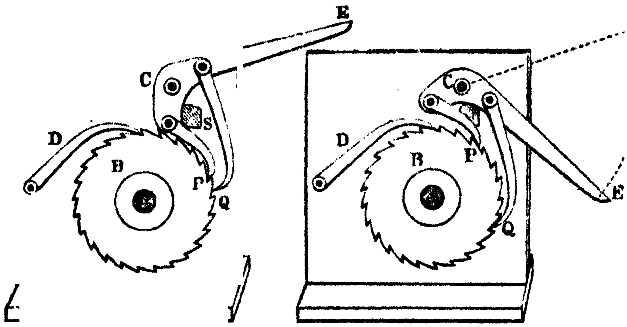


The click may be replaced by a hook turned in the reverse direction, as in the annexed example, which is taken from spinning machinery (fig. 182).

Here the bell crank lever ECD is furnished with the hooks BP, DQ, and by swinging it to and fro through the angle EC*e*, we shall catch up a tooth at the points P and Q alternately, and shall drive the wheel in one direction.

The hooks may both be replaced by pauls turned in the reverse direction, as in the annexed example, which is taken from a model belonging to the School of Mines.

FIG. 183.



Here the lever handle CE swings to and fro through an arc E*e*, and operates upon the pauls P and Q in the manner indicated. The motion of CE is restrained by a stop S, and when CE is being raised as far as it will go, the paul Q is gathered up over the teeth, while the paul P urges the wheel B onward. In like manner, when CE is depressed, the paul Q drives, and P is lifted upwards over the teeth. Thus the wheel advances both when CE rises and when it descends, and it will be found that it moves through two teeth at each stroke. The piece D is an ordinary detent for preventing the recoil of the wheel.

ART. 138.—The construction of mechanism for numbering, or printing consecutive numbers, has wonderfully advanced in late years, and many ingenious improvements have been originated. We do not intend at present to enter upon any details, but there is one contrivance known as a *masked wheel* which ought to be understood.

The object of the masked wheel is to enable a numbering machine to print the same number twice before the unit advances, as in numbering a cheque and its counterfoil.

The masked ratchet wheel consists of two ratchet wheels placed side by side, the teeth in one being the ordinary uniform teeth, while the teeth in the other are alternately shallow and deep. The

main feature is that the bottom of the cut in a shallow tooth is sufficiently removed from the centre to cause the driver of the ordinary wheel to be raised entirely above the teeth of the same, whereby it follows that when the paul rests in the shallow cut it does not operate on the teeth in the other wheel. When the paul rests in the deep cut it acts as a driver to both wheels.

The two wheels ride upon the same axis, but move independently of each other, the ordinary ratchet wheel being connected with the numbering apparatus.

In the drawing, which is taken from an old model belonging to the School of Mines, a pin wheel represents the first ratchet wheel, and the second wheel C has its teeth in pairs, every alternate tooth being cut deeper in the manner stated.

FIG. 184.

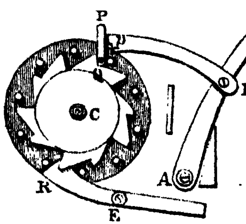
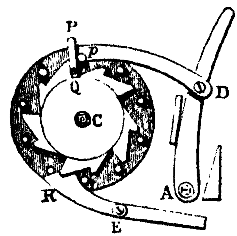


FIG. 185.



The paul PD has a driving pin Q, which must always engage with a tooth of the wheel C, but which may or may not drop low enough to take up a pin on the other wheel.

In fig. 184 the paul has engaged a shallow tooth in the wheel C, and is ready to drive that wheel onwards ; but the paul has its point *p* clear of the pins on the shaded wheel, being masked or prevented from acting by the shallowness of the cut at Q.

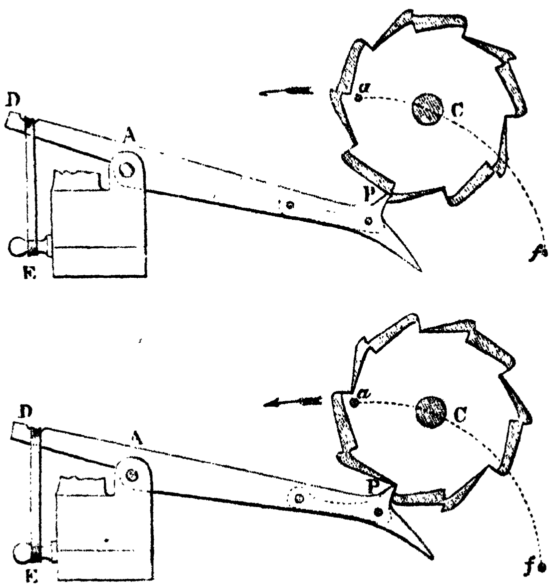
After the paul has driven on the wheel C by the space of a tooth, it will next fall into the deeper cut, when both Q and *p* will engage their respective teeth, and the two wheels will advance together in the manner shown in fig. 185.

Thus two strokes of the paul are required in order to advance the pin wheel by one tooth.

We subjoin two sketches of a masked ratchet wheel as constructed in a small numbering machine.

The wheels are shown in elevation, one behind the other, and having a common axis at C. The unshaded disc represents an ordinary ratchet with uniform saw-shaped teeth, whereas the tinted wheel has deep and shallow teeth alternately. In the machine referred to the method of advancing the ratchet is peculiar, the driving paul is stationary, and the axis of the ratchet wheels is swung round parallel to itself in a circular arc, as indicated by the dotted line *af.*

FIG. 186.



The driving paul is in the form of a lever, DAP, having a fulcrum at A and a projection at P, which engages with the teeth of the ratchets. The shape of the end of the lever is something like a boot, and the pointed end acts as a driver under other conditions, with which we are not concerned at present. An elastic india-rubber band, DE, presses the paul P into contact with the ratchets.

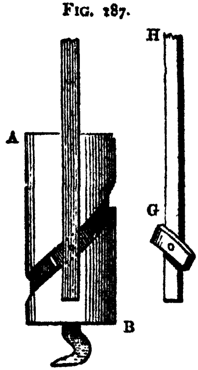
As the wheels sweep to and fro from *f* to *a*, and back again,

the piece P catches the masking or shaded wheel, and moves it on through the space of one tooth. On the next stroke the piece P falls upon a deeply cut space in the masking wheel, and being broad enough to engage both wheels, it carries them on together through the space of a tooth. Thus the unshaded ratchet is moved through the space of a tooth at each alternate stroke in the direction of the arrow.

ART. 139.—The converse of the contrivance described in Art. 68 may also be used to convert reciprocating into circular motion; that is to say, the bar may be employed to turn the screw barrel, instead of the screw driving the bar: but such an arrangement gives rise to a great increase of friction, and is only met with when a small amount of force is to be exerted.

There is a well-known instance in light hand drills, called Archimedean drills, where the rotation is derived by pushing a nut up and down a rod, upon which a screw of rapid pitch is formed, the drill rotating in opposite directions as the nut rises and falls.

FIG. 187.



This movement was at one time proposed by Sir J. Whitworth, in order to obtain a reversing motion of the cutter in planing machinery.

Here a rod, HG, was provided with a sort of tooth, G, which fitted exactly into a groove in the form of a screw-thread traced upon the cylinder AB. As the rod moved up and down it reversed the position of the cutter, and enabled it to act while the table was moving in either direction.

This reversing motion has not been used, but another has been employed in the place of it, where an endless catgut band, which runs round a pulley capable of vibrating through a given angle, is carried on to a small pulley attached to the top of the tool box, the cutter turning through two right angles when the band is made to traverse a small space in opposite directions by the action of the vibrating pulley.

This band is kept stretched by a tightening pulley, and a

second turn round one or more pulleys in the circuit will always prevent the band from slipping.

ART. 140.—When the diameter of a ratchet wheel is increased, the curvature of the rim is diminished, and the ultimate form of a portion of the enlarged wheel is that of a straight bar. Such a ratchet bar is often useful, and we have seen an example in the machine for cutting conical boxwood plugs.

It has long been observed that every movement which is constrained to take place in one direction, and cannot be reversed, is that of a ratchet wheel or bar. Accordingly, the analogy between a common suction pump and a ratchet bar is complete. When the pump is at work the water must rise, and can only descend by some leakage or imperfection.

The mechanism of a pump consists of a bucket with a valve opening upwards, which we may call A, and a fixed valve B, in the pump barrel, which also opens upwards. Our object is to point out to the student that the column of water passing through the pump may be viewed as a vertical ratchet bar, that the driving paul is the same thing as the pump bucket and valve A, while the detent is the lower valve called B.

It is clear that the valve B acts as a detent, for it allows the water to pass freely upwards, but will not permit its descent. It is equally clear that the bucket and valve A act as a driving paul, for during the descent of the bucket the valve A opens, and the hold upon the column of water is obtained at a lower point. The action is precisely that of a driving paul when slipping over the teeth of a bar, and holding on at a lower point. On the ascent of the bucket the column is lifted just as the driving paul lifts a rigid ratchet bar.

CHAPTER V.

ON THE TEETH OF WHEELS.

ART. 141.—We propose now to enter into a mathematical investigation of the forms of teeth adapted for the transmission of motion or force in combinations of wheelwork, and we have already stated the general nature of the problem.

It is required to shape the teeth or projections upon the edges of two circular discs in such a manner that the motion resulting from the mutual action of the teeth upon the discs or wheels shall be precisely the same as the rolling action of those definite circles known as the pitch circles of the wheels in question.

Commencing with flat plates or discs, which, in the case of light wheels, such as are used in clockwork, would be castings in brass or gun metal, having light rims of sufficient substance to allow of the cutting away of the material so as to shape the teeth, we should settle in the first instance the exact size of the pitch circles, and next the number and pitch of the teeth which we meant to employ.

The last inquiry involves only a very elementary knowledge of geometry: we have merely to find out how many times we can repeat the space occupied by the pitch of a tooth upon a circle of given diameter, so as quite to fill up the circumference without any error in excess or defect.

ART. 142.—It will be seen at once that the following very simple equation connects the diameter of a pitch circle with the number of teeth and their pitch.

Let D be the diameter of the pitch circle in inches,

P the pitch of a tooth *in inches*,

n the number of teeth upon the wheel.

Then $nP =$ circumference of pitch circle $= \pi D$,

where $\pi = 3{\cdot}14159$, or $= \frac{22}{7}$ approximately.

$$\text{Hence } n = \frac{\pi}{P} \times D, \text{ or } D = \frac{P}{\pi} \times n.$$

In order to save trouble, definite values are assigned to P, such as $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{7}{8}$, 1, $1\frac{1}{8}$, $1\frac{1}{4}$, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, &c., and the values of $\frac{\pi}{P}$, $\frac{P}{\pi}$, are calculated and registered in a table of which we give a specimen.

	$\frac{\pi}{P}$	$\frac{P}{\pi}$
2	1·5708	·6366
$2\frac{1}{4}$	1·3963	·7162
$2\frac{1}{2}$	1·2566	·7958
$2\frac{3}{4}$	1·1333	·8754
3	1·0472	·9549

Thus, let $P = 2\frac{1}{2}$ inches, we find in the table that

$$\frac{\pi}{P} = 1{\cdot}2566 \text{ and } \frac{P}{\pi} = {\cdot}7958.$$

Suppose a wheel of 88 teeth, and $2\frac{1}{2}$ inch pitch, to be in course of construction, and that we require to know the diameter of the pitch circle.

$$D = \frac{P}{\pi} \times n = {\cdot}7958 \times 88 = 70{\cdot}03 \text{ inches.}$$

Or, again, if the diameter of the pitch circle be 70 inches, and the number of teeth of $2\frac{1}{2}$-inch pitch be required,

$$n = \frac{\pi}{P} \times D = 1{\cdot}2566 \times 70 = 87{\cdot}96 = 88 \text{ very nearly.}$$

In practice it is more easy to treat of the subdivision of a straight line than of the circumference of a circle, and it is the custom therefore to suppose the diameter of the pitch circle to be divided into as many equal parts as there are teeth upon the wheel, and to designate $\frac{D}{n}$ as the diametral pitch in contradistinction to P, the circular pitch.

Further, let $\frac{D}{n} = \frac{1}{m}$ where m is a whole number,

$$\text{now } \frac{D}{n} = \frac{P}{\pi}, \therefore \frac{1}{m} = \frac{P}{\pi}, \text{ or } P = \frac{\pi}{m}.$$

Thus the readings of the legs give the proportionate diameters of the wheels and pinions, and hence also the proportionate numbers of teeth in the same.

ART. 144.—As far as we have gone at present, we have simply determined the relation between the pitch of a tooth and the circle upon which it is formed : it will be necessary to consider also how much of this arc called the pitch is to be occupied by the solid tooth, and how much by the adjoining empty space : we must arrange also the depth of the tooth and the depth of the open space. But without touching upon these points at present, it may be convenient to work out the problem of shaping the teeth in such a manner that the wheels shall roll with perfect accuracy upon each other, precisely as the ideal pitch circles, which we have already referred to, would move by rolling contact.

Two principal propositions on which we rely have already been demonstrated, viz.:—

1. *When two circles roll together, their angular velocities are inversely as the radii of the circles.*

2. *When two arms or cranks are connected by a straight link, the angular velocities of the arms will be inversely as the segments into which the direction of the link divides the line of centres.*

ART. 145.—The latter proposition applies equally when PQ meets CB between the centres C and B, as in the case now to be dealt with ; and we must remind the student that geometers have shown that every curved line may generally at each point be supposed to possess the curvature which would be found in a circle of definite size called the *circle of curvature* of the point in question. In the modern editions of Newton's 'Principia,' the circle of curvature at any point of a curve is defined as being *that circle which has the same tangent and curvature as the curve has at the point in question.*

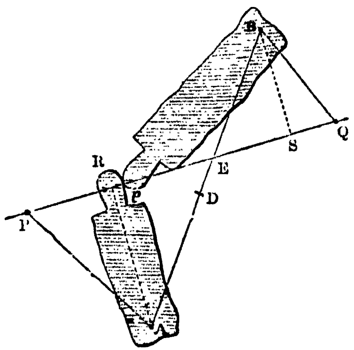
It is also capable of proof that no other circle can be drawn whose circumference lies between the curve and its circle of curvature, starting from the point considered.

In order to find this circle of curvature at any point P of a curve, we first draw the tangent at P, we then take a very small arc PQ of the curve terminating in P, and from the other extremity Q of the arc we draw QR perpendicular to the tangent at

P, and meeting it in the point R; the diameter of the circle of curvature will then be the limiting value of the ratio $\frac{(\text{arc PQ})^2}{\text{QR}}$.

ART. 146.—We must now assume that this circle can be determined, and in fig. 188 we will make A and B the centres of motion of two pieces provided with teeth of some determinate form, which are in contact at the point p.

FIG. 188.



Draw now PpQ, a common perpendicular to the surfaces of the teeth at p; and let P and Q be the centres of the circles of curvature of the curves which touch at p: draw also AR, BS, perpendicular to PQ.

Then, in the first instant of motion, PQ *may be regarded as constant*, because it is the distance between the centres of two ascertained circles which do not vary in size for a very small amount of sliding motion of the curves upon each other; and therefore the angular velocities of the two pieces will be identical with those of AP and BQ.

But we have just proved that

$$\frac{\text{angular vel. of AP}}{\text{angular vel. of BQ}} = \frac{\text{BS}}{\text{AR}} = \frac{\text{BE}}{\text{AE}};$$

$$\text{whence}\quad \frac{\text{angular vel. of piece A}}{\text{angular vel. of piece B}} = \frac{\text{BE}}{\text{AE}}.$$

In order to connect our investigation with this case of linkwork motion, we have only to remember that an imaginary combination of the two arms BQ, AP, connected by a link PQ is always supplied, and that although the separate parts of this combination are continually changing, yet it is always present as a whole, and gives us the means of comparing the angular velocities of the pieces Ap and Bp at every instant.

Suppose it to be required that the angular velocities of the

two pieces shall be the same as those of the pitch circles of radii AD, BD, which is the case in wheelwork.

We must now form the curves so that E shall coincide with D, and shall never leave it during the motion; in other words, the common perpendicular to the surfaces of any two teeth in contact must always pass through the point of contact of the two pitch circles.

If the teeth can be formed so as to satisfy this condition the problem will be fully solved, and we proceed to give the solutions which have been devised by the ingenuity of mathematicians.

There are two curves which will be of great assistance to us, which are the following:—

1. An *epicycloid* is a curve traced out by a point, P, in the circumference of one circle, which rolls upon the convex arc of another circle. This curve is represented by the dotted line in figure 189.

FIG. 189.

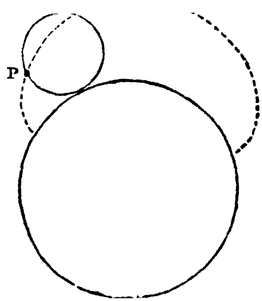
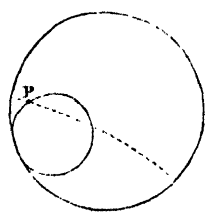


FIG. 190.

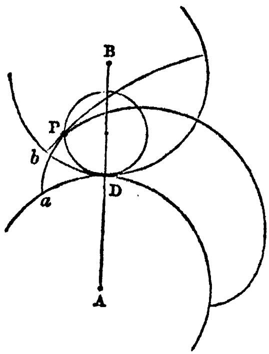


2. A *hypocycloid* is a curve traced out by a point, P, in the circumference of one circle, which rolls upon the concave arc of another circle. This curve is represented by the dotted line in figure 190.

ART. 147.—Conceive now that we are about to form these two curves, the one an *epicycloid* upon the outside of the pitch circle A, the other a *hypocycloid* upon the inside of the pitch circle B: we will employ the *same* generating circle in both cases, and will suppose that A and B represent the pitch circles of two discs upon which teeth are to be carved out. Our object is to show that teeth shaped according to these curves will answer the purpose.

Having drawn the curves, bring the circles together till the *two circles touch in the line of centres and the two curves in another point* P, as shown, when it will be found that the common perpendicular at the point of contact of these curves passes through D.

FIG. 191.



The truth of this statement will be evident from the consideration that when the curves touch, the describing circle may be taken as being ready to generate either the one curve or the other. Now the describing circle cannot do this unless it be resting upon both circumferences indifferently, that is, resting upon the point where the circumferences of the pitch circles touch each other, in which case the common perpendicular to the curves at P passes through D.

But this is the very condition which we are seeking to fulfil, because if it maintains the teeth will move the discs upon which they are shaped with a relative velocity, which is represented by the ratio $\frac{BD}{AD}$, and we have shown that the relative velocity of the two pitch circles is also $\frac{BD}{AD}$, hence the relative velocity of the discs furnished with teeth is precisely the same as that of the pitch circles, and the problem which we are investigating is completely solved.

ART. 148.—We have now obtained two curves which satisfy the geometrical requirements of the problem, and it remains to put our theory into practice.

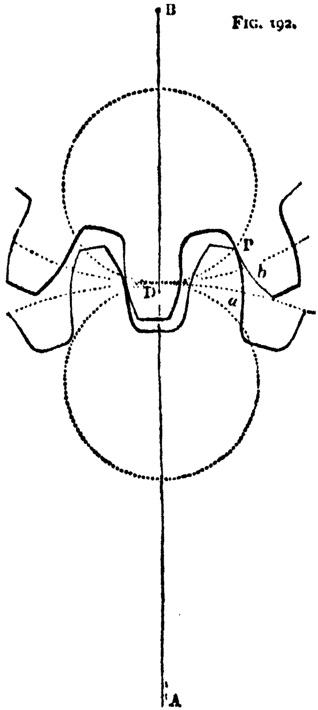
The epicycloid and hypocycloid which form the acting surfaces of two teeth must be produced by one and the same describing circle.

Let A and B be the two pitch circles.

Take a circle of any convenient size less than either A or B, as indicated by the small dotted circle, and with it describe an epicycloid upon A and a hypocycloid upon B.

Let these curves determine the acting surfaces *a*P, *b*P, of two teeth in contact at P; then the tooth *a*P will press against *b*P so that the perpendicular to the surfaces in contact at P shall pass through D, and the relative angular velocities of two pieces centred at A and B, and furnished with these teeth, will be the same as those of the two pitch circles.

FIG. 192.



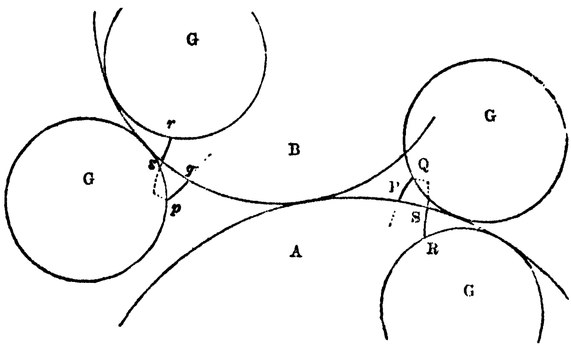
As the wheels rotate, we find that the point of contact P travels along the upper small dotted circle starting from D. In the same way the points of contact of teeth to the left of ADB travel along the lower dotted circle up to D, and it is, therefore, essential to form the teeth in the manner which we are about to describe by somewhat extending our construction. We have now to make complete teeth upon both wheels, and to provide that either A or B may be the driver.

As far as we have gone we have described the point of a tooth upon A and the flank of one upon B, and have supposed A to drive B. If the conditions were reversed, and B were to drive A, we should have to obtain from *one* describing circle the curves suitable for the point of a tooth upon B and the flank of one upon A. This describing circle is not necessarily of the same size as the former one, but it is very advantageous to make it so, and we shall therefore assume that the teeth upon A and B are formed by the *same* describing circle.

Let the describing circle G trace PQ, SR upon A, and *pq*, *sr* upon B, then the complete teeth can be made up as shown in the diagram on the next page (see fig. 193).

By preserving a constant describing circle, any wheels of a set of more than two will work together, as, for example, in the case of change wheels in a lathe.

FIG. 193.

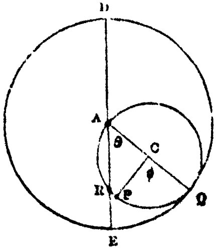


It remains to discuss the character of the teeth as dependent upon changes in the form or configuration of the hypocycloidal portion of the curves.

ART. 149.—If we trace the changes in form of the hypocycloid, as the describing circle increases in size until its diameter becomes equal to the radius of the circle in which it rolls, we shall find that the curve gradually opens out into a straight line.

It is indeed a well-known geometrical fact that when the diameter of the circle which describes a hypocycloid is made equal to the radius of the circle within which it rolls, the curve becomes a straight line.

FIG. 194.



Let C be the centre of the describing circle at any time, and let P be the corresponding position of the describing point (fig. 194).

Suppose that P begins to move from E, so that the arc PQ shall be equal to the arc EQ.

Join CP, AE; let EAQ $=\theta$, PCQ $=\phi$.

Then PQ $=$ arc EQ, or

But $AE = 2\ CQ$, $\therefore CQ \times \phi = 2CQ \times \theta$ or $\phi = 2\theta$.

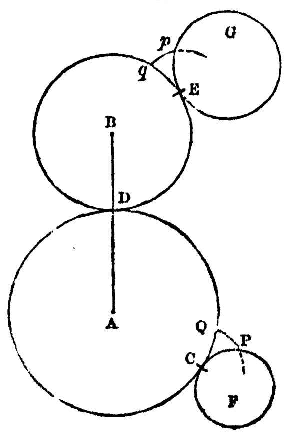
Now ϕ cannot be equal to 2θ unless P coincides with R in the line AE, in which case the diameter EAD is the path of P.

This property of a hypocycloid is taken advantage of in Wheatstone's *photometer*, where an annular wheel is constructed, and a second wheel of half its diameter is made to run very rapidly upon the internal circumference: a small bead of glass, silvered inside, is attached to a piece of cork fitted on this internal wheel. The bead would give the images of two lights held upon either side of it. When the wheel revolves these small images or spots of light become luminous lines of light, whose brilliancy can be compared, and made equal, by shifting the apparatus towards the weaker light. This contrivance is a philosophical toy, it is not used.

ART. 150.—The *first* particular case of the general solution is the subject of the present article.

It will be remembered that the hypocycloid determines the flank of the tooth upon either wheel: if, therefore, the radius of the circle describing the hypocycloid be taken in each case to be *half that of the corresponding pitch circle*, the teeth will have straight, or *radial flanks*, as they are commonly called.

FIG. 195.



The method of setting out the teeth is the following:—

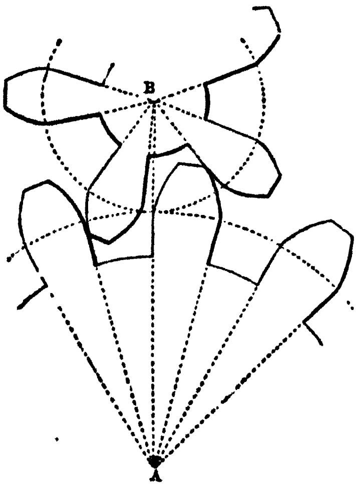
Let A and B be the centres of two pitch circles which touch in the point D.

Let a circle, F, *whose diameter is equal to BD*, roll upon the circle A, and generate the epicycloid QP: this curve determines the form of the driving surface of the teeth to be placed upon A.

Let another circle, G, *whose diameter is equal to AD*, roll upon the circle B, and generate the epicycloid *qp*: this curve determines the driving surface of the teeth to be placed upon B.

Here of necessity the describing circle is not of the same size when tracing out the points of the teeth upon A and B ; but, by reason that the same circle gives the point upon A and the flank upon B, or conversely, and that the flanks in each case are straight lines, the condition in Art. 146 is still fulfilled.

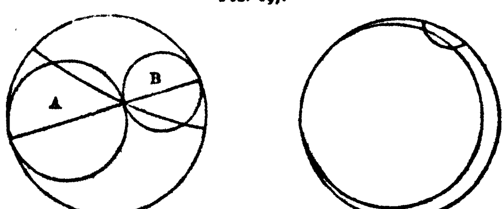
FIG. 196.



The annexed figure shows us these teeth with *radial flanks*, the straight edges of the teeth pointing towards the centres of the respective pitch circles.

ART. 151.—As the circle describing the hypocycloid goes on increasing until it becomes equal to the circle in which it rolls, the curve passes from a straight line into a curve, and finally degenerates from a small half-loop shown in the sketch down to an actual point.

FIG. 197.



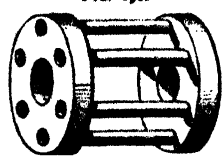
It appears also that the same hypocycloid is generated by each of the circles A and B, which are so related that the sum of their diameters is equal to the diameter of the circle in which they roll.

ART. 152.—The *second* particular case of the general solution occurs when the hypocycloid degenerates to a point: we then

obtain a wheel with pins in the place of teeth, and derive a form which is extensively used in clockwork.

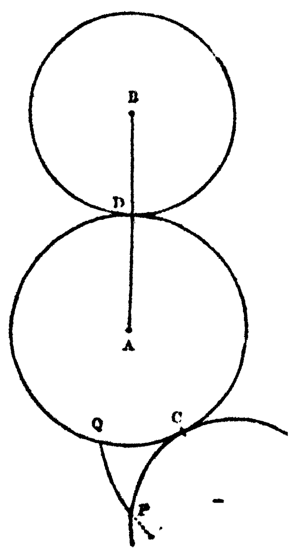
There is a very old form of pin wheel, called a lantern pinion, where the pins are made of round and hard steel wire, and are supported between two plates, in the manner shown in the sketch. This form has been much used by clockmakers, because it runs smoothly, and has the merit of combining great strength with durability.

Fig. 198.



The pin must have some sensible diameter, but we will first suppose it to be a mathematical point.

Fig. 99.



We have just seen that when the hypocycloid becomes a point, the describing circle must be taken equal to that within which it is supposed to roll.

As before, let A and B be the centres of two given pitch circles which touch each other in the point D.

Let a circle, F, equal to B, roll upon the circle A, and generate the epicycloid PQ.

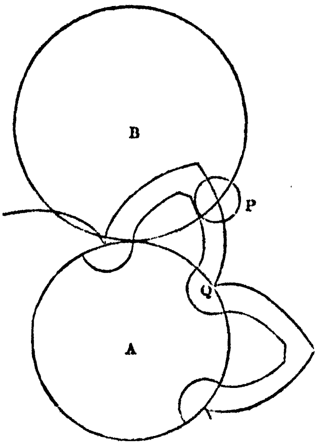
This curve will determine the acting surface of the teeth to be placed upon A, which will work against pins to be placed at equal intervals on the circumference of the circle B.

Thus we shall have epicycloidal teeth upon the driver, working with hypocycloidal teeth on the follower, but these latter teeth are pins, or mere points theoretically, instead of being curved pieces of definite form. Here it is perfectly apparent that the condition upon which we rely is again fulfilled.

The pin must have some size, and we shall take into account the size of the pin by supposing a small circle, equal to it in

sectional area, to travel along the theoretical path of the point, and to remove a corresponding portion of the curved area occupied by the epicycloids.

FIG. 200.



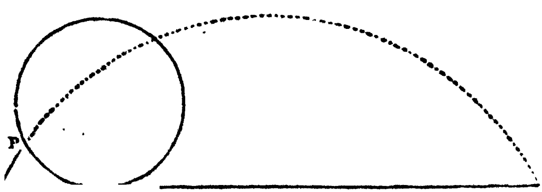
Assume that QP represents the acting surface of a tooth which drives before it a point, P (fig. 200).

Make P the centre of a circle equal to the size of the pin: suppose this circle to travel along PQ, having its centre always in the curve: remove as much of the tooth as the circle intercepts, and the remainder will give the form of the working portion.

We shall presently find that in practice the pins are always placed upon the *driven wheel*, and as this rule is never broken, for reasons to be stated hereafter, we shall assume it to exist when we come to apply our solution to the case of a rack and pinion.

ART. 153.—If either of the wheels becomes a rack, that is, straightens into the form of a bar, the radius of the pitch circle must be infinitely large; and we shall now take up the inquiry as to the changes introduced into the shape of the teeth by this transition from the circle into a straight line.

FIG. 201.



The curve which we have called an *epicycloid* changes into a *cycloid* when the rolling circle runs along a straight line instead of upon the outer circumference of another circle.

It is, in fact, the curve described by a point in the rim of a wheel as it runs along a level road or rail.

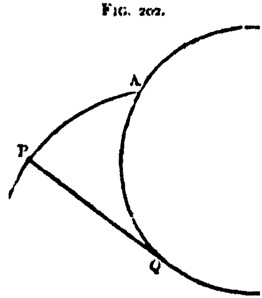
It is shown in fig. 201, and possesses some very interesting properties with reference to the swing of a pendulum; it is, therefore, a curve very familiar to those who study mechanics.

So far as the general solution in Art. 148 is concerned, the changes will be the following. Conceive that the circle A is enlarged till it becomes a straight line; then the circle G, which rolls upon the inner and outer circumferences of the circle A, tracing thereby the points and flanks of the teeth upon A, will in each case generate the same curve, viz., a *cycloid.*

Thus the teeth upon B will remain as before, and each face of a tooth upon the rack A will be made up of two arcs of cycloids meeting in the pitch line.

ART. 154.—In Art. 150, where the teeth have *radial flanks,* the matter is not quite so simple, for the describing circles which give the radial flanks are in each case to be of one-half the diameter of the pitch circle in which they respectively roll; and here one of the pitch circles is infinite, whence it follows that a circle half its diameter is infinite also, or may be regarded as a straight line.

FIG. 202.



The curve traced out by one extremity of a straight line rocking upon the circumference of a given circle, is, of course, the same as that described by one end of a string PQ, which is kept stretched while it is unwound from the circumference of the circle. The end of the line, or the end of the string, is at first at the point A in the curve AP, and the curve is traced out while the line rocks in one direction, or during the unwinding of the string.

This curve AP is a very well-known curve, and is called the *involute* of a circle. We have met with it before, and we proceed to show that in the case of a *rack and pinion having teeth with radial flanks,* the driving surfaces of the teeth upon the pinion will be the *involutes of the pitch circle of the pinion* in question.

ART. 155.—To make this matter clear, we refer to fig. 203, and observe that the circle F, rolling upon a straight line, generates a cycloid and gives the form of the driving surfaces of the teeth upon the rack: the circle G becomes infinite, and Ep changes to a straight line. The change which the rest of the construction undergoes is simply the substitution of the involute qp for the corresponding epicycloid, the circle G having passed into a straight line.

FIG. 203.

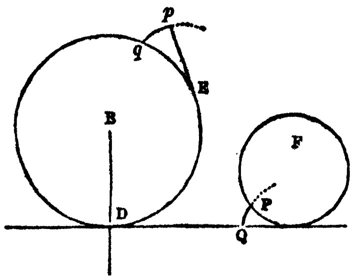
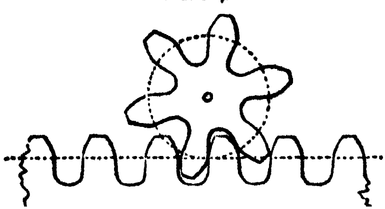


FIG. 204.



The change is scarcely visible to the eye, but the form of the teeth is shown in the diagram, where the curved portions in the rack are cycloids, the radius of the describing circle being half that of the pitch circle of the pinion, and the curves upon the pinion are the involutes of its own pitch circle.

ART. 156.—Where pins are substituted for teeth in either the rack or the pinion, we construct in accordance with the rule that the *pins are always placed upon the follower.*

FIG. 205.

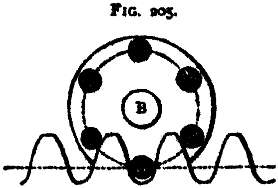
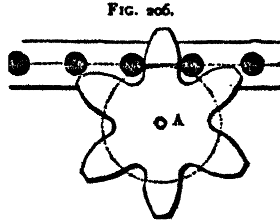


FIG. 206.



1. Let the rack drive the pinion.

Here the circle A becomes infinite, and the curve PQ passes

into a cycloid, so that the teeth upon the rack are cycloidal, as shown in fig. 205.

2. Let the pinion drive the rack.

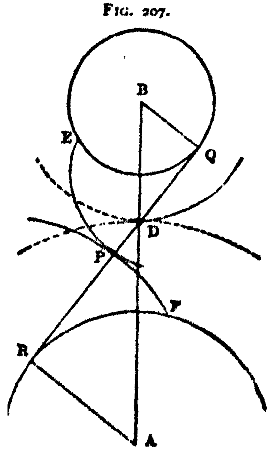
Here the circle B becomes infinitely large, and CP changes into a straight line, the curve PQ passing into the involute of a circle, with the result exhibited in fig. 206, where the teeth of the driver are the involutes of a circle and are known as *involute teeth.*

ART. 157.—The last case to be brought before the reader is derived from a property of this involute of a circle, and the teeth are very easily obtained, but are not used in practice, on account of their being unsuited for the transmission of any considerable forces.

We proceed to show that the geometrical requirements of our construction are fulfilled completely by involute curves.

Let A and B represent the centres of two pitch circles touching at the point D, as shown by the dotted lines, and with B as a centre, and any line BQ, less than BD, as radius, describe another circle. Through D draw DQ touching this smaller circle, draw AR perpendicular to QD produced, and with centre A and radius AR describe a circle touching QR in the point R.

FIG. 207.



If now we take any point P in QR, and describe the involutes EP and FP by winding two portions of strings, such as PQ and PR, back again upon their respective circles, we shall have two forms of imaginary teeth in contact, viz., EP and FP, such that

(1) These teeth have a common perpendicular to their surfaces at P, viz., RPQ.

(2) This perpendicular cuts the line of centres in a fixed point D.

But these are the conditions which we are seeking to fulfil.

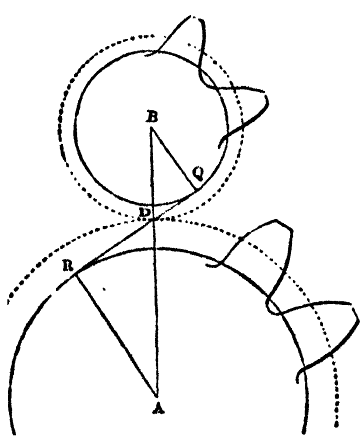
No more direct illustration of our leading proposition could be conceived than this one.

The lines PR and PQ are the respective radii of curvature of the involute curves in contact at P, while RQ, which is equal to RP + PQ, is the link of constant length connecting the arms AR and BQ.

The angular velocities of AR and BQ are therefore as BQ to AR, or as BD to AD, and this ratio remains constant so long as the curves EP, FP remain in contact.

ART. 158.—In order to construct the teeth we must draw our pitch circles touching in D, and then select some angle BDQ at which to draw the line RDQ. When this angle is determined, we obtain the circles of radii BQ, AR, by dropping perpendiculars upon RDQ from the centres A and B, and we then describe teeth of the required pitch by constructing the involutes of these two circles respectively.

FIG. 208.



We observe, of course, that a great latitude is introduced from the circumstance that AD and BD remain constant while AR and BQ may have different values.

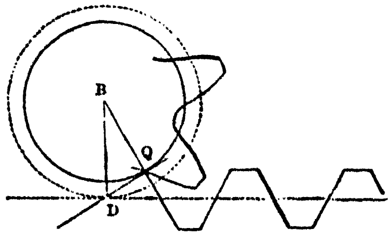
In teeth of this kind there is no difference between the point and the flank: the whole of each edge of a tooth is one and the same curve, viz., the involute of one of the two arbitrary circles.

And further, the points of contact of two teeth must lie either in the line RDQ, or in a second line passing through D, and touching both the circles upon the opposite sides.

ART. 159.—To adapt this solution to the case of a rack and pinion, we note that one of the circles becomes infinite, and, further, that the involute of the infinite circle of radius AR is a straight line perpendicular to its circumference, or perpendicular

to QD. Hence the teeth of the rack are straight lines perpendicular to the direction of QD.

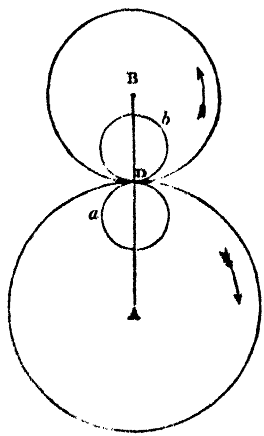
FIG. 209.



The direction of DQ is arbitrary; but when it has once been assumed, the radius BQ will be determined, and involute teeth can be formed upon B, the teeth of the rack being straight lines inclined to the pitch line at an angle equal to BDQ.

ART. 160.—There are now sundry general points for consideration. We may inquire, where does the action of two teeth begin, and where does it leave off?

FIG. 210.



Referring to the solution in Art. 148, we observe that if the motion takes place in the direction of the arrows, and the describing circle be placed so as to touch either pitch circle in D, the contact of two teeth commences somewhere in *a*D, travels along the arc *a*D*b*, and ceases somewhere in D*b*.

Since *a*D lies entirely without the pitch circle B, it is clear that the action in *a*D is due solely to the fact that the teeth upon B project beyond the pitch circle B, and similarly that the action in D*b* depends upon the projections or *points* of the teeth upon A.

It is further evident that the greater the number of teeth upon the wheels, the closer is their resemblance to the original pitch circles, and the more nearly is their action confined to the neighbourhood of the point D.

By properly adjusting the amount to which the teeth are allowed to project beyond the pitch circles, and also their num-

bers, we can assign any given proportion between the arcs of contact of the teeth upon either side of the line ADB.

Where the teeth upon B are pins, there is comparatively very little action before the line of centres, and there would be none at all if the pins could be reduced to mere points, as in that case there would be nothing projecting beyond the pitch circle B.

Again, since the line DP in Art. 147 is a perpendicular to the surfaces in contact at P, it follows that the more nearly DP remains perpendicular to ADB, the less will be the loss of the force transmitted between the wheels.

Here we have an additional reason for keeping the arc of contact as close as possible to the point D. There is a sensible loss of power as soon as the line DP differs appreciably from the direction perpendicular to AB.

It is on this account that involute teeth are not used in machinery calculated to transmit great force. The line RPDQ in Art. 157 is always inclined to the line ADB at a sensible angle, and a direct and useless strain upon the bearings of the wheels is the result.

ART. 161.—In combinations of wheelwork, the accurate position of the centres must be strictly preserved. All the solutions given above, with one exception, entirely fail if there be any error in centring the wheels; they are totally vitiated if anything arises to deprive them of their geometrical accuracy. The exception occurs with involute teeth: the position of the centres determines the sum of the radii of the pitch circles, and the wheels will work accurately as long as the teeth are in contact at all.

We see too that teeth with radial flanks are not suitable for a set of change wheels; the describing circles of one pair of wheels are derived directly from their pitch circles, and cannot be adapted to any other pair in the series.

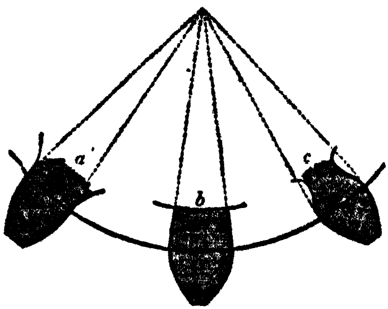
Where, however, the solution in Art. 148 is employed, the describing circles *may* be made the same for all the pitch circles, instead of varying with each one of the series, and in that case any pair of wheels will work truly together.

As regards the strength of the teeth, we remark that this quality is influenced by the size of the describing circle.

If the diameter of the describing circle be less than, equal to,

or greater than the radius of the pitch circle, we shall have the flanks as shown in the sections *a*, *b*, *c* of the sketch.

Fig. 211.



It is evident that a small describing circle makes the teeth strong, and that it would be unwise to have them weaker than they are with radial flanks. The form of involute teeth being somewhat similar to that of a wedge, the teeth of this character are usually abundantly strong.

It will be proved, when we treat of rolling curves, that the surface of one tooth must always slide upon that of another in contact with it, except at the moment when the point of contact is passing the line of centres.

This matter should be well understood, the teeth are perpetually rubbing and grinding against each other; we cannot prevent their doing so: our rules only enable us so to shape the acting surfaces that the pitch circles shall roll upon each other.

Nothing has been said about the teeth rolling upon each other; it is the pitch circles that roll; the teeth themselves slide and rub during every part of the action which takes place out of the line of centres.

Since, then, the friction of the teeth is unavoidable, it only remains to reduce it as much as possible, which will be effected by keeping the arc of action of two teeth within reasonable limits.

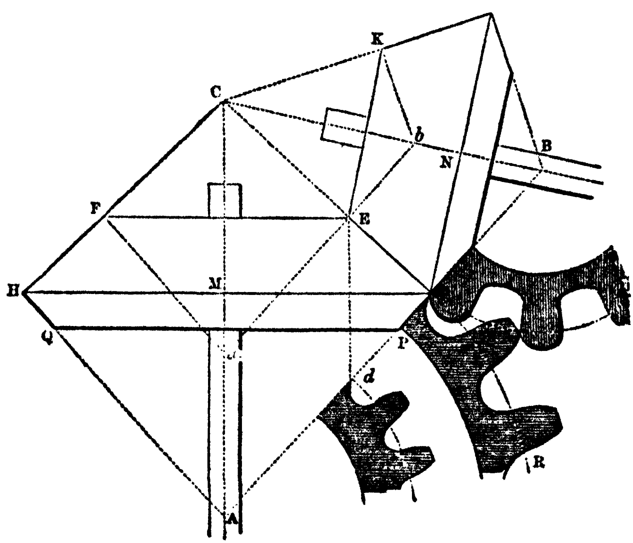
Generally, the friction before a tooth passes the line of centres is more injurious than that which occurs after the tooth has passed the same line: the difference between pushing a walking-stick

along the ground before you and drawing it after you has been given as an illustration of the difference between the friction before and after the line of centres; but this difference is less appreciable when the arc of contact is not excessive.

Where a wheel drives another furnished with pins instead of teeth, the friction nearly all occurs after the line of centres; hence such pin wheels are very suitable for the pinions in clock-work.

ART. 162.—When the axes are not parallel we must employ *bevel wheels*, the teeth upon which are formed by a method due to Tredgold.

FIG. 212.



Let FEDH, KEDL, represent the frusta of two right cones, whose axes meet in C, and which are therefore capable of rolling upon each other.

Let it be required to construct teeth upon two bevel wheels which shall move each other just as these cones move by rolling contact.

Draw ADB perpendicular to DE, meeting the axes of the cones in the points A and B.

Suppose the conical surfaces, HAD, BDL, to have a real existence, and to be flattened out into the circular segments DR, DS: these segments will roll upon each other just as the circular base HD rolls upon the circular base DL.

Hence these segments will serve as pitch circles, upon which teeth may be constructed by the previous rules: such teeth may be formed upon a thin strip of metal, and their outline can then be traced upon the surface of the cone terminating in A.

Similarly, if bEa be drawn perpendicular to ED, the circle of radius Ad = Ea will be the pitch circle for the teeth upon the conical surface EaF. The teeth will taper from D to E, and the intermediate form will be determined by a straight line moving parallel to itself, and originally passing through the points D and E.

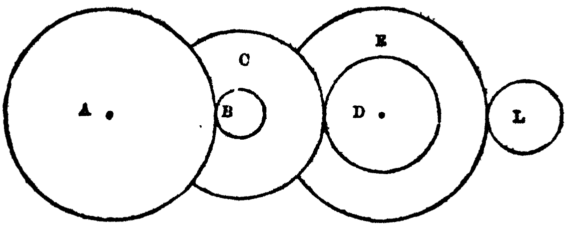
It is stated in Buchanan's account of this method that 'the length of the teeth, the friction of them, and the peculiar advantages of the different modes of forming them, may be considered on the developed pitch lines in the same manner as if they were the pitch lines of spur wheels; consequently every remark that applies to the one, applies to the other. Indeed, the only difficulty in this construction of the teeth of bevelled wheels consists in applying the patterns correctly to the conic surface whereon the ends of the teeth are to be described.'

CHAPTER VI.

ON THE USE OF WHEELS IN TRAINS.

ART. 163.—When a train of wheels is employed in mechanism, the usual arrangement is to fasten two wheels of unequal size upon every axis except the first and last, and to make the larger wheel of any pair gear with the next smaller one in the series.

FIG. 213.



Let A be the driver, L the extreme follower, and conceive that L makes (e) revolutions while A makes one revolution;

$$\text{then } e = \frac{\text{number of revolutions of L in a given time}}{\text{number of revolutions of A in the same time}}.$$

It will be convenient to distinguish (e) as the *value of the train*, and the ratio which it represents may be at once found when the numbers of teeth upon the respective wheels are ascertained.

Suppose that A, B, C, D, &c., represent the numbers of teeth upon the respective wheels, thus we infer from the condition of rolling that

$$\frac{\text{number of revolutions of B in a given time}}{\text{number of revolutions of A in the same time}} = \frac{A}{B};$$

and similarly for each pair of wheels:

$$\therefore e = \frac{A}{B} \times \frac{C}{D} \times \frac{E}{F} \times \&c., \ldots \frac{K}{L}.$$

It may frequently simplify our results if we regard e as positive or negative according as A and L revolve in the same or in opposite directions: thus, in a train of two axes, e would be negative, and in a train of three axes it would be positive.

The comparative rotation of wheels is estimated in various ways, thus:

Let N, n be the numbers of teeth upon two wheels, such as A and B.

R, r their radii.

P, p their periods of revolution.

X, x the number of revolutions made by each wheel in the same given time.

It is easy to see that

$$e=\frac{N}{n}=\frac{R}{r}=\frac{P}{p}=\frac{x}{X}.$$

Note.—A belt and a pair of pulleys supply a mechanical equivalent for spur wheels: the belt may be open or crossed, and in either case

$$\frac{\text{the number of revolutions of B in a given time}}{\text{the number of revolutions of A in the same time}}=\frac{\text{diameter of A}}{\text{diameter of B}}.$$

The crossing of the belt merely reverses the direction of one of the pulleys. Whence it follows that two pulleys with a crossed strap are equivalent to two spur wheels in gear, but that if the strap be open the combination is equivalent to three spur wheels.

Ex. Suppose that we have a train of five axes, and that

1. A wheel of 96 drives a pinion of 8.

2. The second axis makes a revolution in 12 seconds, and the third axis in 5 seconds.

3. The third axis drives the fourth by a belt and a pair of pulleys of radii 20 and 6 inches.

4. The fourth axis goes round twice while the fifth goes round three times.

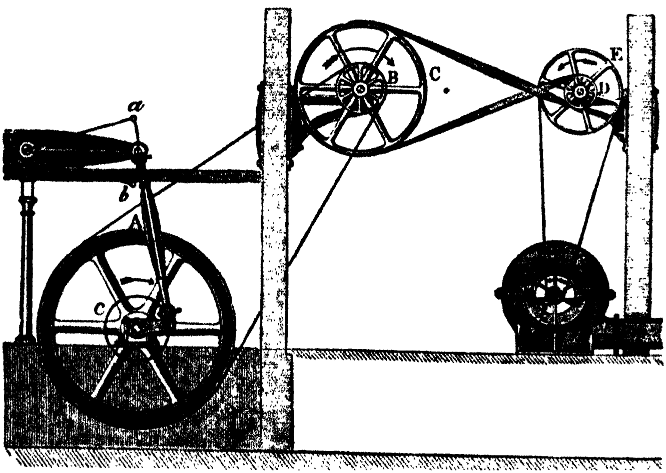
$$\text{Here } e=\frac{96}{8}\times\frac{12}{5}\times\frac{20}{6}\times\frac{3}{2}=144,$$

or the last axis makes 144 revolutions while the first axis goes round once.

ART. 164.—An example of the communication of motion direct from the fly-wheel of an engine to a rotating fan by means of pulleys and bands is given in one of Sir J. Anderson's diagrams.

Here the beam of the engine vibrates through the arc *ab*, and the crank pin at the end of the connecting rod describes a circle, the diameter of which is to that of the fly-wheel A as 4 to 12, or as 1 to 3. Let the mean pressure on the crank pin = 6000 lbs., then the tangential pressure at circumference of fly-wheel is equal

FIG. 214



to 2000 lbs. It will be seen that the motion is carried from the fly-wheel A to the pulley B by an open strap, then from the pulley C to D by means of a crossed strap, then from E by an open strap to F, the fan. In each case there is an increase of the speed of rotation of the driven pulleys, and a corresponding decrease in the driving pressure.

According to the numbers set out on the diagram the fly-wheel makes 20 revolutions per minute, and the fan makes 1600 revolutions in the same time, the rate of increase being arrived at by a comparison of the respective diameters of the drivers and followers.

In like manner the tension of each band may be deduced

from the principle of work, by observing that the product of the tension of a band into its linear velocity is constant. Whence, linear velocity of band on C : linear velocity of band on B :: 8 : 3.

Therefore, tension of band on $C = \frac{3}{8} \times 2000$ lbs. $= 750$ lbs.

In like manner, tension of band on $E = \frac{2}{5} \times 750$ lbs. $= 300$ lbs.

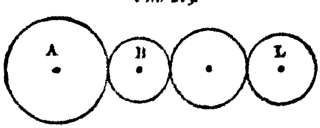
The results are tabulated on the diagram as follows :—

	Diameter	Revolutions	Pressure
A	12	20	2000
B	3	80	2000
C	8	80	750
D	2	320	750
E	5	320	300
F	1	1600	300

ART. 165.—It is very obvious that a wheel and pinion upon the same axis is a combination equivalent to a lever with unequal arms, and modifies the force which may be transmitted through it, and, further, that a single wheel is equivalent to a lever with equal arms, and produces no modification in the force which may pass through it.

So, therefore, when any number of separate wheels are in gear, no two wheels being upon the same axis, they are equivalent to a single pair of wheels, viz., the first A, and the last L: the intermediate wheels act as carriers only, and transfer the motion through the intervening space.

FIG. 215.



This also appears from the formula, where we find that

A .. B .. C .. K A

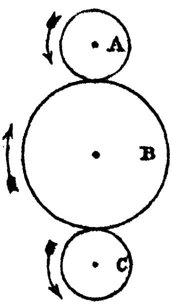
which is the same result as if A and L were alone concerned in the movement.

Art. 166.—If, however, a single wheel, such as B, be interposed between two other wheels A and L, although B will not modify the force transmitted, nor alter the velocity, it may be useful in changing the direction in which the wheel L would otherwise revolve. An intermediate wheel so introduced is technically called an *idle wheel*, and we give instances where this intermediate wheel serves a very useful purpose in causing two other wheels to rotate in the same direction with precisely the same velocity.

1. The student will remember the peculiar heart-shaped cam for driving the needle bar in a sewing machine, as well as the combination of two cranks and a link for giving motion to the shuttle, and he will find on looking back that in each case the driver was a pin rotating in a circle.

The machine from which these movements were taken illustrates a combination of three spur wheels in gear, the central wheel being the driver, and the other two wheels being equal and revolving in the same direction with equal velocities.

Fig. 216.



There is a small fly-wheel driven by hand, on the axis of which is the spur wheel B, and the object being to cause two parallel axes at A and C to rotate in the same direction, and at the same rate, it is arranged that equal spur wheels, A and C, shall both gear with B, in the manner shown by the pitch circles marked in the sketch.

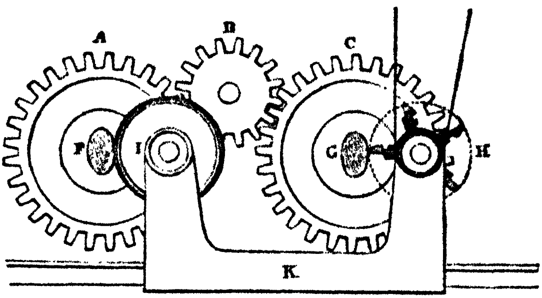
2. The Blanchard turning-lathe, of which a portion is shown in the sketch, is used for shaping the spokes of wheels, gun-stocks, shoe-lasts, and other pieces of an arbitrary form, which no one could imagine, until the method was explained, as being the sort of objects that would probably be turned in a lathe.

But the solution is, that this copying principle admits of endless application, and it will be seen that if we place two lathes side by side, and cause the actual cutter in the one to copy exactly the

form which an imaginary cutter is tracing out upon a model in the other, we shall reproduce upon a piece of wood placed in the second lathe the precise pattern which exists as the copy.

In the drawing the mandrels of the two lathes are shown at F and G, the dark oval at F representing a section of the spoke of a wheel, and being, in fact, an exact copy in iron of the thing to be manufactured. The spoke F is attached to the wheel A, while B is an *intermediate* wheel or driver, and C is another wheel of the same size as A.

FIG. 217.



The unfinished spoke is placed parallel to the copy, along the axis of the wheel C, and the function of the intermediate wheel, or driver B, is to cause the material to revolve in the same direction and at the same rate as the pattern.

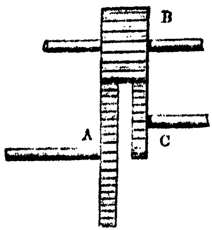
A sliding frame, K, carries a tracing wheel, I, with a blunt edge, which is kept pressed against the pattern by a weight or spring, and also contains the cutters, H, which are driven at a speed of about 2,000 revolutions per minute by an independent strap.

The circle described by the extremities of the cutters is precisely the same size as the circle of the tracer, and it follows that the exact form which the tracer feels, as it were, upon the pattern, will be reproduced by the whirling of the cutters against the material, G, and that the spoke may be completed by giving a slow motion to the combination in a direction parallel to the axis of the pattern.

Sometimes the tracer and cutters are mounted upon a rocking

frame, instead of upon a slide rest, but the principle of the machine is not changed thereby.

Fig. 218.



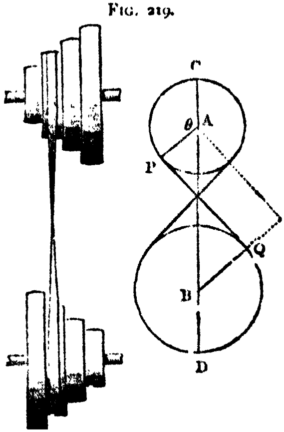
An intermediate wheel may also be useful when two parallel axes are so close together that there is not space for the ordinary spur wheels.

In such a case the axes A and C may be connected by a third wheel, B, and will of course revolve in the same direction.

The wheel, B, is elongated so as to gear with both A and C, and is called a *Marlborough wheel.* The axes might also be connected without wheelwork, as we shall see hereafter.

Art. 167.—*Speed pulleys* are so called because they allow of the transfer of different velocities of rotation from one shaft to another: they are much used in engineers' factories.

Fig. 219.



They are made in a series of steps, as shown in the diagram, one pulley being the counterpart of the other, but pointing in the opposite direction.

If the steps be equal, as is commonly the case, the sum of the radii of each pair of opposite pulleys will be a constant quantity.

It is a geometrical fact that when two circles are placed with their centres at a given distance, and are so related that the sum of their radii remains constant, an endless crossed band connecting both the circles will not vary in length in the smallest degree during the change in the actual diameter of each circle.

Hence a crossed strap will fit any pair of the pulleys in our series with perfect exactness.

The proof is the following:—

Let A and B represent two pulleys whose radii are AP and

BQ, and assume that $AP+BQ$ remains unchanged while AP and BQ respectively increase and diminish,

and let $AP=x$, $BQ=y$, $PAC=DBQ=\theta$.

$$\text{Then}\quad CPQD = x\theta + y\theta + PQ,$$
$$= (x+y)\,\theta + PQ.$$

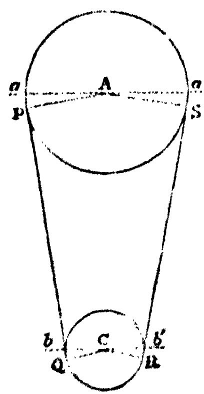
But it is clear that if AP be increased by a given quantity, and BQ be diminished by the same quantity, we shall not change the length of PQ, by reason that the alteration will only cause PQ to move through a small space parallel to itself between the lines AP, BQ, which are also parallel.

Also $x+y$ is constant by hypothesis,

$\therefore$ CPQD remains unaltered in length so long as our condition holds good.

It may be interesting to examine this matter a little further, and to find an expression for the length of an open band connecting two given pulleys. We will assume that we are dealing with step pulleys, the sum of the radii being $2a$ in every case, and x being the depth of the step or steps, or the quantity by which either radius differs from the assumed value of the semi-sum of the radii.

Fig. 220.



$$\text{Let } AP = a + x,$$
$$CQ = a - x,$$
$$AC = c,$$
$$PAa = \theta = QCb,$$

and let $l =$ length of the band PQRS.

Then the curved portions of the band resting upon the pulleys are $(\pi + 2\theta)(a+x)$ and $(\pi - 2\theta)(a-x)$ respectively.

$$\therefore\ l = (\pi + 2\theta)(a+x) + PQ$$
$$+ (\pi - 2\theta)(a-x) + RS,$$
$$= 2\pi a + 4\theta x + 2PQ.$$

$$\text{Now } \frac{PQ}{AC} = \cos\theta,$$

$$\therefore\ PQ = AC\cos\theta = c\cos\theta,$$

$$\text{also } \frac{AP-CQ}{AC} = \sin\theta,$$

$$\therefore\ 2x = AC\sin\theta = c\sin\theta,$$

$$\therefore\ l = 2\pi a + 2c\theta\sin\theta + 2c\cos$$

It is evident that l is no longer constant, and that it must necessarily change when x or θ changes: still the variation of length may be so little as to be disregarded under the ordinary proportions occurring in a workshop.

Since θ would seldom represent an angle so large as 10°, and we have pointed out in Art. 111 how small a difference exists between sin θ and θ within even larger limits, we will assume that $\sin\theta = \theta$,

$$\text{then } \cos\theta = 1 - 2\sin^2\frac{\theta}{2} = 1 - \frac{2\theta^2}{4} = 1 - \frac{\theta^2}{2}.$$

$$\text{Therefore } l = 2\pi a + 2c\theta^2 + 2c\left(1 - \frac{\theta^2}{2}\right),$$

$$= 2\pi a + 2c + c\theta^2.$$

Call l' the value of l when $x = 0$, or $\theta = 0$

$$\therefore\ l' = 2\pi a + 2c.$$

$$\therefore\ l - l' = c\theta^2.$$

$$\text{But } 2x = c\sin\theta = c\theta,$$

$$\therefore\ \theta = \frac{2x}{c}.$$

$$\therefore\ l - l' = c\,.\,\frac{4x^2}{c^2} = \frac{4x^2}{c},$$

which expresses the difference of the lengths in a convenient form.

It is apparent at once that l is greater than l'.

It has been stated that this difference is very trifling in many cases, and the following example is an illustration.

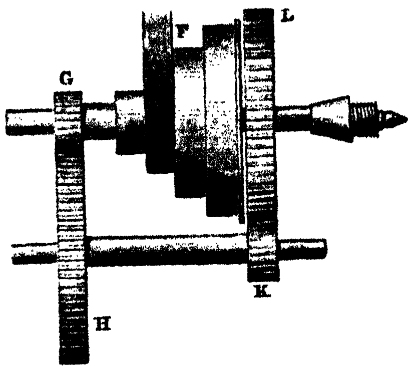
Let the diameters of the steps of the pulleys be 4, 6, 8, 10, 12 inches respectively, and let l be the length of strap for the pair of 12 and 4, while l' is the length for the equal pair of 8 and 8, the distance between the centres of the pulleys being 6 feet.

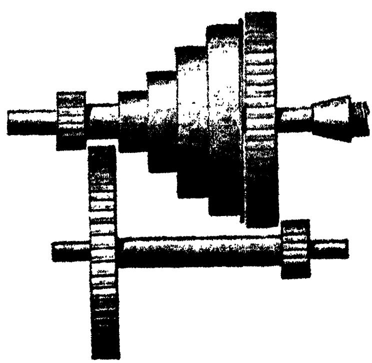
Then $l - l' = 4\,\dfrac{(6-4)^2}{72} = \dfrac{16}{72} = \dfrac{2}{9}$ inch, which is rather less than $\frac{1}{4}$ of an inch.

In practice, open bands are usually preferred to those which are crossed. The latter embrace a larger portion of the circumference, and are therefore less liable to slip, but they rub and wear away at the point where they cross.

ART. 168.—As an example of the use of speed pulleys, we refer to the contrivance sketched in fig. 221, which is to be found in every large lathe, and is useful in other machinery, where it is required to obtain increased power or a diminished speed. It enables the mechanic to change the velocity of the mandrel of the lathe, and gives another simple example of the use of wheels in trains.

There is a driving shaft overhead, provided with a cone pulley, and with fast and loose pulleys, which receive the power from the engine: a second cone pulley, F, is fitted on the spindle of the lathe, and rides loose upon it: to this cone is attached a pinion G, which drives a wheel H, and so the motion is communicated by the pinion K to the wheel L, which is fastened to the mandrel of the lathe, and turns with it. The result is that the wheel L revolves much more slowly than the cone pulley F, and that the speed of the mandrel is reduced by the multiplier $\frac{G \times K}{H \times L}$, where G, K, H, L represent the numbers of teeth upon these wheels respectively.





Where the lathe is worked at ordinary speeds, the wheels H and K are pushed out of gear by sliding the piece HK in the

direction of its axis, as shown in the lower diagram, and the cone pulley, F, is fastened to L by a pin.

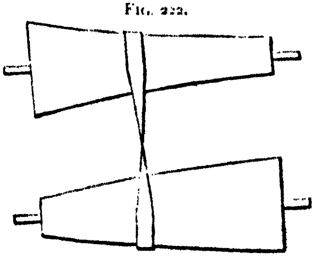
This pin must of course be removed as soon as the slow motion comes into work.

As this movement is very similar to the gearing in a crane, we shall presently examine the application of these trains in raising heavy weights, and shall see how they may be applied so as to reduce velocity, and thereby to increase the amount of force which is called into play.

After what has been stated it is scarcely necessary to point out the express use of *conical pulleys*: they form an obvious modification of step pulleys where the change is continuous instead of being abrupt.

There are two forms, one where the oblique edges of a section are parallel straight lines, and the other where the convexity of one section exactly fits into the concavity of the other.

FIG. 222.



If the band be crossed we have seen that it will retain the same tension in every position upon the cones. If it be open, it will be less stretched at the middle than at either end, according to Art. 167. When the obliquity is small, the difference becomes absorbed in the elasticity or 'sag' of the band; otherwise it must be provided for by giving convexity to one or both of the cones.

The rotation of the upper cone being uniform, it is evident that the rotation of the lower cone will decrease as the strap is shifted towards the right hand.

One of the cones is sometimes replaced by a cylindrical drum, in which case the strap must be kept stretched by a tightening pulley.

As an illustration, we refer to the use of these conical pulleys in the manufacture of stoneware jars and other large earthenware vessels, where a mass of clay is fashioned into the required form upon a rotating table, and the workman varies the speed of the

table according to the requirements of the work by shifting the driving strap along a pair of cones.

ART. 169.—A common eight-day clock affords a familiar illustration of the employment of a train of wheels.

We have marked the disposition of the wheelwork in a clock of this character, and the various wheels are named in the sketch.

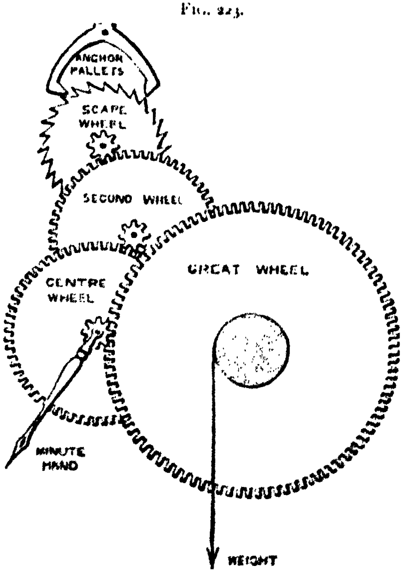
The great wheel turns round once in 12 hours, and may have 96 teeth. Suppose it to engage with a pinion of 8 teeth on the axis or arbor of the centre wheel, this pinion will turn twelve times while the great wheel turns once, and is capable of carrying the minute hand. Let the pendulum swing 60 times in a minute, or be a seconds' pendulum, the scape wheel will then have 30 teeth, and will be required to turn once in a minute.

Hence the value of the train from the centre to the scape wheel should be 60; and in constructing the train we observe that if the pinions on the axes of the second and scape wheels have each of them 8 teeth, the centre and second wheels may have 64 and 60 teeth. In such a case we should have

$$c = \frac{64 \times 60}{8 \times 8} = 60.$$

In order that the clock may go for 8 days, the great wheel must be capable of turning 16 times before the maintaining power is exhausted.

FIG. 223.



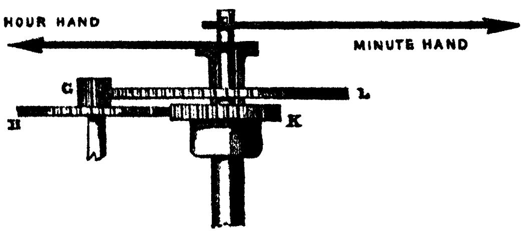
It is easy to see that if the speed of the scape wheel at one end of the train be increased, and if we are at the same time limited in respect of the number of rotations of the great wheel, it will be convenient to introduce a new axis into the train; and, accordingly,

an additional wheel and pinion is found in the train of a watch, where the balance wheel, which performs the function of a pendulum, makes at least 120 vibrations in a minute.

Another illustration of a train of wheels is found in the method of driving the hour hand of a clock or watch, and in order to understand it we have only to observe that in a clock or watch the minute hand is fastened to the arbor or axis of the centre wheel, and that the hour hand is attached to a pipe which fits upon this axis, and derives its motion from the minute hand.

This appears from the diagram, and all we have to do is to connect the pipe and axis by a train of wheels which shall reduce the velocity in the ratio of 1 to 12.

FIG. 224.



The drawing is taken from a small clock, and represents the train of wheels employed. The pinion K, attached to the axis of the minute hand, drives H, whence the motion passes through G to L, and thus to the hour hand, which is fastened to the pipe on which L is fitted, and which corresponds to the mandrel of the lathe. The value of e in the train is given by the equation

$$e = \frac{K \times G}{H \times L} = \frac{28 \times 8}{42 \times 64} = \frac{1}{12}.$$

ART. 170.—The mechanism of a lifting crab for raising weights affords an elementary example of the use of a train of wheels.

The diagram is from Sir J. Anderson's series. On the right hand there is an elevation of the crab, showing the upper shaft, AB, to which the driving lever handles are attached.

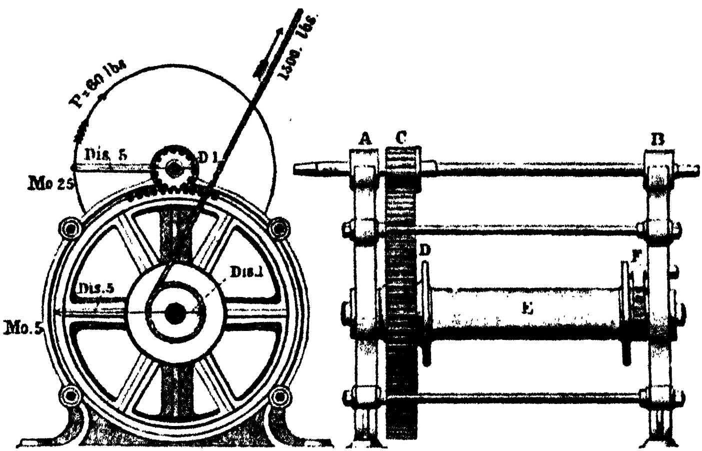
The radius of the circle described by the extremity of the lever handle is to that of the spur wheel C as 5 : 1.

Again, the radius of the spur wheel D is to that of the drum as 5 : 1.

Hence the power is to the weight raised as 25 : 1.

Let P=60 lbs., then resistance overcome by a rope wound round the drum=60 × 25 lbs.=1500 lbs.

FIG. 225.



The diagram is analysed for the use of a teacher. The markings 'Dis. 1,' 'Dis. 5,' &c., indicate the relative distances at which the forces act, and the markings 'Mo. 5,' 'Mo. 25,' give the key to the mechanical advantage gained. Thus, when the rope attached to the weight is pulled in by one foot, the corresponding motion of a point in the circumference of the large pitch circle on the axis of the drum is 5 feet, while the motion of the end of the driving handle is 25 feet. Hence, by the principle of work, $60 \times 25 = x \times 1$, where x is the resistance overcome,

$$\text{therefore } x = 1500 \text{ lbs.}$$

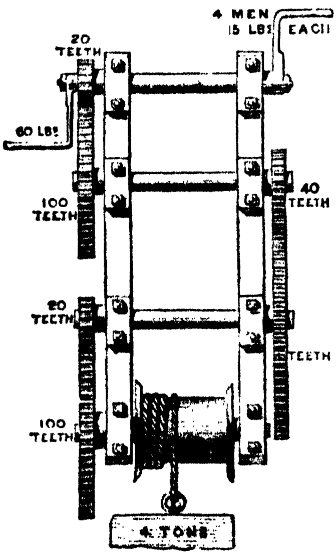
The arrangement of wheelwork in a crane for raising the heaviest weights would be something of the character shown in fig. 226. with this difference, that the wheels would be broader and more massive as we approached the axis on which the weight directly acts.

We take a case in which four men, each exerting a force of 15 lbs., could raise a weight of somewhat more than 4 tons.

As we are only examining the theoretical power of the combination, we will neglect the loss of power by friction.

The men act upon the winch-handles, and the lengths of the arms of these handles are shown as being equal to the diameter of the drum on which the rope or chain is coiled. This gives a leverage of 2 to 1.

FIG. 226.



We next observe that a large and small wheel are placed upon each axis, and calling e the value of the train, we have the relation,

$$e = \frac{20}{100} \times \frac{40}{120} \times \frac{20}{100}$$

$$= \frac{1}{75}.$$

Hence the wheelwork multiplies the power 75 times, while the proportion between the length of the winch-handles and the radius of the drum multiplies the power by 2, and thus we reduce the velocity of the weight which is being lifted 150 times as compared with the rate at which the ends of the handles will move; that is to say, the power exerted upon the weight is 150×60 lbs., or 9000 lbs., which is larger than 4×2240 lbs., or larger than 4 tons expressed in pounds.

ART. 171.—It is a maxim among mechanics that all screws which are required to be perfectly accurate must be cut in a lathe, and there is a geometrical reason for this statement, depending upon the varying inclination of the screw surface at different distances from its axis.

In cutting a screw-thread upon a bolt without using a lathe, we employ pieces of a nut which would exactly fit the screw

when finished, in order to carve out the thread. These pieces, which are called *dies*, are made of soft steel in the first instance, but are afterwards hardened and tempered, and have cutting edges. They are pushed forward by wedges towards the axis of the bolt, during the operation of cutting the thread.

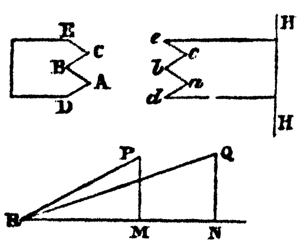
It follows that the angle of a ridge upon the die or cutter begins to trace out the screw-thread upon the bolt. But this angle corresponds to the *inside* line, in the hollow between two ridges, when the screw is completed. We begin, therefore, by tracing out a line, which is slightly different in inclination from the line of the thread that we require. The inclination of the thread, when the cutter begins its work, is not theoretically the same as when it leaves off. The difference is scarcely appreciable, or even recognisable, in small screws; but it exists notwithstanding, and we encounter in screw-cutting a practical difficulty which has never been absolutely overcome.

We can only avoid this difficulty by having recourse to the lathe.

In order to make this statement more intelligible, we refer to the sketch where RPM, RQN represent two right-angled triangles concerned in the formation of a screw-thread of a given pitch.

Let PM=QN, and conceive that the triangle RPM is wrapped round a right cylinder, the circumference of whose base is RM, then RP will form a screw-thread whose *inclination* is the angle PRM. In like manner RQN may be wrapped round a larger cylinder, the circumference of whose base is RN, in which case RQ will be the screw-thread lying at an inclination QRN. Thus, for screw-threads of the same pitch the inclination is less as the cylinder on which the thread is traced becomes greater.

FIG. 227.



Bearing this in mind, let DABCE represent a section of a die which is to be employed to carve out a screw-thread on a cylinder whose axis is HH. The cutting edges at C and A first come

upon the cylinder, and they correspond to the angles of the thread marked *c*, *a*, respectively. They therefore begin by tracing out a thread whose inclination is greater than it should be, and it is manifest that this difficulty is incurable so long as we are operating with ordinary dies.

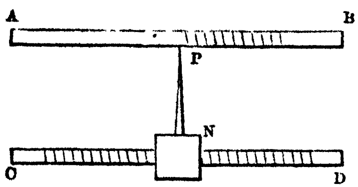
ART. 172.—The principle of construction of the *screw-cutting lathe* will be apparent from the sketch.

Here the *copying principle* receives one of its most valuable applications. The maker of a lathe furnishes a screw, shaped with the greatest care and exactness, and places this screw in a line parallel to the bed of the lathe.

The lathe now carries within itself a copy, which can be reproduced or varied at pleasure, for by means of it we can advance the cutter so as to carve out any screw that we may require.

The screw-thread which forms the copy is traced upon the axis CD, and has a definite pitch assigned to it by the maker.

FIG. 228.



This screw carries a nut, N, and, disregarding the actual construction, we will suppose that the nut, N, is furnished with a pointer, P, capable of tracing a screw-thread upon another axis, AB.

Conceive, now, that AB and CD are connected by a train of wheels in such a manner that they can revolve with any required relative velocities.

Upon each revolution of CD the nut advances through a space equal to the pitch of the screw. If AB also revolve at the same rate as CD, and in the same direction, the point P will describe upon AB a screw-thread exactly similar to that upon CD. If AB revolve more or less rapidly than CD, the pitch of the screw upon AB will be less or greater than that upon CD.

A train which would conveniently connect the axes is shown in fig. 229. Here C is the axis of the leading screw, and A carries the bar which is to be the subject of the operation; it is, in fact, the mandrel of the lathe.

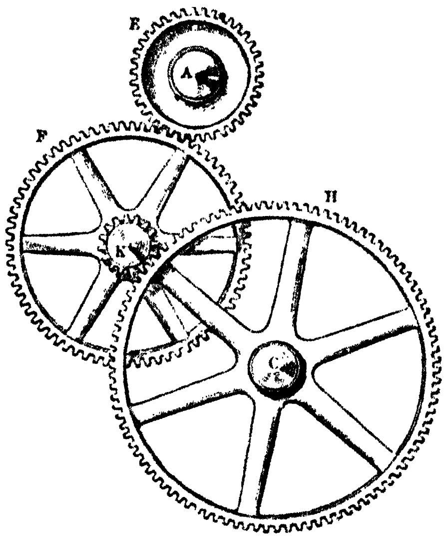
Let E, F, K, H represent the numbers of the teeth upon the

wheels so distinguished, and let e be the value of the train, and suppose AB to make (m) revolutions for (n) revolutions of CD,

$$\text{we have therefore} \quad \frac{\text{pitch of screw on AB}}{\text{pitch of screw on CD}} = \frac{n}{m} = e.$$

$$\text{But} \quad e = \frac{E \times K}{F \times H} \quad \therefore \frac{\text{pitch of screw upon AB}}{\text{pitch of screw upon CD}} = \frac{E \times K}{F \times H}.$$

FIG. 209.



The guiding screw being right-handed, the above arrangement is suitable for cutting *right-handed* screws.

To cut a *left-handed* screw it is essential that AB and CD shall revolve in opposite directions.

Now AB revolves with the mandrel of the lathe, and therefore the direction of the rotation of CD must be reversed. This is effected by interposing an idle wheel between H and K, which reverses the motion of the guide screw, CD, and makes the nut travel in the reverse direction.

There is a double slot or groove upon the arm which carries K, in order to allow the adjustment of this idle wheel.

A set of *change wheels* is furnished with these lathes, and a table indicates the wheels required for cutting a screw of any given number of threads to the inch. The screw upon CD having two threads in an inch, the numbers of teeth to be assigned to E, F, H, K, are given in the table of which a specimen is subjoined.

No. of threads per inch	E	F	K	H
12	60	90	20	120
$12\frac{3}{4}$	60	85	20	90
13	90	90	20	130
$13\frac{1}{2}$	60	90	20	90
$13\frac{3}{4}$	80	100	20	110
14	90	90	20	140

Ex. Let the pitch of the screw upon CD be $\frac{1}{2}$ an inch, and let it be required to cut a screw of $\frac{1}{13}$-inch pitch upon AB, or a screw with 13 threads to the inch.

Here $e = \frac{2}{13}$, which is satisfied in the following manner:

$$\frac{E \times K}{F \times H} = \frac{90 \times 20}{130 \times 90}.$$

In the case of the micrometer screw with 150 threads to the inch, mentioned in the introductory chapter, a lathe is employed for cutting the thread.

The guiding screw has 50 threads to the inch, and the mandrel of the lathe rotates faster than the guiding screw in the proportion of 3 to 1.

This change of velocity is effected by two wheels having these proportions, and connected by an intermediate wheel, the position of the centre of which can be altered so as to suit principal wheels of different sizes.

The cutter which shapes the thread has a fine pointed edge, and the screw is nearly finished in the lathe, but is finally rendered perfect in form by screwing it through a pair of dies. This latter

operation has a tendency to alter the pitch of the screw by permanently stretching the metal of which it is made, and should therefore be resorted to as little as possible.

A screw with 150 threads to the inch, and furnished with a graduated head reading off to hundredths of a revolution, would measure a linear space of $\frac{1}{150 \times 100}$ or $\frac{1}{15000}$th of an inch.

ART. 173.—We pass on now to an enquiry into the construction of a train of wheels for any given purpose; and here it is necessary to point out that mechanicians are tied down by practical considerations, whereby it often happens that an arrangement, which is quite simple and feasible in theory, would nevertheless prove utterly absurd if any attempt were made to carry it out in practice. One very simple example will explain what we mean.

Suppose it to be required to communicate motion from one axis, A, to another, C, and that C is to make 60 revolutions while A makes 1 revolution, as in the clock train. If A be made to drive C directly, it is clear that the number of teeth upon A must be 60 times as great as the number upon C, so that if C have 8 teeth, A must have 480 teeth.

This would involve the use of two wheels side by side, one of which was 60 times as large as the other, to say nothing of the practical difficulty of dividing the larger wheel so as to form the teeth, and accordingly no such combination is to be found in any clock train.

But the insertion of an intermediate axis relieves us at once from the difficulty.

Place such an axis, which we may call B, between A and C, and fasten upon it two wheels of 8 and 60 teeth respectively, give 64 teeth to A and 8 teeth to C, and the value of the train becomes $\frac{64 \times 60}{8 \times 8}$, or 60, the necessary result being obtained with perfect ease and complete simplicity of construction.

ART. 174.—We see, then, that this problem of connecting two axes by a suitable intermediate train of wheels is an arithmetical problem which may, of course, in some cases prove extremely troublesome, and may demand a considerable amount of arithmetical ingenuity.

The value of e being assigned as a fraction, the only thing to be done is to resolve the numerator and denominator into their prime factors, and then to compose the best train which may suggest itself.

Thus, let it be required to connect two axes so that one shall revolve n times while the other revolves once.

Assume some value for n, say 720.

$$\text{Then } e = n = 10 \times 9 \times 8,$$
$$= \frac{80 \times 72 \times 64}{8 \times 8 \times 8},$$

which gives a probable solution for the train.

If any of the factors appear unmanageably large, we may approximate to the value of e by continued fractions, and seek other factors which present less difficulty. If the value of e be an integer, we have seen that it must still be split up into factors, and must be further multiplied and divided by the numbers of teeth in each pinion.

Thus, suppose the two axes are to be connected whereof one revolves in 24 hours, and the other in 365 days 5 hours 48 minutes 48 seconds, as in Mr. Pearson's orrery.

Since 24 hours = 86400 seconds,
and 365 days 5 hrs. 48 min. 48 sec. = 31556928 seconds,

$$\therefore e = \frac{31556928}{86400},$$
$$= \frac{164359}{450},$$
$$= \frac{269 \times 47 \times 13}{10 \times 9 \times 5}.$$

Here 269 is an inconveniently large number, and 5 is certainly too small. The wheel of 269 teeth cannot be got rid of without altering the entire ratio, but the pinions of 9 and 5 teeth may be changed into others of 18 and 10 teeth.

$$\text{Thus we have } e = \frac{269 \times 26 \times 94}{10 \times 10 \times 18}$$

We might have approximated to e by an algebraical process and have derived the fraction

$$\frac{94963}{260}, \text{ as representing } \frac{31556928}{86400} \text{ very closely.}$$

$$\text{But } \frac{94963}{260} = \frac{11 \times 89 \times 97}{4 \times 5 \times 13},$$
$$= \frac{44 \times 89 \times 97}{8 \times 10 \times 13},$$

which avoids the higher number 269, and corresponds to a period of 365 days 5 hours 48 min. 55·4 sec.

Thus we have a train in which the numbers are suitable.

Every possible arithmetical artifice is resorted to in cases of this kind, and the ratio $\frac{269}{1}$ has been dealt with after the following manner, in virtue of the discovery that $269001 = 9 \times 9 \times 9 \times 9 \times 41$, and it is not very difficult to get upon the necessary track, for we see at once that 269001 is divisible by 9 because the sum of its digits is so divisible, and again $\frac{269001}{9} = 29889$, which is again divisible by 9 for a like reason, and thus we soon arrive at the last quotient after all the successive divisions by 9, viz., 41.

$$\text{Since } \frac{269}{1} = \frac{269000}{10 \times 10 \times 10},$$
$$= \frac{269001}{10 \times 10 \times 10} \text{ very nearly.}$$

The numerator can now be split up into 3 factors, which will express the numbers of teeth in the 3 wheels of a train, and we may consider that

$$e = \frac{269}{1} = \frac{269001}{10 \times 10 \times 10} \text{ very nearly,}$$
$$= \frac{81 \times 81 \times 41}{10 \times 10 \times 10},$$

an approximation which would introduce an error of only one revolution in 269000.

Art. 175.—It is also a matter of enquiry to ascertain the smallest number of axes which may be concerned in the transmission of any required motion, since we do not want to employ more wheels than are necessary.

The smallest number of teeth which are to be allowed upon a pinion must be given, as well as the largest number to be allowed upon any wheel.

Suppose that no pinion is to have less than 6 teeth, and no wheel more than 60, and let us trace the values of e.

$$\text{With two axes } e = \frac{60}{6} = 10.$$

If the numerator be diminished, or the denominator be increased, the resulting value of e is lessened, or, in other words, 10 is the greatest possible value of e when two axes are employed.

With three axes the greatest value of e is $\frac{60 \times 60}{6 \times 6}$, or 100, and with four axes it is 1000, and so on.

Let e have some value between 10 and 100; we observe that three axes will suffice, and that each wheel must have less than 60 teeth in order to reduce e from 100 to 60.

$$\text{Thus } e = \frac{48}{6} \times \frac{45}{6} = 60.$$

Again, let $e = \frac{365}{3} = 121\frac{2}{3}$, and suppose 180 and 12 to be the limiting numbers of teeth upon a wheel and pinion respectively. In the train which is about to be composed, we shall now find that this extension of the limits of the numbers of teeth upon the respective wheels and pinions will give us the power of arranging the train without increasing the number of axes.

$$\text{Here } \frac{180}{12} = 15,$$

$$\text{and } \frac{180}{12} \times \frac{180}{12} = 15 \times 15 = 225.$$

Now $121\frac{2}{3}$ is less than 225, and therefore three axes will suffice, as in the train represented by

$$e = \frac{180}{18} \times \frac{146}{12}.$$

We may work out this arithmetical reasoning by the use of symbols, and then our solution will apply to every case which can occur.

Assume now that p represents the least number of teeth upon a pinion, w the greatest number upon a wheel, and let x represent the number of fractions in e.

If all the fractions making up the value of e were equal to each

other and had the greatest admissible value, then e would reach its limiting value, and we should have

$$e = \frac{w}{p} \times \frac{w}{p} \times \frac{w}{p} \text{ . . . . to } x \text{ factors} = \left(\frac{w}{p}\right)^x,$$

$$\text{whence } \log e = x(\log w - \log p),$$

$$\therefore x = \frac{\log e}{\log w - \log p}.$$

Now x will probably be a fraction, in which case the next integer greater than $x + 1$ will represent the required number of axes.

Ex. Let $e = \frac{365}{3}$, $w = 180$, $p = 12$,

$$\therefore \frac{w}{p} = 15 \quad \therefore x = \frac{\log 365 - \log 3}{\log 15},$$

$$= 1 + \text{a fraction.}$$

Now the integer next greater than $x + 1$ is 3, therefore 3 axes will be required.

We observe that it is not necessary to find the actual value of x, but simply to ascertain the integer next greater than it.

ART. 176.—It is sometimes a matter of enquiry how often any two given teeth will come into contact as the wheels run upon each other. We will take the case of a wheel of A teeth driving one of B teeth where A is greater than B, and let $\frac{A}{B} = \frac{a}{b}$ when reduced to its lowest terms.

It is evident that the same points of the two pitch circles would be in contact after a revolutions of B or b revolutions of A.

Hence the smaller the numbers which express the velocity ratio of the two axes, the more frequently will the contact of the same pair of teeth recur.

1. Let it be required to bring the same teeth into contact *as often* as possible.

Since this contact occurs after b revolutions of A or a revolutions of B, we shall effect our object by making a and b as small as possible, that is, by providing that A and B shall have a large common measure.

Ex. Assume that the comparative velocity of the two axes is

intended to be nearly as 5 to 2. And first make $A = 80$, $B = 32$, in which case we shall have

$$\frac{A}{B} = \frac{80}{32} = \frac{5}{2} \text{ exactly,}$$

or the same pair of teeth will be in contact after five revolutions of B, or two revolutions of A.

2. Let it be required to bring the same teeth into contact *as seldom* as possible.

Now change A to 81, and we shall still have $\frac{A}{B} = \frac{5}{2}$ very nearly, or the angular velocity of A relatively to B will be scarcely distinguishable from what it was originally. But the alteration will effect what we require, for now $\frac{A}{B} = \frac{81}{32}$, which is a fraction in its lowest terms. There will therefore be a contact of the same pair of teeth only after 81 revolutions of B or 32 revolutions of A.

The insertion of a tooth in this manner was an old contrivance of millwrights to prevent the same pair of teeth from meeting too often, and was supposed to ensure greater regularity in the wear of the wheels. The tooth inserted was called a *hunting cog*, because a pair of teeth, after being once in contact, would gradually separate and then approach by one tooth in each revolution, and thus appear to hunt each other as they went round

The clockmakers, on the contrary, appear to have adopted the opposite principle.

Finally, we would remind the reader that everything which we have said here about wheels in trains is true, whatever be the directions of their axes. We only care to know the relative sizes of the pitch circles and the directions in which they turn : any part of the train may be composed of *bevel wheels* without affecting our results.

CHAPTER VII.

AGGREGATE MOTION.

ART. 177.—We have seen that every case of the curvilinear motion of a point is of a compound character, resulting from the superposition of two or more rectilinear motions.

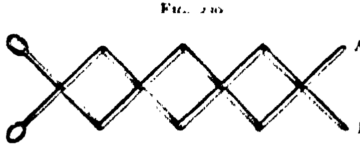
It often happens in machinery that some revolving wheel or moving piece becomes the recipient of more than one independent motion, and that such different movements are concentrated upon it at the same instant of time.

The motion is then of a compound or aggregate character, and we propose to classify under the head of '*Aggregate Motion*' a large variety of useful contrivances.

We commence with two or three simple examples.

The well-known frame called *Lazy Tongs* is a contrivance depending upon aggregate motion.

FIG. 210.



The rapid advance of the ends A and B is due to the fact that these points are the recipients of the sum of the resolved parts of the circular motion which takes place at each angle.

Consider the angular joints at the ends of the first pair of bars which carry the handles: these ends of the bars describe circles, just as the points of a pair of scissors would do. Either of these motions in a circular arc may be resolved as in Art. 7 and one of the components so obtained will be carried to the end of the combination. The same thing happens at every joint of the series, and thus A and B receive the aggregate of all these separate movements.

A wheel rolling upon a plane is a case of aggregate motion:

the centre of the wheel moves parallel to the plane, the wheel itself revolves about its centre, and these two simple motions give the aggregate result of rolling.

Thus, in the case of the driving-wheel of a locomotive, each point on the tyre becomes a fulcrum upon which the rest of the wheel turns, and is for an instant absolutely at rest. The centre of the wheel has the velocity of the train, while a point in the upper edge moves onward with twice that linear velocity. Simple as this matter is, it puzzles some persons when they first think about it.

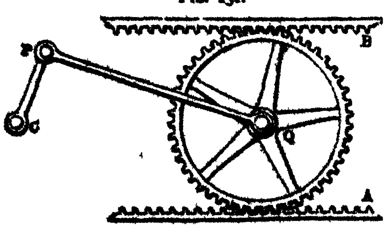
In the same way, if a beam of timber be moved longitudinally upon friction rollers, the travel of the beam will be twice as great as that of the rollers.

So, again, in moving heavy guns, the men employ what is called a *wheel purchase*; that is, they fasten one end of a rope to the spoke of a wheel of the gun-carriage, and make the rope run round the rim. This gives them the leverage of the spokes of the wheel, and the power exerted is exactly one-half of what it would be if the rope were attached directly to the axis of the wheel, in virtue of this principle that the linear velocity of the upper part of the rim is twice that of the centre of the wheel.

In some printing machines the table is driven by a crank and connecting rod, and the length of its path may be doubled by applying the principle under discussion.

Here a wheel, Q, is attached to the end of the connecting rod PQ, so that it can turn freely on its centre, Q.

FIG. 231.



Let the wheel revolve between the two racks A and B, whereof A is fixed to the framework of the machine, while B carries the reciprocating table.

The rack B receives the motion of Q in its twofold character, and moves through exactly twice the space that it would describe if connected simply with the point Q.

The size of the wheel makes no difference in the result, for in

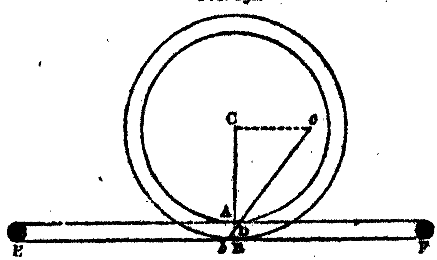
all cases the velocity of a point in the upper edge will be twice that of the centre.

ART. 178.—We may confirm our views of the nature of rolling motion by seeing what would happen if the fulcrum, round which the wheel turns, were raised above the level of the road.

We have now a contrivance by which a carriage may be made to move faster than the horse which draws it, a startling method of stating the fact which has been sometimes adopted. The inventor was a Mr. Saxton, who patented a *Differential Pulley* in the year 1832 (No. 6,351), with a view of obtaining great speed in railway carriages propelled by a rope. By the use of this invention, the consumption of the rope, proposed to be wound up at a stationary engine house, would be much less than if the carriage were attached in the ordinary way.

Let two wheels of different diameters (say as 6 to 7) be centred on a common axis at C, and be fastened together, and let an

FIG. 232.



endless rope be wound round the wheels and pass over pulleys at E and F in the manner shown in the diagram, the rope taking a turn round each of the pulleys.

Conceive now a pull to be exerted on the rope at A, in the direction AF, then the tension of the string will cause an equal and opposite pull to be felt at B in the direction BE, and thus the compound pulley has a tendency to turn about D, the middle point of AB.

This tendency in the pulley to turn about the point D causes the linear motion of C to be very much greater than that of any point in the rope: for example, when B moves through a small

space B*b*, the centre C will advance through C*c*, which upon our supposition is thirteen times as great, so that when one yard of rope is wound up, the carriage will have travelled through 13 yards.

The carriage may be at once stopped by disconnecting the pulleys.

ART. 179.—The *differential screw* is another instance of aggregate motion, and is a favourite with writers on mechanics, inasmuch as it gives theoretically a mode of obtaining an enormous pressure by the action of a comparatively small force.

It is constructed on the following principle: two screw threads of different degrees of inclination are formed upon the same spindle AB, the spindle itself passing through two nuts, whereof one, E, is part of a solid frame, and the other, D, can slide in a groove along the frame. Let P, Q represent the pitches of the screws at E and D; then upon turning AB once the nut D is carried forward through a space P, and is brought back again through a space Q: it therefore advances through the difference of these intervals. (Fig. 233.)

FIG. 233.

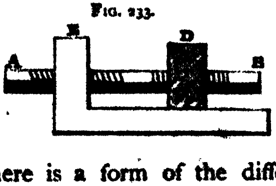
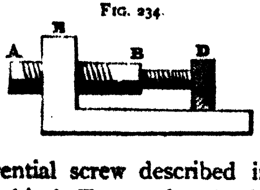


FIG. 234.



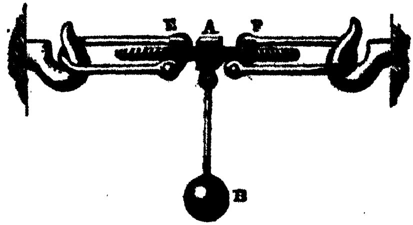
There is a form of the differential screw described in the fifteenth volume of the 'Philosophical Transactions,' which is known as *Hunter's Screw*. Here one screw is a hollow tube acting as a nut for the second screw in the manner shown in fig. 234. The smaller screw is attached to a piece D sliding in the frame, and is not allowed to rotate: upon turning the screwed pipe AB, the piece D will move through a space equal to the difference of the pitches of the two screw threads.

If one screw thread were right-handed and the other left-handed, the nut would travel through a space, P + Q, upon each revolution.

ART. 180.—A right and left-handed screw are often seen in combination, for the purpose of bringing two pieces together.

There is a very common instance in the coupling which is used to connect two railway carriages. Upon swinging round the arm AB, the screws which are moved by it bring the nuts E and F at the ends of the coupling links closer together, or cause them to separate. This is obviously a most convenient arrangement.

FIG. 235.



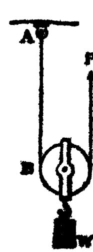
The lever arm and weight at B serve a two-fold purpose: they enable the railway servant to screw up the combination easily, so as to put a pressure upon the buffer-springs, and the weight B prevents the screws from shaking loose during the running and vibration of the train.

There is another instance of the use of a right and left-handed screw in combination which is found in the valve-motion of Nasmyth's steam-hammer.

Here a right and left-handed screw are placed side by side, and are connected by spur-wheels so that they rotate in opposite directions. Two nuts fastened together engage with the separate screws, and both rise and fall at the same time, being both advanced in the *same* direction by screws which rotate in *opposite* directions.

ART. 181.—Any system of pulleys will form an example of aggregate motion.

FIG. 236.



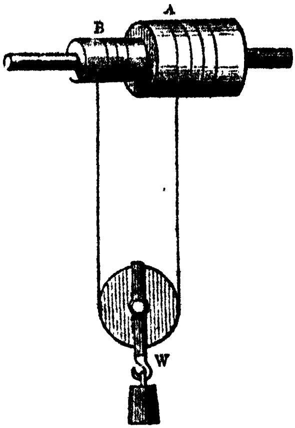
Taking the single movable pulley in fig. 236, it is apparent that when W is raised one inch, the centre of the block rises an inch, and therefore the end P of the line DP is shifted *one* inch.

But at the same time the circular sheave of the pulley runs upon the line AB, just as a wheel runs upon a plane, and by turning on its centre until an additional inch of string has come in contact with it, will transfer the end P through another space of one inch, whereby, on the whole, P moves through *two* inches.

ART. 182.—Another contrivance for lifting heavy weights by a small expenditure of power is the *Chinese Windlass.*

Here a rope is coiled in opposite directions round two axles A and B, of unequal size: the rope is consequently unwound from one axle while it is being wound up by the other, and the weight may rise as slowly as we please.

FIG. 237.



Let R, r be the radii of the axles, then W moves through $\pi\ (R-r)$ upon each revolution of the axles.

The practical objection to this windlass consists in the great length of rope required during the operation.

In the ordinary windlass the amount of rope coiled upon the barrel represents the height through which the weight is raised, whereas here we begin by winding as many coils on the smaller barrel as the number of turns which we intend to make with the winch-handle, and then at the close of every turn a length of rope equal to $2\pi R$ is coiled upon the larger barrel, by which expenditure the weight has only been lifted through $\pi\ (R-r)$.

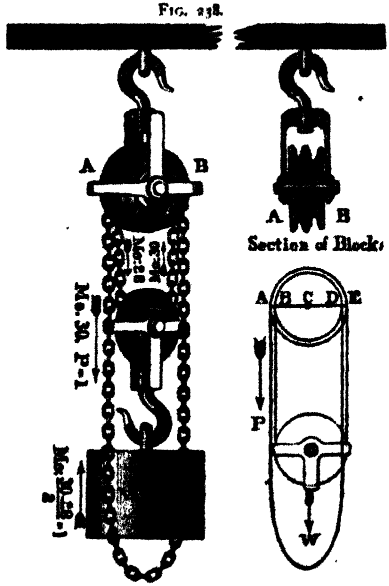
Ex. Let $R = 11$, $r = 10$, then the amount of rope wound up in any number of turns bears the same proportion to the space through which the weight is raised that 22 bears to 1.

This is a sufficient commentary on the invention regarded as a practical contrivance.

ART. 183.—The object of *Weston's Differential Pulley-block* is to avoid this difficulty about the expenditure of rope. In the Chinese Windlass, one end of the rope is supposed to be fastened to the axle A, and the other end to the axle B. If, however, these two ends were brought together, the supply of rope necessary for B might be drawn from that coiled upon A, and the expenditure would be really $2\pi\ (R-r)$. There would be many inconveniencies attending this arrangement in practice, but it has been put into a working shape in the manner shown in the drawing.

In Weston's pulley-block there are two pulleys A and B, nearly equal in size, turning together as one pulley, and forming the upper block : an endless chain supplies the place of the rope, and must of course be prevented from slipping by projections which catch the links of the chain. The power is exerted upon that portion of the chain which leaves the larger pulley, the slack hangs in the manner shown in the sketch, and the chain continues to run round till the weight is raised. The combination is therefore highly effective.

FIG. 238.



Of the detached sketches, one shows a section of the differential block, and the other is intended to explain the mechanical principle involved.

Let C be the centre of the compound block, draw the horizontal diameter ABCDE, and let it be noted that the string or chain is unable to slip upon the surface of either pulley.

Let P represent the tension of AP,

W the weight raised, T the tension of the string or chain at E.

Also let R, r, be the radii of the respective blocks.

$$\text{Then} \quad T \times CE = P \times AC + T \times BC,$$

$$\text{or} \quad T \times R = P \times R + T \times r \quad . \; . \; . \quad (1)$$

$$\text{Also} \quad 2T = W \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

$$\therefore \quad W(R-r) = 2P \times R,$$

$$\text{or} \quad P = \frac{W(R-r)}{2R}.$$

Ex. Let $R = \frac{15}{2}$, $r = \frac{14}{2} = 7$.

$$\therefore \quad P = \frac{W(15-14)}{2 \times 15} = \frac{W}{30},$$

$$\text{or} \quad W = 30P.$$

Regarding the question as an application of the principle of work, the diagram sets out the calculation as follows:—

Since the diameter of the pulley A is 15, we shall assume that a point in the chain passing over A moves through a space 30, or that P has a motion 30. Also the diameter of B is 14, whence it follows that the chain passing over B will have a motion 28 in the opposite direction. *Note.*—In fig. 238, taken from the 'Anderson' series, the symbol 'Mo.' stands for the word 'motion.'

Hence the motion of the chain round the pulley which supports W is (30 − 28), and the motion of W itself is $\frac{30-28}{2} = 1$.

Hence motion of W : motion of P :: 1 : 30,

or W : P :: 30 : 1.

ART. 184.—The subject of *Epicyclic trains* will now occupy our attention, and we shall discuss some of the most useful applications of that peculiar arrangement of wheelwork which is technically so designated.

An epicyclic train differs from an ordinary train in this particular: the axes of the wheels are not fixed in space, but are attached to a rotating frame or bar, in such a manner that the wheels can derive motion from the rotation of the bar.

There are certain fundamental forms which consist of trains of two or three wheels; the first wheel of the train is usually concentric with the revolving arm, and the last wheel may be so likewise.

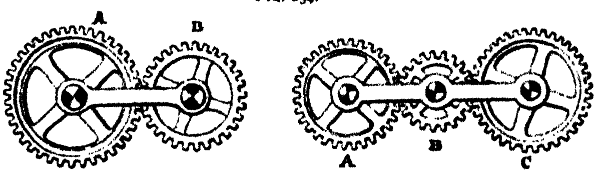
It should, however, be understood that any number of inter-

mediate wheels may exist between the first and last wheels of the train, and that the wheels in the train may derive the whole of their motion from the arm ; or they may receive one portion from the arm and the remainder from an independent source.

The elementary form of a train is exhibited in the annexed diagram, and the peculiarities which result from compounding any independent motion with that which arises from the rotation of the arm will demand some careful and attentive study.

Here it will be seen that the wheel B, or the wheels B and C, are attached to a bar which is capable of revolving about the centre

FIG. 230.



of the wheel A, the axis of this latter wheel being firmly held in one position.

ART. 185.—In order to understand movements of this kind let us take a simple case to begin with.

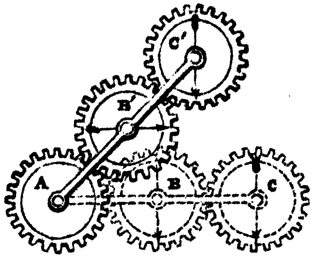
Suppose that there were only two wheels in the train, viz., A and B, and let A be locked so that it cannot rotate ; suppose, further, that A has 45 teeth, and that B has 30 teeth, and let us inquire how many rotations B will make while the arm is carried round once.

We might at first imagine that the wheel B would rotate $\frac{45}{30}$ or $\frac{3}{2}$ times by running round upon A ; but this is only a part of its movement. The wheel B has also been carried round in a circle about A by reason of its connection with the arm, and having turned upon its axis once more on that account, it has really made $\frac{5}{2}$ turns, instead of $\frac{3}{2}$, during one revolution of the arm.

In confirmation of this view, let us consider the case of three wheels, A, B, and C, whereof A and C are equal. As the arm goes round, we conclude that C will turn once in the opposite direction to the arm by the rolling of the wheels, and it will turn once in the same direction as the arm by reason of its connection

therewith; the aggregate result being that C will be carried round in a circle without rotating at all upon its own axis.

FIG. 240.



The motions of the wheels B and C in an epicyclic train are shown in the sketch. The arm is supposed to have revolved through an angle of 45°, and it will be seen that B has turned round through a right angle, while C has not rotated at all.

We propose now to examine the motion by the aid of analysis.

Remembering that there may be any number of wheels in the train, of which A is the first, and L the last wheel.

$$\left.\begin{array}{l}\text{Conceive that the arm makes } a \text{ revolutions} \\ \text{the first wheel A makes } m \text{ revolutions} \\ \text{the last wheel L makes } n \text{ revolutions}\end{array}\right\}\begin{array}{l}\text{during the same} \\ \text{period of time,}\end{array}$$

and let e be the *value* of the train.

Then the first wheel makes $(m-a)$ revolutions relatively to the arm, and the last wheel makes $(n-a)$ revolutions relatively to the same arm, or, in other words, L makes $(n-a)$ revolutions for $(m-a)$ revolutions of A.

Recurring to our definition of the *value* of a train (see Art. 163), we at once deduce the equality

$$e=\frac{n-a}{m-a}.$$

There are three principal cases to consider:—

1. Let A be fixed, or $m=0$,

$$\therefore e=\frac{n-a}{-a}=-\frac{n}{a}+1,$$

$$\text{or } n=a\,(1-e) \text{ and } a=\frac{n}{1-e}.$$

2. Let L be fixed, or $n = 0$,

$$\therefore e = \frac{-a}{m-a}$$

$$\text{or } m = a\left(1 - \frac{1}{e}\right) \text{ and } a = \frac{me}{e-1}$$

3. Let neither A nor L be fixed,
we have now the formula

$$e = \frac{n-a}{m-a},$$

$$\text{whence} \quad em - ea = n - a,$$

$$\text{or} \quad n = me + (1-e)a.$$

In applying these formulæ we must remember that e is positive when the train consists of 3, 5, or an odd number of wheels, and negative when there are 2, 4, or an even number of wheels.

Ex. 1. Let there be two equal wheels, A and B, in the train, and conceive A to be locked, or let A be a *dead wheel*, as it is termed.

$$\text{Here} \quad m = 0, \text{ and } e = -1,$$

$$\therefore n = a(1+1) = 2a,$$

or the wheel B makes two rotations for each revolution of the arm.

Ex. 2. Let there be three wheels, A, B, and C, whereof A and C are equal, and let A be a dead wheel as before.

$$\text{Here} \quad m = 0, e = 1,$$

$$\text{whence} \quad n = a(1-e) = a(1-1) = 0,$$

or C does not turn on its axis at all.

Ex. 3. Take the case first considered, where A is a dead wheel and has 45 teeth, and where B has 30 teeth.

$$\text{Here} \quad e = -\frac{3}{2},$$

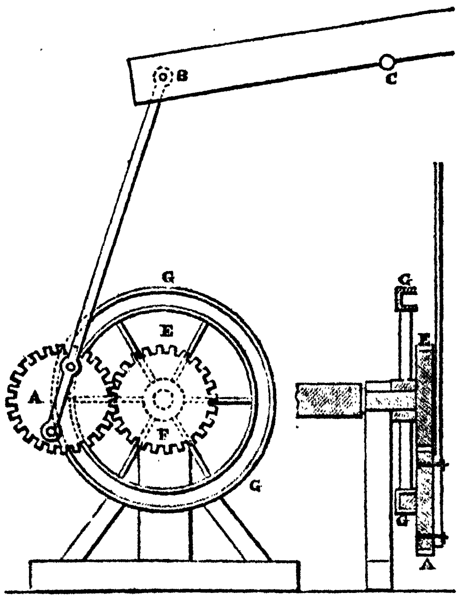
$$\therefore n = a\left(1 + \frac{3}{2}\right) = \frac{5a}{2},$$

the result arrived at by general reasoning.

ART. 186.—The *Sun and Planet Wheels* were invented by Watt, and were used to convert the reciprocating motion of the working beam of an engine into the circular motion of the fly-

wheel. We have already referred to this invention in Art. 37, and have explained the object which it was intended to fulfil.

FIG. 241.



The drawing shows Watt's invention as specified in a patent of 1781 (No. 1,306). CB is the working beam, and AB is the spear or connecting rod; E is a wheel fixed upon the end of the shaft or axis F, which receives the rotatory motion which is communicated to it by a second wheel, firmly fixed to AB in such a manner that it cannot rotate. Behind EB there is a heavy wheel, GG, having a groove or circular channel around its circumference, into which a pin at the back of A enters. The wheels A and E are thus kept in gear, and some such precaution is indispensable, but instead of the wheel with the groove and pin there may be a link connecting A and F. The construction having been described, the specification states that in the working of the engine the connecting rod pulls the wheel A up and down; and since its teeth

are locked in the wheel E, and it cannot turn upon its own axis, it cannot rise or fall without causing E to turn upon the axis F. When the two wheels A and E have equal numbers of teeth the wheel E makes *two* revolutions on its axis for each stroke of the engine.

We may explain the peculiarity as follows. If the discs E and A were fastened together at the point a, and E were to make half a revolution, A would come into the position A′, and the direction of the arrow marked upon it would be reversed. But in the actual motion this arrow retains its first direction, and in order to recover it, the disc A′ must again rotate through 180°, and must carry E round through another half-revolution: so therefore when we recur to the arrangement invented by Watt, E will make a complete revolution while A descends from the highest to the lowest position, or travels half-way round it.

Fig. 242.

If we were to apply our formula $\left(e = \frac{n-a}{m-a}\right)$ we should make L the dead wheel, in which case $n = 0$, and $e = -1$,

$$\therefore \quad -1 = \frac{-a}{m-a},$$

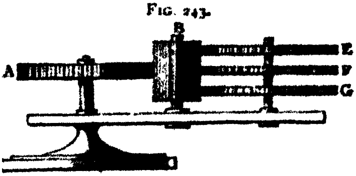
$$\therefore \quad m - a = a, \text{ or } m = 2a,$$

which is the result already arrived at.

Art. 187.—*Ferguson's Paradox* is obtained by placing three wheels upon the axis which usually carries C, and making these wheels very nearly equal to each other, and very nearly equal to A.

Thus let A have 60 teeth, and let the numbers of teeth upon E, F, G, be 61, 60, 59, respectively.

Fig. 243.



The number of teeth upon B is immaterial, and the wheel A is fixed to the stud upon which it rests, and does not rotate with the arm, so that $m = 0$ throughout the motion.

1. Taking the general formula $e = \frac{n-a}{m-a}$, we have, in the case of the wheel E,

$$m = 0, \text{ and } e = \frac{60}{61}, \text{ which is less than unity.}$$

$$\therefore \frac{n-a}{-a} = \frac{60}{61}$$

$$\therefore 61\,n - 61\,a = -\,60\,a,$$

$$\text{whence} \quad n = \frac{a}{61}, \text{ and is positive.}$$

2. For the wheel F; $e = \frac{60}{60} = 1$,

$$\therefore n - a = -a, \text{ or } n = 0.$$

3. For the wheel G; $e = \frac{60}{59}$, which is greater than unity,

$$\therefore \frac{n-a}{-a} = \frac{60}{59}$$

$$\therefore 59n = -60a + 59a,$$

$$\text{whence } n = -\frac{a}{59}, \text{ and is negative.}$$

So that when the arm is made to revolve round the locked or dead wheel A, the wheel E turns slowly in the same direction as the arm, F remains at rest, and G moves slowly in the reverse direction. This combination has formed a rather popular mechanical puzzle.

The general result deducible from Ferguson's paradox is the following :—

Let there be a train of three wheels, viz., A, B, and C, and let A be a dead wheel, and *greater* than C, then the rotation of the arm in a given direction will cause C to rotate in the opposite direction in space.

Whereas when A is a dead wheel and *less* than C, the rotation of C will take place in the same direction as that of the arm.

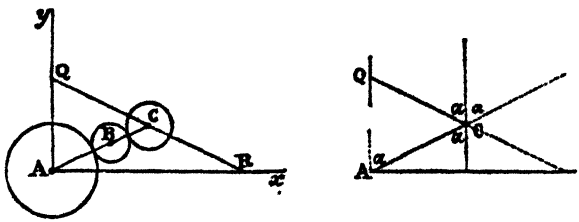
ART. 188.—*Problem.* Let it be required to draw an exact straight line by means of an epicyclic train.

Conceive that ACQ represents a bar made up of two equal straight lines AC, CQ, the point A being a centre of motion, and the point C representing an ordinary rule joint.

Let ACQ be placed so as to coincide with the straight line Ay. Then it has been proved that if AC turns through an angle in one direction, the while CQ turns through the same angle in the opposite direction, the point Q will trace out a portion of the straight line.

The student should refer to Art. 117, where it is shown that if Ax be drawn at right angles to Ay, and QC be produced to meet Ax in R, the triangle QAR lies always in a semicircle whose centre is C, and the property above stated is a necessary consequence of the sliding of QR between Ax and Ay.

FIG. 244.



It only remains for us to demonstrate that the required motion of CQ may be obtained by mounting that line upon the last wheel of an epicyclic train with three axes.

Conceive that three wheels, viz., A, B, and C, are mounted on the arm AC, and are mutually in gear.

Let it be arranged that the axis of the first wheel shall coincide with A, and that of the last wheel with C, and let the diameter of C be half that of A. The size of B is immaterial.

We have now to apply the formula $e = \dfrac{n-a}{m-a}$.

Here there are three wheels, and the number of teeth on C is half that on A, whence $e = 2$.

Also let A be a dead wheel, therefore $m = 0$.

$$\therefore 2 = \frac{n-a}{0-a}, \text{ or } n = -a,$$

whence C rotates in a backward direction at the same rate as the arm rotates in a forward direction.

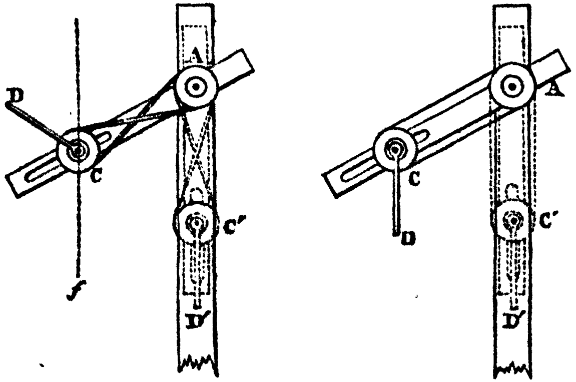
It follows that the point Q will describe the straight line Ay.

In like manner if QC be produced to R, such that CR=CQ, it is apparent that the point R will describe the straight line Ax, which is perpendicular to Ay. Thus the extremities of a straight line QR, which is carried by the wheel C, and bisected at the centre C, will describe straight lines intersecting at a right angle.

As soon as the nature of the primary motion is understood, the proposition now considered becomes a simple deduction therefrom.

ART. 189.—It will of course be understood that models may be readily constructed for illustrating the fundamental propositions in this branch of mechanism without employing toothed wheels. As an example we refer to the diagram, which is taken from a model and is intended to exhibit the results obtained by a train of two or three spur wheels.

FIG. 245.



It will be seen that there is a fixed upright pillar carrying an arm AC, centred at A, and having upon it two round discs or pulleys, A and C. These pulleys are equal, but they may be unequal and of any convenient dimensions, and they are further connected either by crossed or open bands.

When the band is crossed we have an equivalent for two spur wheels in gear, and when it is open the combination is the same as that of three spur wheels.

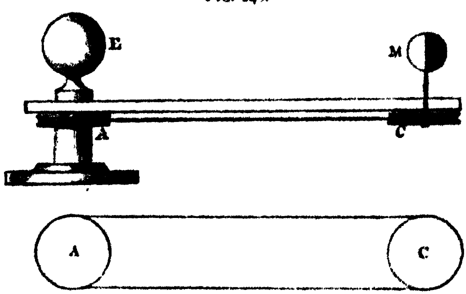
Taking the case where A is a dead wheel and equal to C, the result of moving the arm through a given angle is shown in the sketch. The wheel C carries a pointer D, and the faint dotted lines show the starting position of the arm and of the pointer. When the strap is open, CD remains vertical throughout the motion, but when the strap is crossed it rotates in the same direction as the arm with a velocity ratio of 2 to 1. It follows that the angle DC*f* is equal to twice the angle CAC'.

ART. 190.—Numerous models, designed for illustrating simple astronomical problems, may be formed by properly arranged discs and bands.

Thus, it is quite easy to exhibit mechanically the phases of the moon.

For this purpose a small silvered ball M, representing the moon, is attached by a pipe or hollow stem to a bar capable of being carried in a horizontal plane round a fixed ball, E, intended to represent the earth. Underneath the arm are the driving

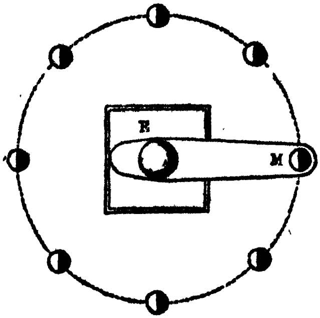
FIG. 246.



pulleys, which consist of (1) a dead wheel A, and (2) a pulley C, equal to A, and connected with it by an open band. A hemispherical black cap, which obscures exactly one half of the surface of M, is attached to a wire passing through both M and the hollow stem, and fastened to C, so as to form part thereof, and to move with it. The sun is supposed to lie on the left-hand side of the diagram, and as the moon is carried round E, the shaded portion lies always to the right hand. It is apparent that the motion of the cap, or of C which directs it, should be that of the third wheel

F in Ferguson's paradox, for which purpose we require that A should be equal to C, and that the band connecting them should be open, the wheel A being a dead wheel.

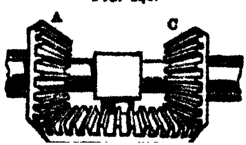
Fig. 247.



As the arm revolves, the disc C moves round in a circular path without at all rotating upon its own axis, and the hemispherical cap takes the various positions shown in the sketch, imitating thereby the shadow which would be caused by a luminous body at a great distance to the left of the globe E.

Art. 191.—An epicyclic train may also be formed by the use of three bevel wheels, A, B, and C, connected as in the figure, and we find now the peculiarity that the wheels A and C turn in opposite directions.

Fig. 248.



The formula already investigated applies equally in this case, and some of the results to be obtained are extremely useful.

Our first example shall be an arrangement whereby the continuation of a piece of shafting may be made to rotate twice as fast as the first portion of it. This forms a simple and easy method of obtaining an increased velocity in a revolving piece, and is used to rotate the coils in some magneto-electric machines.

Thus, let A be a dead wheel, and let B ride loose upon an arm which itself is rigidly attached to the first portion of the shaft, namely, that passing through A, the wheel C being keyed to the other portion which is required to revolve with increased rapidity. As the arm carrying B goes round A, we can easily see that if B were not allowed to rotate at all it would still carry C round once, and that its rotation upon the dead wheel A carries C round a second time, and thus we have an exact reproduction of

the motion of the two equal spur wheels, one of which is a dead wheel, and obtain two rotations of C for each revolution of the arm carrying the intermediate wheel B.

We may of course apply the general formula in the case of bevel wheels just as in that of spur wheels, and the expression $e = \frac{n-a}{m-a}$ gives the result obtained before.

Thus $m = o$, $e = -1$, since A and C revolve in opposite directions.

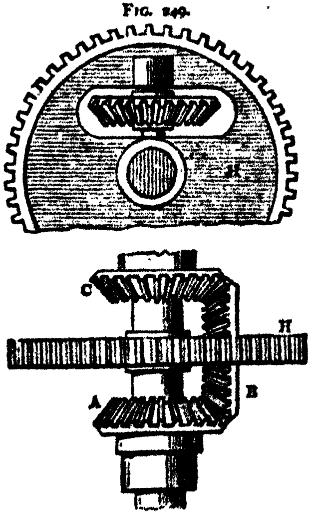
$$\therefore\ -1 = \frac{n-a}{-a},\ \text{or}\ n = 2a,$$

whence C goes round twice for each revolution of the arm.

ART. 192.—An illustration may now be taken from the cotton mills of Lancashire.

During the process of the manufacture of cotton yarn or thread, it is essential to wind the partially twisted fibre upon bobbins, and at the same time this fibre, or *roving*, must not be subjected to any undue strain.

FIG. 249.



The fibre is delivered to the bobbins at a uniform rate, whereas the bobbins get larger as they fill with the material, and hence the winding machinery must be so contrived that the rate of revolution of the bobbin shall slowly decrease upon the completion of each layer of the fibre.

In the year 1826 Mr. H. Houldsworth patented an invention which solves the problem of the *bobbin motion* in the most complete and satisfactory manner.

In the preceding article we have supposed the wheel B to be carried by an arm which is capable of revolving round the axis AC. The better way, however, of suspending B for our purpose is to attach it to the face of a spur wheel, H, as in fig. 249.

Let this be done, and let A be connected with the driving shaft of the engine, so that its rotation shall necessarily be constant.

If now some independent motion be imparted to the wheel H, the result may be calculated from the formula.

Here A, B, C are equal in size, and C rotates in a direction opposite to that of A,

$$\therefore \quad e = -1,$$
$$\therefore n - a = a - m,$$
$$\therefore n + m = 2a,$$

which gives the analytical relation between the angular velocities of A, C, and H.

If we examine this formula, we shall comprehend that the velocity of C may be reduced by altering the velocity of H.

1. For let $a = m$, or let A and H turn at the same rate,

$$\text{then } n + m = 2\,a = 2m,$$

$\therefore n = m$, or C has exactly the same motion as A.

2. Let $a = \frac{3m}{4}$, that is, let H make three revolutions while A makes four,

$$\therefore n + m = 2 \times \frac{3m}{4} = \frac{3m}{2},$$

$\therefore n = \frac{m}{2}$ or C moves half as fast as A.

3. Let $a = \frac{m}{2}$, in which case H makes one revolution for two revolutions of A,

$$\therefore n + m = 2 \times \frac{m}{2} = m,$$

$\therefore n = 0$, or C stops altogether.

We have taken extreme cases, from which it appears that when the velocity of the arm is made less than that of A, the velocity of C is reduced in a twofold degree.

Generally, let $\qquad a = m - \frac{m}{x},$

$$\therefore n = 2a - m = 2m - \frac{2m}{x} - m = m - \frac{2m}{x},$$

or the rate of diminution of n is twice that of a.

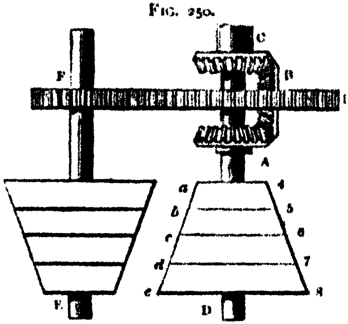
It now becomes easy to obtain any required reduction in the

velocity of C. A reduction in the velocity of H must first be effected by shifting a driving strap along a conical pulley, and the velocity of C will be reduced twice as much as that of H.

Mr. Houldsworth's invention consists, therefore, in imparting to the wheel C two independent motions which travel by different routes, and which, after combination in the manner just investigated, are capable of producing the desired *differential motion.*

ART. 193.—In order to fix our ideas, let us calculate the motion in the following example:—

FIG. 250.



Suppose A, B, C to represent three equal wheels, and let A be fixed to a shaft AD, which carries a conical pulley provided with grooves at *a*, *b*, *c*, *d*, *e*, where the diameters are 4, 5, 6, 7, 8.

EF is another shaft carrying a second conical pulley which is the counterpart of the first, and terminating in a wheel F, whose diameter is half that of H.

A crossed band connects the two cones, and the axis AD is made to revolve with a uniform velocity.

It is required to ascertain the motion of C when the strap is shifted along the conical pulley.

1. Let the strap be placed at *a*, the angular velocity of H will be $\frac{1}{4}$ that of AD, and we have $a = \frac{m}{4}$,

$$\therefore \ n = 2 \times \frac{m}{4} - m = -\frac{m}{2},$$

or C moves in the opposite direction to A, and with half its velocity.

2. Let the strap be at *b*, the velocity of H will be $\frac{5}{14}$ that of AD (here $e = \frac{5}{7} \times \frac{1}{2} = \frac{5}{14}$ according to Art. 163),

$$\text{therefore } a = \frac{5m}{14}, \text{ and } n = \frac{5m}{7} - m = -\frac{2m}{7},$$

hence C still moves in the opposite direction to A, but less rapidly, in the ratio of 2 to 7.

3. Place the strap at *c*, when *e* increases to $\frac{1}{2}$, and *a* becomes equal to $\frac{m}{2}$, $\therefore n = 2 \times \frac{m}{2} - m = 0$,

or C stops altogether, its motion being entirely destroyed.

4. Place the strap at *d*, and we have $a = \frac{7m}{5} \times \frac{1}{2} = \frac{7m}{10}$,

$$\text{whence } n = 2a - m = \frac{7m}{5} - m = \frac{2m}{5},$$

that is, C and A move in the same direction with velocities in the ratio of 2 to 5.

5. Finally adjust the strap at *e*, and the velocity of H will be the same as that of AD.

Here $a = m$, and $n = 2m - m = m$,

or the motion of C is precisely the same as that of A.

The principle of this invention may now be understood, although it is difficult to appreciate such a movement thoroughly without the assistance of a model.

It only remains to present to the student a representation of so much of an actual machine as will embody the cones and the differential train of wheels. The diagram exhibits the manner in which simple elementary movements may be combined together so as to form a train of mechanism, the arrangement of which, before it is properly understood, might appear to be very complex and intricate (see fig. 251.)

The operation of spinning, so far as it is carried on by the mechanism before us, is effected by passing a partially twisted fibre or *roving* through a tube, called a flier, attached to the end of a spindle, and then causing both the flier and the bobbin to rotate with a high velocity. Before the fibre reaches the fliers it is elongated or drawn out by a combination of rollers, moving at different speeds and called *drawing rollers*; it is therefore of necessity fed on at a fixed uniform rate.

The flier and the bobbin both rotate together, and thus twist the roving, but they also rotate at somewhat different speeds, by which arrangement it is provided that the joint operations of twist-

ing the thread and of winding it up upon the bobbin shall go on together.

A bobbin with its spindle and flier is shown in the sketch. It will be seen that the roving passes down through the hollow vertical arm and is carried to the bobbin by a finger; the finger is pressed against the bobbin by the centrifugal action of a small elongated piece which runs down the side of the arm, and which, by its tendency to get as far as possible from the axis of the spindle during its rotation, keeps the finger pressed against the surface of that portion of roving which is already wound upon the bobbin. This part of the apparatus has formed the subject-matter of a most lucrative invention.

As the winding goes on the bobbin rises and falls, and the flier winds the fibre in uniform layers upon the bobbin.

Thus the spindle and flier rotate together, and they are driven by skew-bevels, whereof one is shown at the bottom of the drawing.

The bobbin rotates independently of the spindle, and is also driven by skew-bevels, whereof one is shown just underneath the bobbin. *Note.*—As to skew-bevels, see Art. 239.

These bevel wheels are in direct communication with the spur wheels marked 'to spindles' and 'to bobbins' in the drawing.

It will be understood that the winding on will take place when the spindles and the bobbins move at different velocities, and that either may go faster than the other. We shall take the case in which the bobbins precede the fliers.

Since the spindles with their fliers move at a fixed velocity, while the bobbins are continually filling with the rovings and becoming larger, we infer that the bobbins will require a smaller amount of rotation relatively to the fliers, in order that the winding up of the fibre, which is being fed on at a fixed rate by the drawing rollers, may take place uniformly. Hence, if the bobbin runs in advance of the flier, the speed of revolution has to be diminished as its diameter becomes larger.

Refer now to the sketch, and it will be seen that the power may pass through the combination of bevel wheels to the three spur wheels placed in a line at the extremity of the 'driving axis' and connected with the cone marked as the 'driver.' The driving power then crosses over to the follower, and enters the

combination of bevel wheels by the small pinion upon the axis of the lower cone which gears with the large spur wheel marked H, which latter wheel *rides loose* upon the driving axis.

The combination of four bevel wheels is exactly analogous to that discussed in Art. 191, the two wheels B and B are equivalent to a single wheel, and prevent the one-sided, unbalanced action which would otherwise occur.

The wheel A is fixed to the driving shaft, the wheel C rides loose upon it, but is fastened immovably to the spur wheel marked 'to bobbins,' the function of which has been already explained.

We have to prove that the combination of the two cones with the spur and bevel wheels is capable of gradually reducing the velocity of the bobbins as they fill up with the roving.

Assume that the cones are equal in section where the strap is placed, then the speed of the first cone will be reduced to $\frac{1}{2}$ by the combination of three spur wheels starting from the driving axis, and thus the pinion which drives H will move at $\frac{1}{2}$ the speed of the driving axis.

But H is five times as large as that pinion, hence the velocity of H is $\frac{1}{10}$th that of the driving axis.

The wheel H also rotates in the *opposite* direction to the driving axis.

Take now the formula $n = m\,e + (1 - e)\,a$.

Here $e = -1$, since A, B, and C are equal,

and $a = -\dfrac{m}{10}$ as we have just shown;

$$\therefore n = -m + 2\left(-\frac{m}{10}\right)$$
$$= -m - \frac{m}{5}$$
$$= -\frac{6m}{5}.$$

Hence the speed of the bobbin pinion is to that of the flier pinion as 6 to 5, or 18 to 15, the negative sign merely showing that the loose wheel C revolves in the opposite direction to the driving axis.

The student may be surprised to find that all this apparently reducing arrangement has ended in making the last spur wheel in

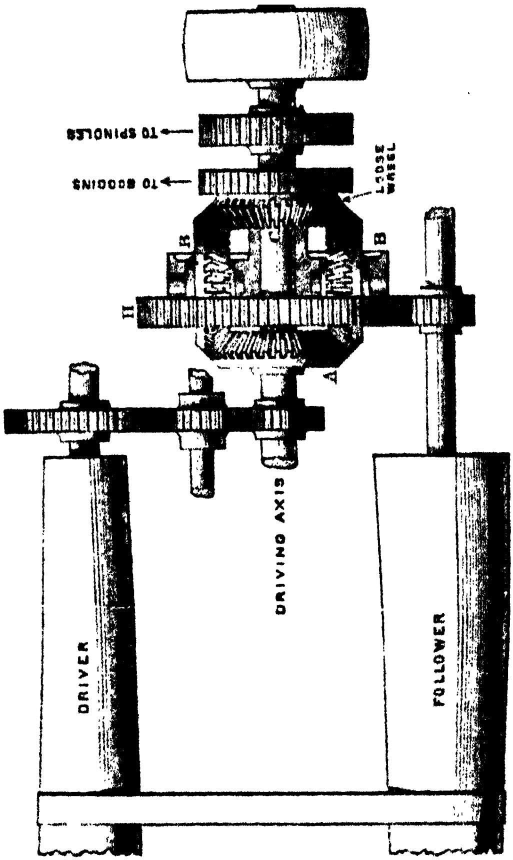
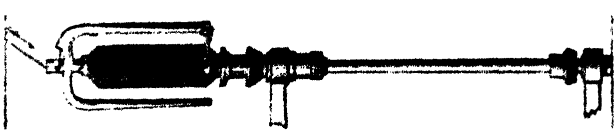

TO SPINDLES
TO BOBBINS
LOOSE WHEEL
B
B
H
A
DRIVING AXIS
DRIVER
FOLLOWER

FIG. 251



the train turn faster than the driving axis; but an explanation is found in the fact that the rotation of H takes place in the opposite direction to the driver, that is, in the same direction as the loose wheel C, and accordingly we shall find that if the velocity of H be reduced we shall also reduce the inequality between the velocity of the bobbins and spindles.

Conceive now that the strap is shifted towards the right hand until the sections of the cones are in the proportion of 2 to 3; that is, nearly as far as the spur wheels.

The velocity of H will be reduced two-thirds, and will become equal to $\frac{1}{15}$th that of the driving axis.

$$\therefore n = -m + 2\left(-\frac{m}{15}\right)$$
$$= -m - \frac{2m}{15}$$
$$= -\frac{17m}{15},$$

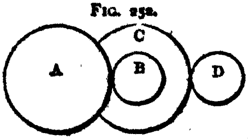
or the relative speed of the bobbin pinion to that of the flier pinion is reduced from 18 to 15, and now stands at 17 to 15.

It is hoped that the complete action of the apparatus is now sufficiently explained, and there is only one refinement in construction which remains to be pointed out. It will be seen that the upper cone is slightly concave and the lower one convex: this configuration is adopted because the absolute increase in the diameter of a bobbin bears a ratio to the actual diameter which is not constant, but is continually diminishing in a small degree. The mechanic must not forget or overlook any material point in working out his design.

ART. 194.—Epicyclic trains may be employed to produce a very slow motion upon the following principle:—

Let A, B, C, D represent the numbers of teeth in a train of wheels in gear arranged as in the diagram.

FIG. 232.



If A = D, and B = C, then A and D will rotate with the same velocity in the same direction; but if the equality between (A, D) and (B, C) be slightly disturbed, we shall produce a small change in the *value* of the train. Suppose, for example, that A is less than D,

or that A=31, D=32; and, again, that B is less than C, or that B = 125, C = 129: then e, the *value* of the train, will be

$$=\frac{AC}{BD}=\frac{31\times 129}{125\times 32}=\frac{3999}{4000}.$$

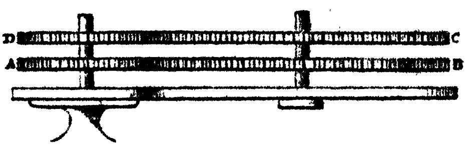
Also, the more nearly the equality is maintained between (A, D) and (B, C) respectively, the more nearly will the angular velocities of A and D be the same, or the more nearly will e be equal to unity.

Thus if B = D = 100, A = 101, C = 99,

$$\text{we have } e=\frac{101\times 99}{100\times 100}=\frac{9999}{10000}.$$

Let us now arrange A, B, C, D in an epicyclic train, and carry back the wheel D so that it shall turn upon the same axis as A. The turning of the arm will then set all the wheels in motion except A, which is to be made an immovable or dead wheel, and we shall have D and A moving relatively to each other just as before, that is to say, D will turn very slowly over A at rest.

FIG. 253.



As an easy example, take wheels of the following numbers viz., A = 60, B = 45, C = 40, D = 65.

$$\text{Then } e=\frac{AC}{BD}=\frac{60\times 40}{45\times 65}=\frac{32}{39}$$

$$\therefore 1-e=1-\frac{32}{39}=\frac{7}{39}.$$

If we now rotate the arm and carry round the train it will be found that D makes one revolution when the arm has been carried through a little more than $5\frac{1}{2}$ revolutions, which is also evident from the formula upon observing that $\frac{39}{7}=5\frac{4}{7}$, which is a little greater than $5\frac{1}{2}$.

So, again, taking the formula $\frac{n}{a} = 1 - e$, and substituting for e the values given previously, we have in the respective examples,

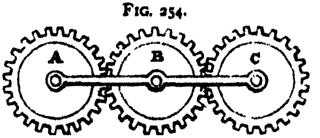
$$\frac{n}{a} = \frac{1}{4000}, \text{ or } \frac{n}{a} = \frac{1}{10000}.$$

Hence the arm will make 4000 or 10,000 revolutions respectively while the wheel D turns round once.

ART. 195.—These examples lead us to compare the movement of any wheel in an epicyclic train with that in another train where the axes are fixed in space, and to regard the subject from a different point of view.

Referring again to the fundamental case, viz., that of three equal wheels, A, B, and C, we have seen that if the arm be fixed, and A makes one turn, the wheel C will also turn once in the same direction. But if the arm revolve round A fixed, the wheel C will apparently run round just as it did upon the last supposition, and yet at the end of a revolution of the arm it will be found that the wheel C has not turned at all.

FIG. 254.



The explanation is that the fixed train gives the *absolute* motion of C due to its connection with A, whereas the epicyclic train exhibits the *relative* motion of C with regard to A, which in this case is nothing, because A and C rotate with equal velocities in the same direction.

The same thing is true with respect to any other wheel in the train, such as B. Thus, when the axes are fixed in space, A and B revolve in opposite directions, and the motion of B *relatively* to A is twice its absolute motion, and thus we account for the fact that in the epicyclic train B will rotate twice while the arm goes round once.

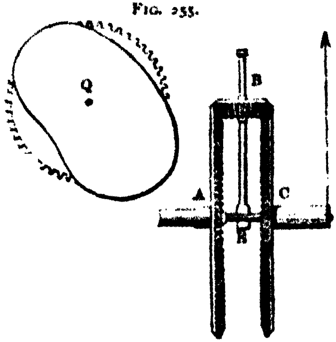
So also in Art. 194 the fixed train gives the absolute motion of D, viz., $\frac{3999}{4000}$ths of a revolution for each revolution of A, and the epicyclic train exhibits the relative motion of D as compared with that of A, viz., $\frac{1}{4000}$th of the movement of A in the fixed train.

ART. 196.—Another illustration of aggregate motion is found in *Equation clocks*. In these nearly obsolete pieces of mechanism

the minute hand points to true solar time, and its motion therefore consists of the equable motion of the ordinary minute hand plus or minus the *equation* or difference between true and mean solar time.

In clocks of this class the hand pointing to true solar time is fixed to the bevel wheel.

FIG. 255.



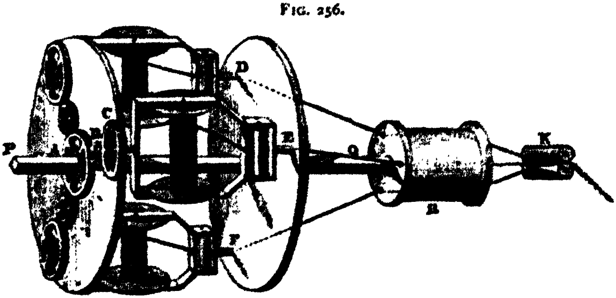
The wheel A moves as the minute hand of an ordinary clock; the intermediate wheel B is fixed to a swinging arm, EB, as in Art. 191, and the position of C will be in advance of that of A when EB is caused to rotate a little in the same direction, and behind that of A when EB is moved in the opposite direction.

Thus, as C goes round during each hour of the day, the hand attached to it may be a few minutes before or behind another showing mean time, and deriving its motion at once from A.

The required motion of EB is obtained from a cam plate, Q, curved as in the diagram, and attached to a wheel which revolves once in a year.

ART. 197.—In the manufacture of rope the operation of

FIG. 256.



'*laying*,' or twisting the strands into a perfect rope, has been effected by special machinery.

The Rev. Edmund Cartwright, the inventor of the power-loom, was also the first inventor of a machine for making rope. The general character of the contrivance will be understood from the sketch, which is taken from the specification of the invention.

The machine itself is called a '*Cordelier*,' and consists of a frame placed upon a horizontal shaft PQ, and terminating in a laying-block R, which serves the double purpose of directing the strands to the rollers at K, where they are twisted into rope, and of forming a support or bearing for one end of the shaft.

Three spool frames carry the bobbins, or spools, which contain the supply of strands, and the strands, as they are unwound from the bobbins, pass through delivery rollers at D, E, and F, and thence onward to the laying top.

All this is simple enough, and might be the invention of any-one; but there is yet a difficulty to be overcome, which we proceed to explain.

Upon examining a rope it will be found that the twist of the rope is always in the opposite direction to that of the strands, and it follows that if the bobbins were absolutely fixed to the rotating frame the strands themselves would be untwisting during the whole operation. This untwisting is provided against in a rope-walk by the use of two machines, one at each end of the walk. The strands are attached to hooks on one of the machines, and these hooks are made to rotate with a velocity which exactly neutralises the twist of the machine which is forming the strands into a finished rope.

In the Cordelier the difficulty is at once removed by the introduction of an epicyclic train. A dead wheel A, so fitted that it remains stationary while the shaft PQ rotates within it, gears with a second wheel B, and this latter with a third wheel C, *equal* to A, whose axis terminates in one of the spool frames. Now we have just proved that in such a train C will run round A without rotating at all upon its own axis, and hence the bobbin may be carried round without in the slightest degree untwisting the strand.

In order to make this matter still more apparent we refer the student to fig. 257, which is intended to show three positions of a spool when rotating in a frame without the intervention of an epicyclic train. It is quite evident that the spool has made one

rotation round an imaginary axis through its centre while rotating once round the centre of the frame.

In fig. 258, on the other hand, where an epicyclic train, with C equal to A, is interposed, the bobbin will take the positions C, C′, C″, during a revolution, and the rotation just referred to will be exactly neutralised.

FIG. 257.

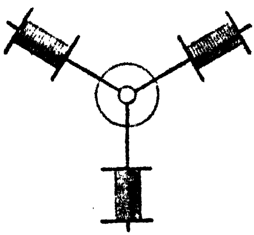
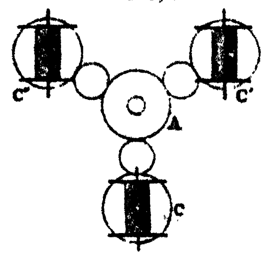


FIG. 258.



ART. 198.—We have stated that the twist of a rope is always in the opposite direction to that of the strands, and it may be asked, Why is this, and what is the reason that a rope does not untwist itself?

The answer is that any single strand or cord, when twisted up, will always tend to untwist in virtue of the elasticity of its fibres, and that each separate strand in a rope exerts this tendency throughout its whole length; but since the twist of the rope is in the opposite direction, the aggregate of all these comparatively feeble forces is felt as a powerful force restraining the whole rope from becoming untwisted.

It follows, therefore, that by putting a little extra twist upon the strands of a rope in the process of laying, the rope itself will become harder or more tightly twisted.

If anyone will try and make a small piece of cord out of three pieces of string he may at once satisfy himself of the correctness of what has been stated.

Take three pieces of string, or fine sash line, thread them through holes in a small plate or disc, to keep them separate, and fasten them together at one end, leaving the other ends free.

Upon twisting the knotted end and slowly advancing the disc, a cord will be made which will untwist as soon as it is handled.

Whereas by continually twisting each individual strand, and allowing the knotted end to turn in the opposite direction to that in which the strands are being twisted, a hard piece of cord may be made which will have no tendency whatever to untwist.

There is a model in the collection belonging to the School of Mines which shows this experiment in a striking manner. (Fig. 259.)

The driving apparatus consists of an arrangement for rotating at the same time three hooks. Each hook, *p*, is formed of a bent piece of wire terminating in an upright portion, *a*, which is threaded into a flat disc A. There is a detached sketch of one of these bent wires. Upon carrying A round in a circle, it will be found that each hook rotates on its own axis. This motion has been explained in Art. 96.

Take now three pieces of braided sash line, which have no twist, and suspend a weight W to each of them. Make a small loop at the opposite end of each line, and hang them all up on one of the hooks.

A small conical block B, having a handle H, and grooved as in the sketch, is held in such a manner as to receive each line and to direct its motion.

The operator now rotates the hook *p*, and allows the block B to descend slowly while the cord is being twisted. But on looking closely at the sash lines it will be found that each weight W is turning on its axis during the whole operation. In truth, the weights are made in the form of long cylindrical bars, in order to permit this movement. The result is that there is no twist whatever remaining on the individual sash lines or strands.

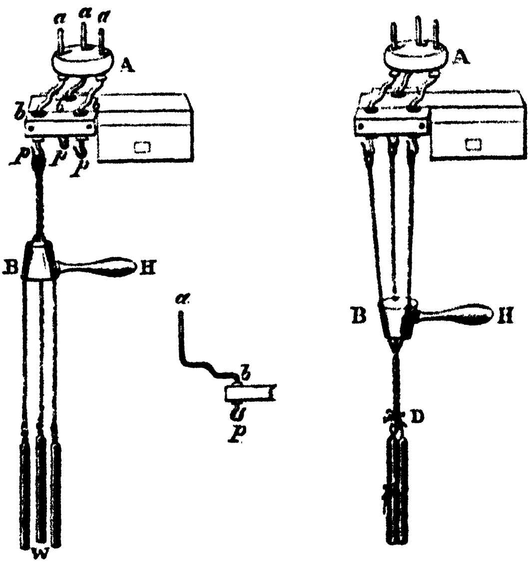
Now remove the block, when it will be found that the cord, which appears to the eye well made and perfect, will at once untwist, and is, in fact, of no use whatever. It is defective in not having any power of retaining the twist which is essential to the hardness and durability of a rope or cord.

The apparatus can, however, be so arranged as to put a strong twist upon each individual strand, in such a manner that the twist shall be retained in the finished cord, and shall act always to twist the same more tightly.

For this purpose the weights are tied together by a piece of string at D, and can no longer rotate separately. Each strand is

hung on a separate hook, and the respective hooks *p*, *p*, *p*, rotate together by the carrying round of the disc A. The block is held differently, being placed as near as possible to D, and it is moved slowly upwards while the cord is being made. This is exactly the operation performed in a rope-walk, except that the strands are carried along in a horizontal line.

FIG. 259.



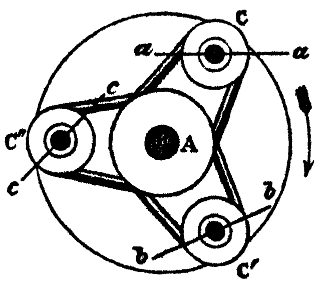
There is no difficulty about twisting the cord, for the surplus twist put upon the strands causes the weights W to go round together underneath the block B, and a well-formed cord is made as the block rises. When completed, the string at D may be untied, the strands may be taken from the hooks, but there is no untwisting. On the contrary, the cord will bear handling, and is quite hard and durable.

ART. 199.—Many years ago Captain Huddart incorporated the invention of the Cordelier into some useful machinery for manufacturing rope, and he employed the same epicyclic train, but made the wheel C smaller than A in the proportion of 13 to

14, as in the case of the wheel G in Ferguson's paradox. The result was that a slight additional twist, or *forehard*, as it is termed, was given to the strands of the rope.

Among the apparatus belonging to the School of Mines is a hand machine for manufacturing fine hard cord, resembling whip-cord. It is, in fact, a miniature Cordelier, and instead of the epicyclic train of wheels for keeping the bobbins parallel during the rotation, there is a single dead wheel or grooved pulley, A, and three strong india-rubber cords, connecting A with the separate axes C, C′, C″, on which the bobbins are placed.

FIG. 260.



Each of the grooved pulleys, C, C′, C″, is of smaller diameter than the wheel A, and therefore turns slowly backward in the opposite direction to that in which it is carried. This fact is made clear in the sketch, for the arrow indicates the direction of rotation of the frame carrying the bobbins, and the dark lines, *aa*, *bb*, *cc*, show the manner in which the respective bobbins rotate backward. That is, when C arrives at C′, the line *aa* will have turned into the position *bb*, and when it arrives at C″, the same line will have turned into the position *cc*.

Whereas, if the pulleys C, C′, C″ were each equal to A, the line *aa* would have remained parallel to itself throughout the motion.

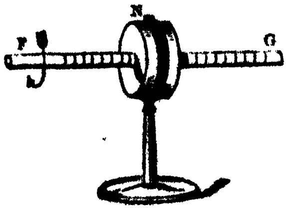
In Mr. Smith's wire-rope machine, which is described in the papers of the Institute of Mechanical Engineers for the year 1862, the bobbins are placed one behind the other in the axis of a revolving frame, and have simply a slow unwinding motion on their axes as the wire strands are run off; the important result being that the rate of manufacture is greatly increased. There is no question as to the superiority of this arrangement in a mechanical point of view, for the process of laying has no tendency to twist the strands when the bobbins themselves lie in the axis of rotation of the frame which surrounds them.

ART. 200.—A further illustration of aggregate motion occurs in machinery for drilling and boring.

In a drilling machine the spindle which carries the cutting tool revolves rapidly, and at the same time advances slowly in the direction of its length.

The movement is obtained upon an obvious principle, which may be stated as follows:—

FIG. 261.

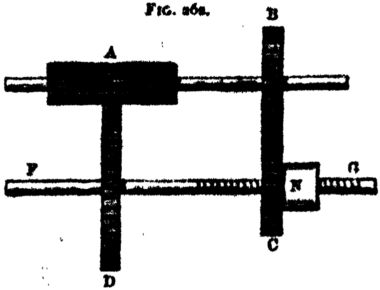


Conceive a nut, N, to be placed upon a screw-bolt, FG, and to be so held in a ring or collar that it can rotate freely without being capable of any other motion.

If the nut be fixed, and FG be turned in the direction of the arrow, it is clear that the bolt must advance through the nut. If, again, the screw be prevented from turning, and the nut be made to rotate in the same direction as before, the bolt will come back again. And, finally, if by any contrivance different amounts of rotation be impressed at the same time upon the nut and the screw, the bolt will receive the two longitudinal movements simultaneously, and the aggregate motion will be the sum or difference of these component parts.

ART. 201.—Suppose the wheels D and C to be attached to the bolt and nut respectively, and to be driven by the pinions A and B, which are fixed upon the same spindle; and let A, B, C, D represent the numbers of teeth upon the respective wheels.

FIG. 262.



If (a) be the number of rotations made by either A or B while the nut fixed to C makes m rotations, and the wheel D makes n rotations,

$$\text{we shall have } \frac{m}{a} = \frac{B}{C}, \text{ and } \frac{n}{a} = \frac{A}{D}.$$

Therefore (a) rotations of A will cause a travel of the bolt FG through a space

$$= (m - n) \times \text{pitch of the screw}$$
$$= a\left(\frac{B}{C} - \frac{A}{D}\right) \times \text{pitch of the screw.}$$

ART. 202.—We shall proceed to examine the construction of

FIG. 263.

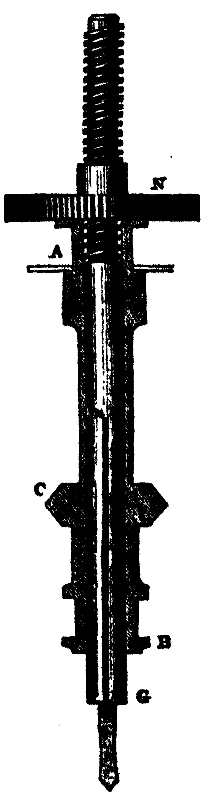
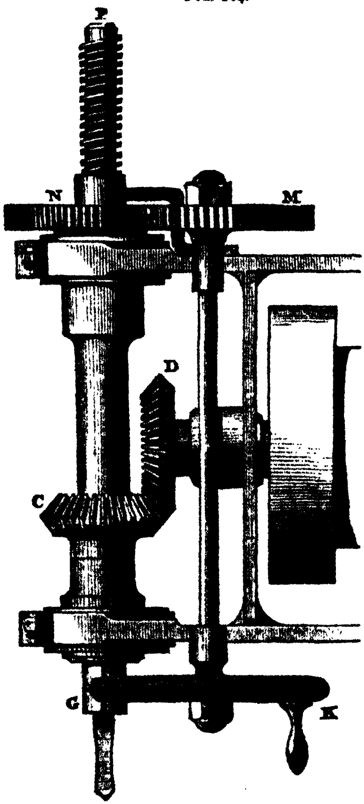


FIG. 264.



a small *Drilling Machine*, which may be worked either by hand or by steam-power, but is *not self-acting*.

The general arrangement of the machine is shown in fig. 264. The power is applied to turn the bevel wheel D, which again drives C, and causes the case or pipe containing the drill spindle to rotate. This provides for one part of the motion, viz., the rotating of the drill spindle, and the hand wheel K drives the spur wheels M and N, and advances the drill into the work in a manner which we shall endeavour to make clear.

The drill spindle is formed in two pieces, as shown in fig. 263, and the upper or screwed portion does not rotate with the lower cylindrical portion which carries the drill, but simply moves it up and down by means of a collar without interfering with its rotation. The screwed piece works in a nut forming the boss of the wheel N, and is prevented from rotating by a feather sliding in a groove or slot which runs along the whole length of the screw, and which cannot be seen in the view given in the drawing, the feather itself being fixed in a stop-collar at N.

Hence the rotation of the wheel N, by reason of its connection with the hand wheel K, will raise or depress the whole spindle as required.

The rotation of the drill spindle is provided for by cutting a groove *m n* in the lower part of it, and attaching a corresponding projection or feather to the inside of the pipe AB. This allows the spindle to move lengthways in the pipe, and ensures its rotation just as if it were a part of the tube in which it is held.

A machine of this construction might easily be made self-acting, as in a very useful form manufactured by Messrs. Smith, Beacock, and Tannett. Here the screwed spindle is not employed, but a rack and pinion is substituted for it, and the pinion is slowly raised or depressed by an endless screw and worm wheel set in motion by a hand wheel similar to K.

The self-acting portion consists of a small cone pulley, which draws off a motion of rotation from the driving shaft, and the axis of this pulley is fitted with a second endless screw and worm wheel placed just over the hand wheel, and which can be slid into gear so as to produce the self-acting motion.

Thus the same slow rotation may be given to the driving pinion on the axis of the hand wheel, by the steam-power, which is otherwise given to it directly by the workman; the cone pulley of course

providing for varying amounts of feed according to the requirements of the work.

ART. 203.—A Drilling Machine by Mr. Bodmer, of Manchester, is made self-acting in the following manner :—

The drill spindle (fig. 265) has a screw-thread traced upon it. A groove is cut longitudinally along the spindle, and a projection upon the interior of the boss of the wheel D fits accurately into the groove.

Thus the spindle can traverse through the wheel D, although the spindle and wheel must turn together.

FIG. 265.

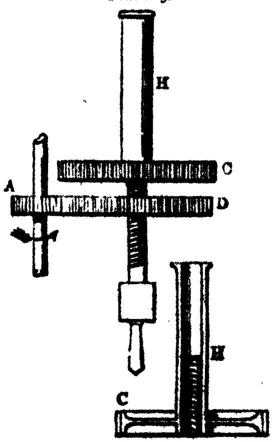
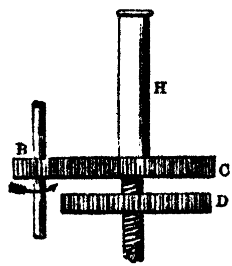


FIG. 266.



A nut H, in the form of a pipe, and having a wheel, C, at the bottom of it, receives the spindle. This wheel and pipe are shown separately in section.

If a pinion, A, turning in the direction shown by the arrow, engage the wheel D, it will screw the spindle rapidly out of the pipe H, and bring it down towards the work.

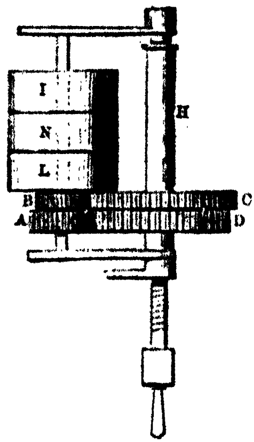
Suppose a second pinion, B, turning in the same direction as A, to act upon C, it will move the nut instead of the screw, and the drill spindle will rise rapidly so long as it is prevented from rotating. (Fig. 266.)

Thus far we have provided for bringing the spindle down to its work, and for raising it up again. It remains to apply the principle of aggregate motion, and to cause the drill spindle to become the recipient of these two movements in a nearly equal

degree, and thereby to ensure the slow descent accompanied by a rapid rotation, which is required in process of drilling.

The result of the combination is shown in fig. 267, where the wheels A and B are moved together: the wheel A tends to depress the spindle, the wheel B tends to raise it, and, since A is greater than B, the spindle descends by the difference of these motions, having further the motion of rotation given by the wheel A.

FIG. 267.



The motions of A and B are obtained from the driving pulleys I, N, and L.

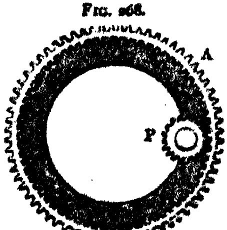
I is an idle pulley, N drives A, and L drives B. When the strap is on N the drill descends to the work, when the strap is on L it ascends from the work, and when the strap is partly on N and partly on L the drilling proceeds.

The practical objection to this movement is that the rate of feed is invariable so long as the train of wheels remains the same. It may be thought better to control the feed by means of a cone pulley, where the strap can be readily shifted so as to change the advance of the cutter.

ART. 204.—A *Boring Machine* would be employed to give an accurate cylindrical form to the interior surface of a steam cylinder.

In the annexed example the boring cutters are attached to a frame which rides upon a massive cast-iron shaft or boring bar, and rotates with it: this frame is further the recipient of a slow longitudinal movement given by a screw.

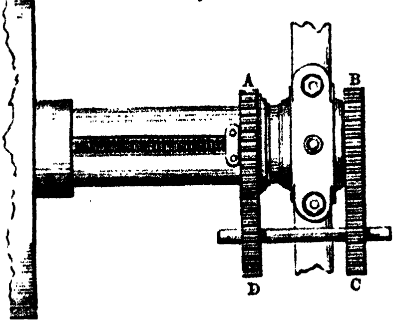
FIG. 268.



An annular wheel, A, shaped as in the diagram, rides loose upon the bar, and drives a pinion, P, at the end of the feeding screw which advances the cutters, the boring bar being recessed in order to receive the screw.

It is quite apparent that as long as the rotation of the wheel A is identical with that of the boring bar, the pinion P will not turn at all; and, further, that a slow motion will be impressed upon P if the rotation of A be made to lag a little behind that of the bar.

Fig. 269.



A spur wheel, B, is keyed to the bar, a small shaft fixed at the side carries the wheels C and D, and thus motion is imparted to A, the driver of the feeding screw. Let the numbers of teeth upon B, C, D be 64, 36, 35, and let the wheel A have 64 teeth, both upon the outside and the inside of its circumference, the pitch of the screw being $\frac{1}{2}$ an inch, and the number of teeth upon the pinion being 16.

$$\text{Here } e = \frac{B \times D}{C \times A}, = \frac{64 \times 35}{36 \times 64}, = \frac{35}{36}.$$

That is, A loses $\frac{1}{36}$th of a revolution for every complete rotation of the boring bar.

At the same time the pinion P moves through $\frac{1}{36} \times \frac{64}{16}$ or $\frac{1}{9}$ of a revolution, and the cutter advances through $\frac{1}{9} \times \frac{1}{2}$ an inch or through $\frac{1}{18}$th of an inch.

Art. 205.—This slow rotation of the screw which advances the boring head may be obtained in a more simple manner by a combination which virtually embodies the sun and planet wheels of Watt.

Conceive that two wheels, A and B, of 40 and 80 teeth respectively, are attached to the bar CAB, which has a centre of motion at C.

If the bar be carried round C, and A be made a dead wheel, the effect of depriving A of the rotation due to its connection with the arm will be to cause B to rotate relatively to the arm just as if the axes of both wheels were fixed in space.

The movement is shown in the diagram, where A has turned

through half a right angle from its first position *relatively to the arm*, while the arm itself has been carried through a right angle.

The student will distinguish between the *absolute* and *relative* rotations of B; the absolute amount of the rotation of B is one right angle and a half.

This also appears from the formula, viz. $e = \frac{n-a}{m-a}$.

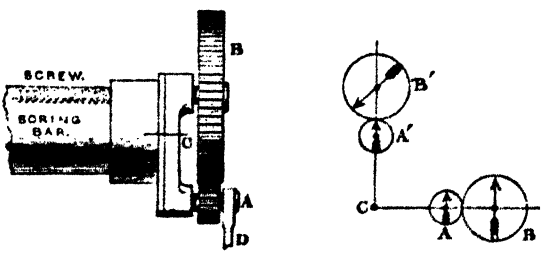
Substituting the values $m = 0$, $e = -\frac{1}{2}$, we have $-\frac{1}{2} = \frac{n-a}{-a}$.

$$\therefore n - a = \frac{a}{2}.$$

But $(n-a)$ represents the number of rotations of the wheel B relatively to the arm while the latter is making (a) revolutions, and the analysis therefore shows that the angular velocity of B relatively to the arm is half that of the arm itself, and also that both rotations take place in the same direction.

Further, it must be noted that the position of C makes no difference in the result, which will be the same if the point C be somewhere between A and B.

FIG. 270.



In the application of this movement to the boring machine, the centre of motion is between the axes of the wheels, in the line marked C in the diagram, and the numerical value of e is less than $\frac{1}{2}$, probably about $\frac{1}{3}$.

The wheel B is placed upon the axis of the screw which advances the boring cutters, the rotating arm being now a part of the solid end of the boring bar; the wheel A rides upon a separate stud, and is attached to a bar AD of some convenient

length which passes through and rests upon a fork in an independent upright support placed at some little distance from the machine.

As the wheel A is carried round the axis of the boring bar this rod slides a little to and fro in the fork, and controls the wheel A so as to render it impossible for it to rotate, or, in other words, to make it a dead wheel.

The wheel B will now turn slowly under the action of A so far as its position relatively to the boring bar is concerned, and upon our supposition, the screw will advance the boring cutters by a space equal to its pitch in five complete revolutions.

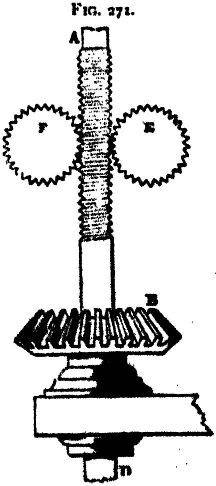
This would give a feed dependent upon the pitch of the screw, which could of course be varied at once by changing the wheels A and B.

ART. 206.—Sir J. Whitworth's *Friction Drilling Machine* is an elegant application of the principle of aggregate motion.

AD is the drill spindle, which is driven in the usual manner by the bevel wheel B.

E and F are two worm wheels embracing the screwed portion of the spindle upon opposite sides. They are of peculiar construction, being hollowed out so as to fit against the small screwed spindle, and they work with a V-threaded screw upon AD.

FIG. 271.



If E and F be prevented from turning, they will form a nut through which the spindle will screw itself rapidly.

If E and F be allowed to turn quite freely, the drill spindle will set them in motion, and the nut will be virtually eliminated. The drill spindle may then be regarded as the recipient of two equal and opposite motions: it is depressed by screwing through the nut, it is elevated by the turning of the wheels.

If the rotation of the wheels be in any degree checked by the application of friction, the equality is destroyed, and the drill spindle descends to a corresponding extent.

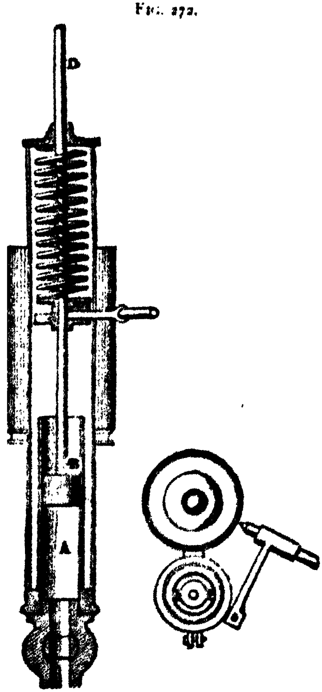
A friction brake, regulated by a screw, restrains the motion of E and F, and gives a perfect command over the working of the machine.

When B is at rest the worm wheels act upon the screwed part of the spindle just as a pinion does upon a rack, and the drill can be rapidly brought down to the work.

This method of converting a screw and worm wheel into a rack and pinion is quite worthy of attentive consideration: it is employed in the well-known lathes by the same firm.

ART. 207.—*Watt's Indicator* is an instrument used to ascertain the actual horse-power of a working steam-engine. The principle upon which it is constructed is the following:—

FIG. 172.



A pencil oscillates through the space of a few inches in a horizontal line, with a velocity which always bears a fixed ratio to that of the piston, whereby its motion is an exact counterpart upon a very reduced scale of the actual motion of the piston in the steam cylinder; and at the same time it is the subject of a second movement in a vertical line, which is caused by the pressure of the steam or uncondensed vapour in the cylinder, and occurs whenever the pressure of the steam or vapour upon one and the same side of the piston of the engine becomes greater or less than that of the atmosphere.

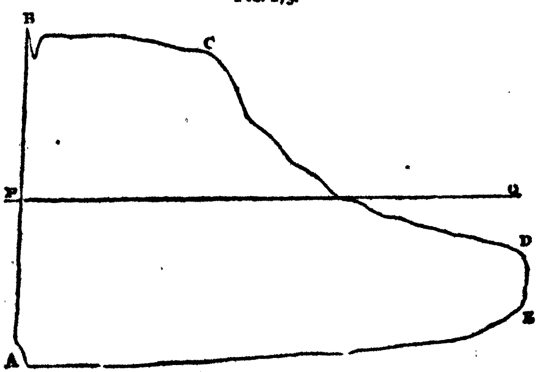
Under the influence of these independent motions the aggregate path of the pencil will be a curve which is capable of interpretation, and which affords a wonderful insight into actions which are taking place in the interior of the cylinder.

An excellent early form of the apparatus is known as McNaught's indicator, and consists of a small cylinder, A, fitted with a steam-tight piston, B. The piston rod, BD, is attached to a spiral steel spring, which is capable of extension and compression within definite limits, and is enclosed in the upper part of a tube which carries the cylinder A.

The pencil is attached to a point in the rod BD, and traces the indicator diagram upon a piece of paper wrapped round a second cylinder by the side of the first.

The cylinder, A, is freely open to the atmosphere at the top, and a stopcock admits the steam from below when required. The indicator is usually fixed upon the cover at one end of the steam cylinder of the engine. When the stopcock is opened and the lower side of B is in free communication with the interior of the cylinder, the pressure of the steam will be usually greater or less than that of the atmosphere: if it be greater, B will rise against the pressure of the spring, and if it be less, the pressure of the atmosphere upon the upper surface of B will overcome the resistance of the spring and cause the pencil to descend.

FIG. 273.



At the same time, the cylinder which carries the paper is made to turn with a motion derived at once from that of the piston in the engine, but much less in degree, and thus a curve is traced out somewhat of the character represented above.

Here PQ is the atmospheric line, and is the path of the pencil when the pressure of the steam is equal to that of the atmosphere, or when the spiral spring is neither extended nor compressed.

As the steam enters the cylinder, the piston may be supposed to be descending, and the pencil to be describing the upper portion of the curve: when the piston returns, the pencil moves to the left through DEA, and thus the diagram is traced out. We may examine this matter with more particularity as follows: the steam is admitted when the piston reaches the top of its stroke, and the pencil rises with a rapid motion from A to B; the full pressure of the steam is then maintained while the pencil, recording a portion of the travel of the piston, moves from B to C; at C the steam is cut off, and the pencil falls gradually as the steam expands with a diminishing pressure; at D the steam pours into the condenser, and the fall becomes sudden; from E to A the cylinder is in full communication with the condenser, and the pencil describes a line somewhat inclined to the line PQ, the position and form of which depend upon the perfection of the vacuum in the condenser.

The strength of the spiral spring being ascertained, the curve tells us exactly the number of pounds by which the pressure of the steam urges the piston onward during every inch of its path in one direction, and the amount of resistance which the uncondensed vapour or gases existing in the condenser oppose to its passage in the other direction. The area of the curve, therefore, affords an estimate of the work done in the engine during one complete stroke, and is a graphic representation of the same. The engineer estimates this area by simple measurement in the most direct manner which occurs to him, and the actual indicated horse-power is obtained by multiplying the work done in one stroke by the number of strokes made in a minute, and then dividing by 33,000, the number of foot-pounds which form the measure of rate of work called a horse-power.

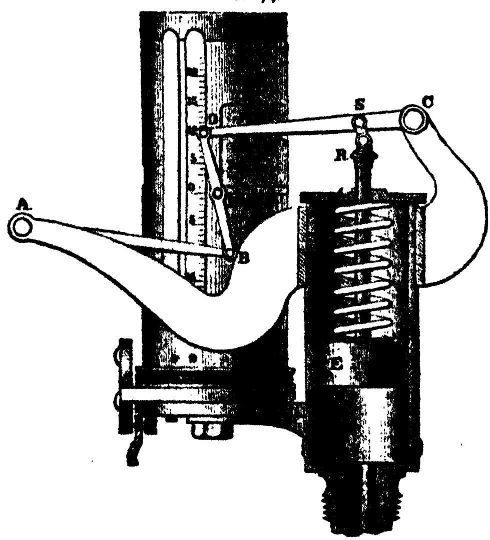
Art. 208.—The object of the indicator being to ascertain the exact pressure of the steam or vapour in the cylinder at each point of the stroke of the piston, it has been found to be a great advantage to diminish as much as possible the play of the spring which controls the pencil. In this way the vibration and irregu-

larity of motion of the pencil is greatly reduced. But the play given to the spring determines the height of the diagram, and we do not wish to reduce this, but rather the contrary. It is not easy to reconcile these contradictory requirements, but, nevertheless, a form of indicator has been invented by Mr. Richards which solves the difficulty, and has become most deservedly popular.

It is an ingenious application of the combination of two bars and a link forming a parallel motion, and will be understood at once from the drawing, which is taken from a small model representing very closely the essential parts of an actual instrument.

The parallel motion bars AB and CD carry the pencil, which traces out upon a drum a copy of the vertical movement of the piston E of the indicator, but magnified by reason of the attachment of the piston to a point S near the fulcrum of the bar CD.

FIG. 274.

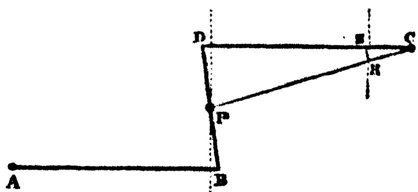


The principle of the apparatus is precisely the same as that which we have already explained, and the only difference consists in the application of the parallel motion bars to enlarge the diagram.

The drum derives its motion from any part of the engine whose movement is coincident with that of the piston, and the spiral spring can be changed so as to suit different engines. The connecting link is not set perpendicularly to the bars AB, CD, but makes an angle with them as shown, an artifice which causes the pencil to describe a line free from any sensible curvature.

The parallel motion is set out in a separate diagram, in order that it may be thoroughly understood.

FIG. 275.



The link RS is parallel to BD when the motion begins, and it remains parallel throughout, for R and P are both constrained to describe vertical straight lines.

Hence we have the pantograph in a disguised form.

Also, travel of P : travel of R = CD : CS.

In the indicator as constructed the movement of R is magnified about *four* times.

It should be understood that the frame carrying the motion bars is attached to a collar which can be rotated on the cylinder, whereby the pencil is readily brought up to the paper or removed from it.

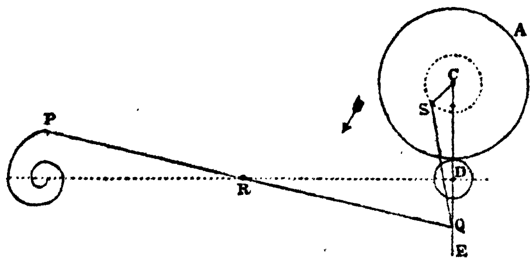
ART. 209.—There is a curious movement derived from the employment of a dead wheel in a train which has been applied by Mr. Goodall in a machine used for grinding glass into powder by the action of a pestle and mortar. The pestle is made to sweep round in a series of nearly circular curves contracting to nothing, and then expanding again so as to command the whole surface of the mortar.

We shall show that the contrivance is merely a solution of the problem of obtaining an *expanding and contracting crank.*

Let CQE be a crank whose centre of motion is D; conceive

that D is a dead wheel on the same axis, and that A is a larger wheel riding upon one end C of the crank arm.

FIG. 276.



Suppose, further, that a piece Q, capable of sliding along CE, is attached by a link SQ to a point S in the circle A, which is not its centre; and, finally, that a pencil at P is connected with Q by a link PRQ constrained always to pass through a fixed point R.

As the circle A and the crank CQ travel together round the dead wheel D, it has been proved in Art. 195 that the wheel A will turn relatively to the arm just as it would do in an ordinary train with fixed axes. Hence the point S travels slowly round in the dotted circle, thereby causing the point Q to move to and fro along CQ.

It may be arranged that Q shall start from D, and it will travel along CE through a space equal to twice CS. When Q is at D, the point P is motionless, whereas, while Q moves further from D, and continually sweeps round, by virtue of its being a point in a revolving crank, it is evident that P will trace out an expanding spiral, which will return again to nothing when Q is pulled back to D by the action of the wheel A.

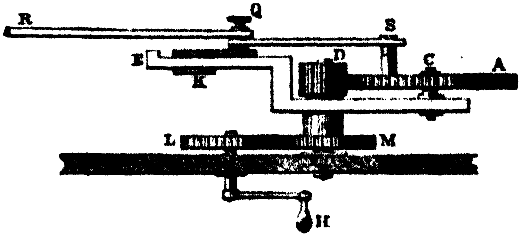
It now only remains for us to consider what would be the actual construction of the apparatus.

The drawing is taken from a model, and not from the machine itself.

The crank CDE is a bar whose parallel arms are connected by a vertical piece, and which carries the wheel C upon one arm and the sliding piece KQ upon the other arm. This crank is driven by the spur wheels L and M connected with the handle H; it

therefore rotates round the axis of the dead wheel D, and carries C and KQ upon opposite sides of the vertical axis through D.

FIG. 277.



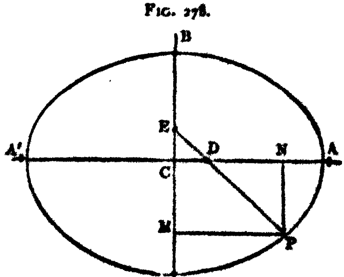
The link SQ connects the wheel C with the piece KQ, and this latter piece is again connected by QR with the pestle, it being provided that QR shall pass through a guide at some fixed point about half-way between Q and the mortar. The pestle is swung from a ball and socket joint at some convenient height above P.

The rotation of the crank round the dead wheel causes C to turn slowly upon its own axis, the point S therefore travels slowly round C, hence the end Q of the connecting rod QRP is sometimes at a distance from D, and at other times is exactly over it, and during the whole time Q is a part of the crank DQ, and sweeps round with the arm. Thus the required motion is provided for.

ART. 210.—The *oval chuck* affords an instance of aggregate motion. It is based upon the following property of an ellipse, which is taken advantage of in constructing elliptic compasses for drawing the curve.

Let ACA′, BCB′, represent two grooves at right angles to each other, and traced upon a plane surface; PDE, a rod furnished with pins at D and E. If this rod be moved into every possible position which it can assume while the pins remain in the grooves, the point P will describe an ellipse.

FIG. 278.



Draw PN perpendicular to AC, and PM perpendicular to CB′.

$$\text{Let CN} = x,\ \text{PE} = a,$$
$$\text{PN} = y,\ \text{PD} = b.$$
$$\text{Then } \frac{x}{a} = \frac{\text{PM}}{\text{PE}},\ \text{and } \frac{y}{b} = \frac{\text{PN}}{\text{PD}} = \frac{\text{EM}}{\text{PE}},$$
$$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{\text{PM}^2 + \text{EM}^2}{\text{PE}^2} = 1,$$

which is the equation to an ellipse.

In drawing an ellipse we should fix the paper and move the rod over it, but in turning an ellipse in a lathe we should fix the describing tool and move the piece of wood or metal underneath it ; thus the conditions of the problem become changed, and the construction is modified accordingly.

An equivalent for the grooves ACA′, BCB′ may be arrived at as follows :—

Describe a circle about E of radius larger than ED, and let two parallel bars, QR, ST, be connected by a perpendicular link HK, equal in length to the diameter of the circle, and thus form a rigid frame embracing the circle, and capable of moving round it.

FIG. 279.

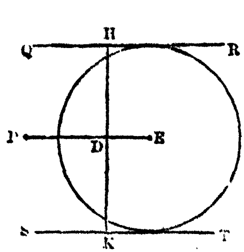
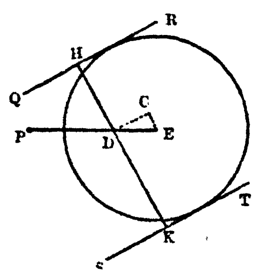


FIG. 280.



As the frame moves round the circle we must provide that HK shall pass through D in every position as represented in the diagram.

If we draw DC parallel to QR, and EC parallel to HK, it is easy to understand that the imaginary triangle DCE in fig. 280 is exactly the same as the triangle DCE in fig. 278 and exists

throughout the motion ; and that whereas we formerly moved the bar EP over a fixed plane and described an ellipse, so now we have arranged to obtain the same motion with a fixed bar and a movable plane, and shall trace out precisely the same curve.

This is a very good example of aggregate motion. The plane upon which the ellipse is traced is the subject of two simultaneous movements : by one of them a line, HK, in the plane is made to revolve round D as a centre, and by the other the same line receives a sliding motion in alternate directions through D.

Thus an oval, or more properly an ellipse, may be turned in the lathe.

CHAPTER VIII.

ON TRUTH OF SURFACE AND THE POWER OF MEASUREMENT.

ART. 211.—The subject matter comprised under the title of this chapter is so large that it cannot be discussed fully, and all that can be done is to present a brief sketch of some important facts connected with it.

We have to describe a method of mechanical measurement, founded upon truth of surface, which is probably the reverse of that which most persons would form for themselves. The idea of measurement is commonly associated with optical contrivances, whereby, if we desired to measure some minute interval of length, say in ten-thousandths of an inch, we should naturally proceed to the task armed with powerful lenses or microscopes, and relying mainly on the sense of sight. It would be something quite novel and unexpected to discover that the sense of touch would do more for us than the eye, and that, in mechanical measurement, it is more easy to feel minute differences of size by the aid of surfaces properly prepared and adjusted, than it is to recognise and compare such differences in the field of view of a microscope.

We shall presently refer to a measuring machine, used in the construction of difference gauges, wherewith a workman can readily test gradations of size differing by $\frac{1}{10000}$th of an inch, and may in special cases carry on the operation as far as $\frac{1}{20000}$th or $\frac{1}{50000}$th of an inch.

Inasmuch as a machine of this kind is a piece of constructive work perfected by the aid of other machines, such as the lathe and the planing machine, in which truth of surface is all important, it will be useful to consider in the first instance the method of originating a plane metallic surface.

Every one knows that a plane surface is an ideal thing, which

the geometrician arrives at by an operation of the mind, but which has no real existence.

In the year 1840 Sir J. Whitworth brought to the notice of engineers a new method of preparing metallic surfaces, and he submitted specimens of cast-iron plates so prepared, which he called *true planes*.

A 'true plane' is commonly spoken of in the workshop as a 'surface plate,' and is made of cast iron, being ribbed at the back and resting on three points of support. The truth which it possesses is of course approximate, and its surface, when carefully examined, will be found to consist of a vast assemblage of minute bearing faces which lie very nearly in the same geometrical plane, the object aimed at being to distribute these bearing faces as nearly as possible at equal distances from each other.

The preparation of a standard surface plate, and the method of employing it for the multiplication of other identical plane surfaces, has been a distinct invention in mechanics, the importance of which can scarcely be over-estimated, and we proceed to give an account of it.

ART. 212.—Up to the year 1840, the process relied upon for obtaining plane surfaces on metal plates, and indeed the only one practically used, had involved the operation of grinding two plates together with emery powder and water. While the operation was going on the plates under preparation were occasionally compared with a standard plate or plane, but the standard plate was imperfect, the method of comparison was uncertain, and the smooth surface given by the process of grinding was entirely deceptive if regarded as an evidence of truth.

The operation of grinding fails for the obvious reason that the action of the powder cannot be restricted to those parts only which are in error, and it is clear that a complete control over the successive removal of any portions of the surface which are believed to be out of truth is the first thing to be sought for. Such a control can be obtained by the use of a scraping tool, which, acting like one tooth of a file, is competent continually to remove portions of the metal in the form of a fine powder or dust, and it will be shown that by a systematic method of comparison it becomes possible to produce planes which rival in

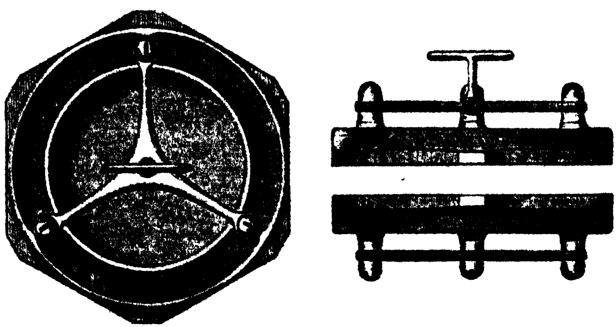
accuracy, though not in polish, the brilliant mirror presented by the surface of mercury when in absolute repose and undisturbed by any ripples.

A surface plate is always made of cast iron, and has usually taken the form of a rectangular plane table, ribbed at the back, and resting upon *three supports* or bearings.

Three supports are necessary in order that the plate may remain under like conditions wherever it is placed. If it were supported on four legs, and the foundation gave way under any one leg, the plate would become distorted and untrue, but with three legs it must take an equal bearing under all circumstances. It may be thrown out of level but it will not be distorted. The same rule applies in lifting a plate : it must be hung from three points.

The form to be assigned to the plate becomes material when the question of lifting or supporting it comes under consideration. In order to avoid distortion, a surface plate ought to press equally on its three supports, or be lifted by equal tensions, and for this purpose its centre of gravity should coincide with that of the triangle formed by joining the points of support or of suspension. Inasmuch as a rectangular plate of uniform thickness has no symmetry of form with regard to a triangle, it follows that some polygonal form is theoretically preferable, and accordingly

FIG. 281.



a hexagonal plate, such as that shown in the diagram, has bee adopted in later years by Sir J. Whitworth. The drawing repre

sents a surface plate in plan and elevation, the circular ribs employed for strengthening the plane table being apparent, as well as the legs or supports.

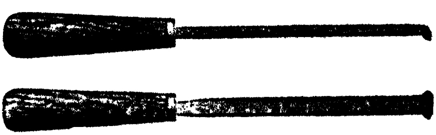
A tripod frame binds the legs together, and the plate can be slung by a handle screwed into the tripod, whereby it is subject to the same mechanical conditions whether it is resting on the supports or is suspended by the handle.

Having decided on the form of the plate and the method of supporting it, we have next to describe the operation of obtaining a plane surface.

ART. 213.—A number of good castings having been made, three plates are prepared in the planing machine by levelling the bosses or supports and by planing over the surfaces.

The first step is to ascertain the most perfect plate of the three chosen, which is done by laying a good straight edge upon each surface, and noting that which agrees most nearly with it. [A straight edge is a flat steel bar, on the thin edge of which a plane surface has been formed. In practice the surface of a straight edge is a true plane, but we shall consider it as less perfect than the plane we are about to originate.] Let us call the three selected plates by the letters A, B, C, and let A be chosen as the primary model. The operation of scraping is now relied upon for the continual correction and improvement of the surfaces.

FIG. 282.



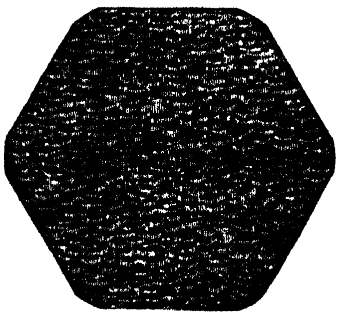
A scraping tool, as shown in elevation and plan in fig. 282, is forged from a bar of steel, the cutting edge being formed by grinding two facets at an angle of about 60°. The edge is carefully set on an oil-stone, and for finishing tools the angle of the scraper may be increased up to 120°.

The workman holds the tool in his right hand, pressing the edge on the surface to be scraped with his left hand, and moves the tool to and fro through a small space, thus taking off very

small quantities of metal in the form either of minute shavings or of fine powder according to the degree of force exerted.

It has been stated that scraping is a very delicate application of the principle of filing, the edge of the scraper acting as if it were one tooth of a very fine file, the object being to detach from any portion of the surface as much or as little of the metal as may be desired, and to confine the operation to the precise spot which may be in error.

FIG. 283.



In order to present some idea of the result of scraping on the appearance of a surface, the annexed drawing has been engraved from a photograph, and shows the peculiar mottled appearance which is due to the multiplied action of the scraping edge in every direction as the work advances.

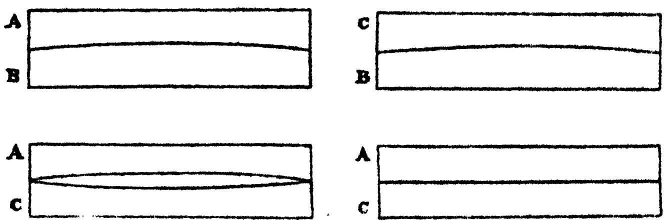
ART. 214.—We have now to endeavour to arrive at a perfect coincidence between three surfaces, A, B, and C.

It is a geometrical fact that if three surfaces can be brought into exact coincidence when compared and interchanged in every position, each of them must be a plane surface. Thus *three* standard planes must be produced in the endeavour to form *one* only.

Taking two of the plates, as A and B, the method is to smear the face of A very lightly with red ochre and oil : the plate B is then lowered upon A, so that the colouring matter may adhere to the surface of B wherever there is contact. B is then scraped at all those points to which the colouring matter has adhered, A in its turn being wiped clean, and B being coated with the mix-

ture, after which the operation is reversed, and A is submitted to the scraper. The process is continued until the contact between A and B is made as perfect as possible, the bearing points being so numerous and evenly distributed that the entire surface appears reddened. But although A and B are in perfect contact throughout, it may be that B is convex and A concave, as in fig. 284. Take now the third plate C, and bring it into perfect coincidence with B by scraping, and then compare A and C together as in the lower diagram, when the error, if any, will be manifest by the failure of absolute contact as indicated by the colouring matter.

FIG. 284.



To bring A and C nearer the truth, equal quantities must be scraped away from both surfaces at the points of contact. When this has been done with all the skill the mechanic may possess, and A and C are brought into coincidence with each other, as in the sketch, the next step is to bring B up to both. The art here lies in getting B between A and C in the probable direction of the true plane.

Taking now B as an improved standard for comparison with A and C, the process commences *de novo*, and is carried on in a regular series of comparisons, which result in a gradual approach towards absolute truth. At length the inherent imperfections of the material and of the tools render it impossible to proceed further, and the most watchful care is necessary in order to guard against the introduction of fresh errors; the penalty for scraping off the slightest excess from any part being the performance of the difficult task of lowering the entire surface to the same extent.

ART. 215.—The conditions which a surface plate should fulfil are the following :—

1. The bearing surfaces should all lie in the same plane.

2. They should be distributed as nearly as possible at equal distances from each other.

3. They should be sufficiently numerous for the particular application intended.

When a surface plate is completed the bearing faces will be found to have acquired a degree of polish, from continual rubbing, and they will appear as numerous bright spots dotted over the plane. The plate itself has not the appearance or surface of a polished mirror, but it possesses, nevertheless, a considerable power of reflecting light, and might, for example, be used as a reflector for throwing on one side an image of the carbon points of an electric lamp.

If two well-finished surface plates be wiped with a dry cloth and laid one upon the other, the upper plate will appear to float and become buoyant, as if some lubricating matter existed between them. If the upper plate be somewhat raised and allowed to fall, there will be no metallic ring, but the blow emits a peculiar muffled sound. These effects have been attributed to the presence of a cushion of air. If, again, one plate be carefully slid over the other so as to exclude the air, and then pressed together, it will be found extremely difficult to separate them. Here again it has been supposed that the plates adhered by atmospheric pressure.

Dr. Tyndall has upset this hypothesis as to the adherence of two surface plates by atmospheric pressure, and has described his experiments in a lecture given at the Royal Institution.

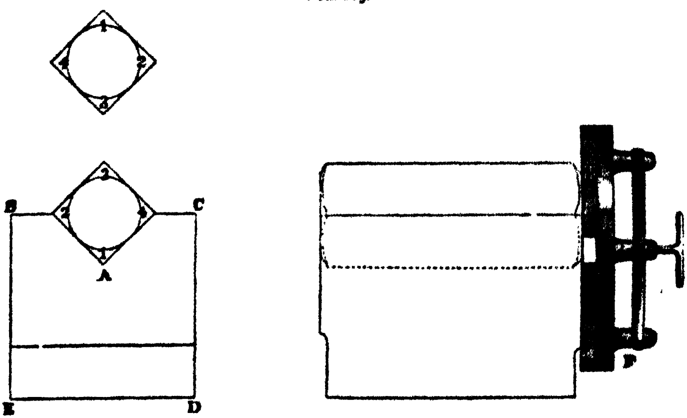
Having obtained a contact between two small hexagonal Whitworth plates, he found that the plates remained adherent in the best vacuum obtainable by a good air-pump. The vacuum was still further improved by filling the receiver with carbonic acid, and absorbing the residue with caustic potash. In this way the atmosphere was reduced until its total pressure on the surface of the hexagon amounted to only half a pound. The lower plate weighed three pounds, and to it was attached a mass of lead weighing twelve pounds. Though the pull of gravity was thirty times the pressure of the atmosphere there was an excess force, and the weight was supported.

It is also noticed that the amount of vacuum formed in the receiver of an ordinary air-pump has little or no power in diminishing the floating effect which is observed when one plate lies upon the other. The floating is just as apparent under the exhausted receiver as in the open air.

The conclusion at which Dr. Tyndall arrives is that the plates adhere by the molecular attraction of the bearing points brought into close contact by reason of the near approach to absolute truth of surface.

ART. 216.—We pass on to describe the method of producing a rectangular bar with plane sides and plane ends, which may be taken as the type of bars hereafter to be used in the measuring machine.

FIG. 285.



A right-angled groove with plane faces is prepared and made as true as possible by straight edges and squares. A rectangular bar, fitting the groove as shown in fig. 285, is also prepared and rendered approximately true. The bar is then laid in the groove, and the faces of the groove are brought by scraping into coincidence with those of the bar. The bar is next rotated through $\frac{1}{4}$th of a revolution, so that the angle marked (1) takes the position of that marked as (2) in the diagram, and the surfaces are again compared and corrected. This goes on till a coincidence is ob-

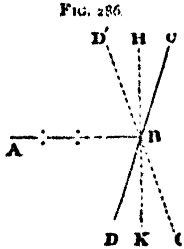
tained in every position, when the bar and groove must both have their surfaces at right angles.

The geometrical principle is that the four interior angles of a quadrilateral figure are equal to four right angles, and if these angles be equal, each must be a right angle.

Having obtained a bar with plane sides, it remains to work up the ends into true planes at right angles to the axis of the bar. For this purpose one end of the grooved rectangular block ABCDE is made plane, and as nearly as possible perpendicular to the axis or edge of the groove. The bar is laid in the groove, and a trial plane F is brought into contact with the plane end of the block and with the end of the bar. The latter surface is then made a true plane, and occupies the position 1 2 3 4 in the diagram. The bar is now rotated through half a revolution, so as to take the position 3 4 1 2, and we have a ready method of testing whether the plane end is or is not perpendicular to the axis of the bar.

If the line CBD, fig. 286, be nearly but not quite at right angles to AB, and if the system ABCD be rotated through 180° about AB as an axis, the lines BC and BD will take the respective positions BC′ and BD′, and the true perpendicular to AB will bisect each of the angles CBD′ or C′BD. In other words, the true perpendicular lies in HK, which is half way between the two positions occupied by CD.

Fig. 286.



Applying our proposition to test the direction of the plane end of the bar, we shall probably find that one of the angles of the end-face is alone in contact with the trial plane after the rotation. This shows that both the plane ABEDC and the end of the bar are out of truth, and an equal quantity must be taken from each, the directions of the plane surfaces being shifted through half the supposed angle of error. The operation is laborious, as it involves an alteration in the direction of the whole surface of a plane.

Having brought the trial plane into perfect coincidence with the end of the bar when the angles (1) and (3) are successively uppermost, the same thing has to be repeated with the angles (2) and (4), and if the coincidence be perfect in every position during

the rotation, it follows that the plane end of the bar must be truly perpendicular to its axis.

The final operation is to remove the bar from the groove, and rub it lightly on a surface plate.

ART. 217.—Every mechanic is familiar with measurement by callipers, whereby to judge of differences of size by the degree of tightness which is felt when pieces of metal are passed between the legs of the instrument. Supposing it were required to compare two spindles, for the purpose of making one an exact copy of the other. Calling the original A, and the copy B, the workman would adjust the callipers until he had obtained a certain degree of tightness when passing them over the surface of A. He would then gradually reduce B, until, to his sense of touch, the feeling of tightness of the instrument was alike when passing it over A or B indifferently.

The comparison is not easy, for there are many sources of error. The ends of the callipers are blunt edges, imperfectly shaped, and it is difficult to hold the legs of the callipers in a plane exactly perpendicular to the axis of the cylinder which is being tested. The transverse section of a cylinder is a circle, whose diameter gives the measure of the cylinder, but an oblique section of a cylinder is an ellipse, whose major axis is greater than the diameter of the cylinder. When the callipers are so held as to embrace a transverse section the observation is correct; whereas when they embrace the opposite arcs of an oblique section the observation is worthless. Although a skilful workman may use callipers so as to obtain very good results, it is evident that he struggles against an inherent defect which cannot be remedied.

In like manner callipers have been constantly employed for the conversion of line into end measure. In effecting this interchange the workman lays his callipers upon the graduations of a foot rule, and reads off the interval as nearly as he can. He then shapes some piece of metal which is to be made of given dimensions until the callipers pass over it with an estimated degree of tightness, and he thus transfers an interval on the rule to some part of a solid body.

There are three difficulties in the way, and one is very serious.

1. It is impossible to determine how nearly the end of each

leg of the callipers coincides with the centre of the corresponding line of graduation of the rule.

2. There is the difficulty of holding the instrument in a plane perpendicular to the surfaces under measurement.

3. It is uncertain to what extent the legs of the instrument may spring or yield under the variable pressure of contact.

Some years ago a standard inch was unknown in the workshop, and the graduated foot-rules were often incorrectly divided.

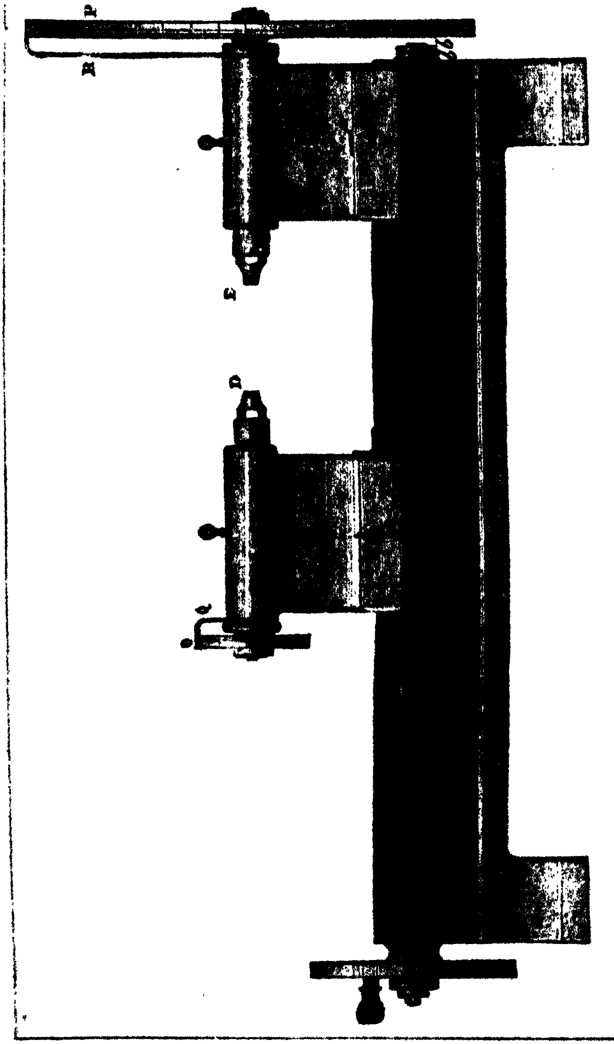
The subject of mechanical measurement remained in a state of uncertainty and confusion until Sir J. Whitworth applied himself to the construction of end measure standard bars, as well as of a machine for end measuring.

Before commencing that task he had satisfied himself that the only practicable mode of measurement suited for the workshop should be one founded on truth of surface and the sense of touch —the delicacy of the nerves of feeling being, in fact, a thing quite disregarded and neglected by all others who had applied themselves to improving mechanical measurement. He undertook the novel task of showing how to measure minute differences of size which were simply felt, and not observed or recognised by the eye, and he constructed a machine for reproducing in a much more perfect manner the ordinary callipers of the workshop.

Conceive now that the ends of the legs of the callipers are replaced by two parallel true planes fixed at a distance equal to the diameter of the cylinder, and that these planes can be caused to approach or separate by almost insensible intervals. We have then a measuring machine of the highest delicacy and precision. The sense of touch can be tested in a manner that will surprise any one who approaches the subject without previous training or experience, and differences of ten thousandths of an inch become palpably evident. Taking the workshop measuring machine as constructed by Sir J. Whitworth, we find that its general appearance is that of a small turning lathe.

1. There is a cast-iron bed formed of two parallel cheeks, connected at the ends, and stiffened by intermediate ribs, the bed itself being supported upon standards or feet placed at the respective ends. These feet consist of narrow longitudinal fillets, two being placed at one end of the frame and one at the other,

Fig. 287.



Elevation of the Workshop Measuring Machine.

so that the machine rests on three supports, just as if it were an ordinary surface plate.

2. The upper part of each cheek is spread out into a flange having one inclined and one vertical face, so as to form together a guide for the headstocks B and C, whereof C is fixed and B is capable of being traversed along the bed by means of a hand-wheel F attached to a quick threaded screw which lies between the two cheeks of the bed, and is supported on cylindrical bearings at each end.

The guiding faces of the flanges are carefully scraped and made true planes, as are also the surfaces of the headstocks which rest upon them; and since the intersection of two planes is a straight line, it follows that the headstock B traverses in a definite direction along the plane bed of the machine, and that any point in it describes a close mechanical approximation to an exact straight line.

3. A hole is bored from end to end of the upper part of each headstock, great care being taken to make it truly cylindrical, and with its axis parallel to the bed of the machine. Also the two cylindrical holes have a common axis.

Each bore is then fitted with a cylindrical sliding piece, the diameter of the sliding piece being slightly less than that of the internal surface according to a difference previously determined and tested by difference gauges. The method of doing this will be explained hereafter.

Each cylindrical sliding piece is prevented from turning round in the headstock by a narrow key which forms a feather, and is recessed, partly into a longitudinal groove cut along the under side of the sliding piece and partly into the headstock itself. The cylinders terminate in the true planes D and E, which are circular in form, and as nearly as possible perpendicular to the axes of the respective cylinders.

4. The measuring plane D is movable by a screw with a graduated hand-wheel, the object of which is to put the plane approximately into any required position.

The actual measurement is performed on the side of the headstock C. Here the sliding cylinder which terminates in the true plane E is adjusted by a screw with twenty threads to the

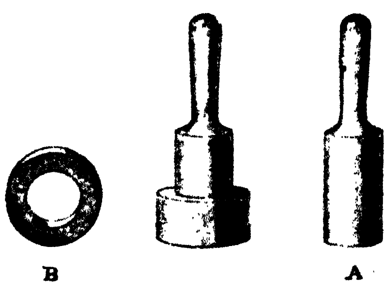
inch, and terminating in the hand-wheel P, having 500 divisions on its rim. A pointer R registers the motion of the wheel, and it follows that a movement of the rim of the wheel through the space of one graduation traverses the plane E through $\frac{1}{10000}$th of an inch.

Since the large wheel P is 11·8 inches in diameter, and has 500 graduations, it follows that the space travelled over in passing from one graduation in the rim to the next in order is 741 times that through which the measuring plane is shifted. In other words, if the measuring plane advances by $\frac{1}{10000}$th of an inch, a graduation in the rim of the large wheel advances by $\frac{741}{10000}$ths of an inch, which is rather more than $\frac{1}{13}$th of an inch, an interval that can be readily observed without assisting the eye by lenses.

In this machine there is space for measuring circular gauges up to 6 inches in diameter, and bars up to 12 inches in length. But for ordinary purposes, and for the construction of small gauges, a much smaller instrument will suffice, the working parts, however, being the same.

ART. 218.—We have now to speak of the construction of case-hardened cylindrical gauges, or standards, for comparison in the workshop, and the drawing shows the two principal forms.

FIG. 288.



1. There is an external gauge A, cylindrical in form, with a handle, and of varying diameters from $\frac{1}{10}$ inch up to 2 inches.

2. There is an internal gauge B, also cylindrical, and exactly fitting upon A.

These gauges are made in pairs, and serve to test the diameter

of any solid cylinder, or of any cylindrical opening having the same diameter.

In the collection of the School of Mines are two of these external or plug gauges, and one internal or collar gauge, whereof the respective diameters are one inch, one inch less $\frac{1}{10000}$th inch, and one inch.

Calling them in the order as above stated by the letters A, A′, and B, we find these results :—

1. Wipe A and B, which are exact 1-inch gauges, with a clean dry cloth, and endeavour to pass one over the other. We cannot do it, for the surfaces at once bite together, and we should say that the plug was too large for the collar.

Now rub a very little of the finest oil upon the surfaces of A and B, and it will be found quite easy to pass B upon A. If A be held in a vertical position, B will slowly sink from the top to the bottom of the cylinder. The smoothness and yet tightness of the fit is most remarkable, the oil preventing the adhesion and jamming together of the metallic surfaces.

2. We next proceed to test A′ and B, remembering that the diameter of A′ is $\frac{1}{10000}$th of an inch less than the internal diameter of B, and we begin by wiping the surfaces so that all oil is removed, and they are perfectly dry and clean.

It will now be found that B fits quite loosely on A′. If the gauge A′ be held in a vertical position, B will fall freely from the top to the bottom.

Again rub some oil upon the surfaces, which will fill up the vacant space, and A′ will pass through B very smoothly, but some slight resistance will be felt.

By handling these gauges the difference of fit due to a difference of $\frac{1}{10000}$th of an inch becomes very apparent.

The practical value of difference gauges is well understood. For instance, the cylinder of the moving headstock of a lathe requires as good a fit as possible, but that means that a true and proper allowance should be made in the size of the parts working together, and Sir J. Whitworth states that in the case of an ordinary lathe the hole in the headstock should be $\frac{1}{5000}$th of an inch larger than the cylinder.

In like manner gauges would prove to be of great service in

carrying out the manufacture of an axle, the journal being made to a standard gauge, and the bearing being bored out so as to fit a difference gauge somewhat larger in size, and of the precise difference in diameter, which experience has shown to be necessary, regard being had to the conditions under which the axle is to work.

It may therefore be conceded that every manufacturer should be in a condition to produce difference gauges for use in his workshop.

ART. 219.—Sir J. Whitworth has constructed a measuring machine with rectangular plane bars in place of the sliding cylinders, and with a higher mechanical multiplier, whereby he has measured up to $\frac{1}{1000000}$ of an inch.

We have not space to describe the apparatus fully, but may state that the sliding bars are rectangular, with faces made truly plane, and that they move in right-angled V-grooves, whose surfaces are also true planes.

The ends of the measuring bars are circular planes, each about $\frac{3}{10}$ths of an inch in diameter, and the utmost care is taken to ensure that these plane ends shall be truly perpendicular to the respective axes of the bars. We have described the method of securing this result.

Of the two measuring bars one is advanced by a screw having twenty threads to the inch, and terminating in a graduated hand-wheel with 250 divisions on its rim. This gives a quick motion of $\frac{1}{5000}$th of an inch for each graduation.

The other measuring bar is actuated by a combination of screw-gearing. There is first a screw with twenty threads to the inch, which terminates in a worm-wheel having 200 teeth upon its rim. Then the worm wheel is itself driven by a tangent screw carrying a hand-wheel with a micrometer graduation of 250 divisions upon its circumference.

It follows that the rotation of the tangent screw through one division will advance the bed screw of twenty threads by a space equal to

$$\frac{1}{250} \times \frac{1}{200} \times \frac{1}{20} \text{ of an inch,}$$

or $\frac{1}{1000000}$th of an inch.

Results of this character could be extended as far as we pleased in theory, but not in practice. The accuracy and truth of the pieces upon which we rely are so severely tested that the power of human execution soon fails, and hence we can appreciate the interest which this apparatus has awakened.

From the dimensions of the wheels in this machine it has been found that a motion of ·000001 inch in the measuring plane is equivalent to a motion of ·04 inch on the rim of the graduated hand-wheel, whence it follows that the machine magnifies the motion 40,000 times, or that the eye observes a graduation to traverse over a space 40,000 times as great as that which is being measured.

In order to estimate the degree of tightness between the plane face of a measuring bar and the corresponding plane surface of an object under measurement, a so-called *feeling piece* or *gravity piece* has been employed. The gravity piece consists of a small plate of steel with parallel plane sides, and having slender arms, one for its partial support, and the other for resting on the finger of the observer.

One arm of the piece rests on a part of the bed of the machine and the other arm is tilted up by the fore-finger of the operator. The plane surfaces are then brought together, one on each side of the feeling piece, until the pressure of contact is sufficient to hold it supported just as it remained when one end rested on the finger. This degree of tightness is perfectly definite, and depends on the weight of the gravity piece but not on the estimation of the observer.

In this way the expansion due to heat when a 36-inch bar has been touched for an instant with the finger-nail may be detected.

Also the movement of ·000001 inch has been indicated by the gravity piece becoming suspended instead of falling, and the piece has fallen again on reversing the tangent screw through two graduations, representing ·000002; showing the almost infinitesimal amount of play in the bearings of the screws.

For the coarser measuring machine the workman relies upon the sense of touch for feeling the size of the body which is being passed between the measuring planes.

ART. 220.—The national standard of length is a rectangular bronze bar, 38 inches long, and 1 square inch in transverse section. A cylindrical hole, $\frac{3}{8}$ inch in diameter, is sunk near each end to the depth of $\frac{1}{2}$ an inch; a second small hole is then bored at the bottom of the larger one for the reception of a gold plug, forming a table $\frac{1}{10}$ inch in diameter, on which three fine lines are engraved at intervals of $\frac{1}{100}$th of an inch in directions transverse to the length of the bar. The distance between the two middle lines is the length of the standard, and the object of excavating the holes is to obtain a measurement along the axial line of the bar.

This is an example of what is termed *line measure*, and line measure bars are compared by the aid of fixed microscopes, with optical contrivances for reading to $\frac{1}{50000}$th of an inch

In the year 1834 Sir J. Whitworth obtained two standard yards in the form of line measure bars, and by the aid of microscopes transferred the mean distance between each pair of engraved lines to a rectangular end measure bar, as nearly as he could accomplish the task. At the same time he constructed a *millionth* measuring machine for the reception of the bar.

It now became comparatively easy to subdivide the yard into feet, and for this purpose three bars were prepared, each a little longer than one foot. A temporary abutment was then raised in the bed of the measuring machine, and the bars were reduced and tested until (1) they became respectively of the same length, and (2) they filled up the length of the standard yard when placed end to end.

Further subdivisions of the foot were made, and finally a standard inch was arrived at.

Again, standard end measure bars gave birth to standard cylindrical gauges, and thus the mechanical measures adopted in the workshop have been originated and have been reproduced by the employment of end measure bars and a good measuring machine.

CHAPTER IX.

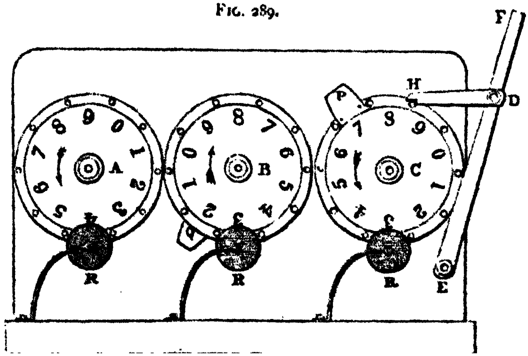
MISCELLANEOUS CONTRIVANCES.

We propose to examine in our concluding chapter various miscellaneous pieces of mechanism, and certain special contrivances which are of frequent occurrence in machinery, and with which a student of applied mechanics ought to render himself familiar.

Art. 221.—The invention of *counting wheels* is due to the celebrated Cavendish, who constructed a piece of apparatus for registering the number of revolutions of his carriage wheel. This apparatus is deposited in the Museum of George III. at King's College.

There is but one guiding principle in this branch of mechanism, however varied may be the details of the separate parts.

Fig. 289.



Each wheel of a series, A, B, C, &c., possesses ten pins or teeth, and it is contrived that one tooth only of C shall be sufficiently long to reach those of B; similarly B is provided with one long tooth which is capable of driving A.

Thus C goes round ten times while B makes one revolution, and so on for the other wheels; in this way the series is adapted for counting units, tens, hundreds, thousands, &c.

In fig. 289 the arm EF imparts rotation to the first, or *ratchet wheel*, by means of the paul HD; the number now registered is 988; after two vibrations of the arm the zero of C will reach the highest point, the tooth P will drive B through the space of one tooth; and the number registered will be 990; after ten more vibrations of the arm, P will again advance B, and at the same instant Q will move A, and will bring its zero up to the highest point: the three wheels will now register 000, having passed the number 999, which is the last they can give us.

The wheels are retained in position by the rollers R, R, R, mounted upon springs. As each roller is forced in between two pins, it not only acts as a detent, but also adjusts the wheel in its right position. Mr. Babbage employed this contrivance in his calculating machine.

ART. 222.—A small counting apparatus is attached to every gas-meter used in houses, and registers the number of cubic feet of gas consumed; here, however, the step by step motion is not employed, the dial plates are fixed, and a separate pointer travels round each dial respectively.

The pointers are placed upon the successive axes of a train of wheels, composed of a pinion and wheel upon each axis, the number of teeth on any wheel being ten times that upon the pinion which drives it. Suppose, for example, that the pointer on the plate registering thousands completes a revolution and adds ten thousand to the score, its neighbour on the left will have moved over one division on the dial registering tens of thousands, and thus an inspection of the pointers throughout the series will at once indicate the consumption of the gas.

These index-fingers move alternately in opposite directions, being attached to the successive axes of a train of wheels; the figures upon the counting wheels are also placed in the reverse order on every alternate wheel.

As we are only concerned with the counting apparatus, it is not necessary to explain the manner in which the flow of gas through the meter sets the train of wheels in motion, but we may

point out that there is no ratchet wheel employed, and that the flow of gas keeps up a constant rotation in an endless screw, which starts the train and maintains it in action.

A reliable counting apparatus which will record the exact number of impressions made by a printing machine is indispensable in some public departments, and it is found that the best result is arrived at by combining a ratchet wheel having a few deep well-cut teeth with the train of wheels used in the gas-meter.

The practice is to place upon the axis of the first or ratchet wheel carrying the units a pinion of 10 teeth gearing with a wheel of 100 teeth, then another pinion of 10 drives a wheel of 100 teeth, and so on, as far as we please. The train of wheels cannot fail to record the hundreds, thousands, &c., accurately; the only possibility of a mistake occurs with the units, but if the paul be carried well over the teeth of the ratchet, and if the wheel itself be driven at each advance a little beyond the point necessary to give another unit, if, in other words, the movement should be a little over-pronounced, the register will be perfectly exact.

In order to avoid the objection that the successive wheels turn in opposite directions, an idle wheel is interposed between each alternate pinion of 10 and its wheel of 100. All the pointers then revolve in the same direction as the hands of a clock.

ART. 223.—Where it is intended to print the figures registered, as in the numbering of bank-notes, the step by step motion is essential, and, further, each wheel must carry the letters upon its edge, and not upon the face; the apparatus employed is the same in principle as that of Cavendish, but the construction differs, the wheels being placed side by side and close together.

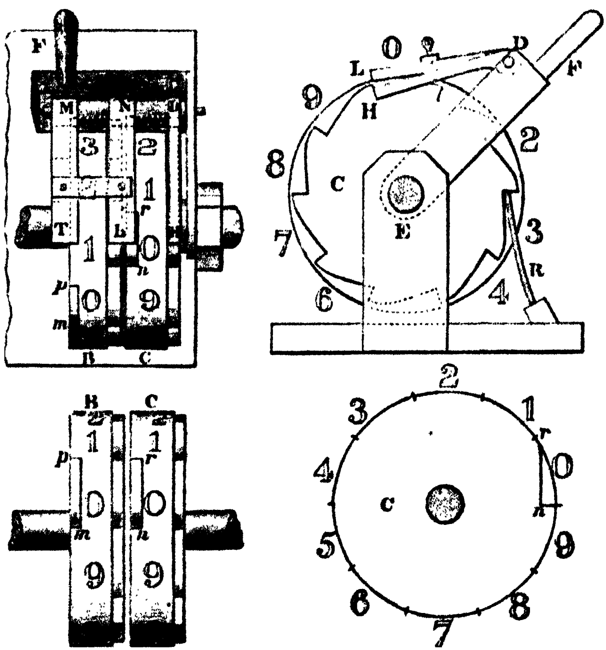
In order to present a fair idea of the construction of a numbering machine, that is of a machine designed for printing consecutive numbers, we refer in the first instance to a rough model belonging to the School of Mines, and shall afterwards give some description of a complete apparatus.

In the model, the wheels are flat cylindrical discs, having the numerals, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, painted upon their edges. On one side of each disc a ratchet wheel with 10 teeth is carved out, while on the opposite face of the disc only one nick or cut *rn* is formed. The cut *rn* is adjacent to the numeral 0, and is in-

tended to serve the same purpose as the projecting tooth in the previous arrangement.

The drawing shows the complete ratchet in side elevation, and the position of the teeth with reference to the numbers on the disc. Also it will be noticed that the first driver DH is a slender bar which encounters the teeth of the ratchet on which it works, whereas the second driver NL is twice as broad as DH, and encounters both the ratchet on the second numbering wheel, and also that slice of the rim of the first disc on which the nick *rn* is situated.

FIG. 290.



It is apparent that so long as NL is resting on the rim of the first disc at any part except where the cut *rn* is formed, it will be held above the teeth of the second ratchet, and will be inoperative, but that when NL falls into *rn* it can drive the second wheel.

Thus let the unit wheel mark 9, and the second wheel mark 0, the number read on the first two wheels being 09, meaning 9. At the next stroke NL falls both into *rn* opposite the numeral 0, and into the second ratchet at the part opposite the numeral 1, whereby NL advances both wheels by the space of one tooth, and the number 10 takes the place of 09.

In like manner there is a second cut or nick *pm* adjacent to the numeral 0 on the second wheel, and on the opposite side to the complete ratchet. This determines the period when the next driving paul MT shall advance the third numbering wheel, and thus the series is continued.

It follows that the consecutive advance of the respective wheels may be provided for by the employment of ratchets, having alternately ten teeth and one tooth respectively, and placed in regular order upon the numbering wheels, as shown in the diagram.

ART. 224.—We can now explain with more particularity the construction of a numbering machine, a considerable portion of which is set out in the annexed diagrams.

The mechanism is automatic or self-acting, the operator grasps the handle H, and moves it to and fro as far as it will go; each time that he does so the type is inked, the numbering wheels are adjusted, and an impression is taken.

In order to comprehend the operation we may point out that the principal working parts are the following:—

1. The handle H, centred at C, and provided with an arm carrying the printing wheels. The central axis of these wheels is at P, and since P is rigidly attached to the handle and C is also a fixed point, it follows that CP is of constant length.

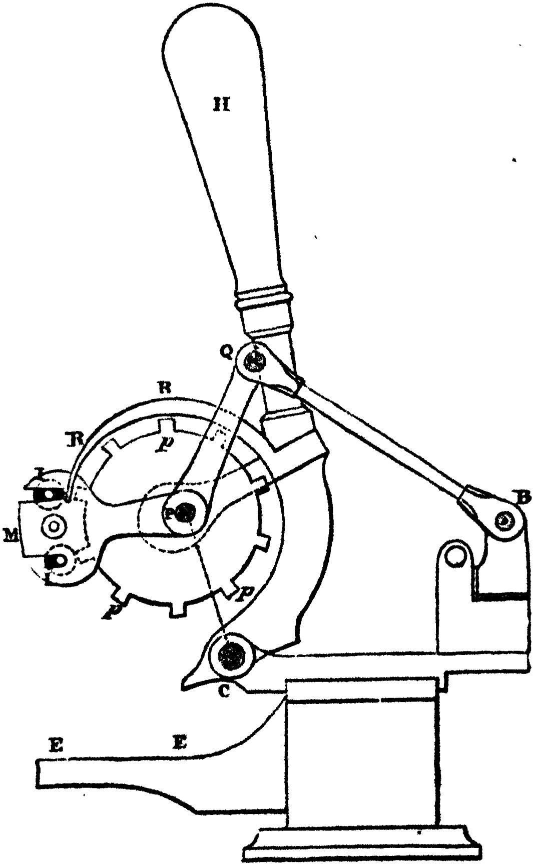
2. The two cranks, CP, BQ, with the connecting rod PQ, whereof CP is an imaginary line, but BQ and PQ control the inking apparatus. It has already been stated that advantage may be taken of the different positions of PQ in the above combination, and an example is here afforded, as will be seen immediately.

3. The numbering wheels lie side by side, and the projecting portions, marked *p*, *p*, are the successive numerals, 0, 1, 2 . . . 9.

4. The inking rollers, marked I, I, work in slots in the arm

PM, and are pulled towards P by elastic bands, not shown in the drawing. Connected with the inking rollers is the circular table

FIG. 291.



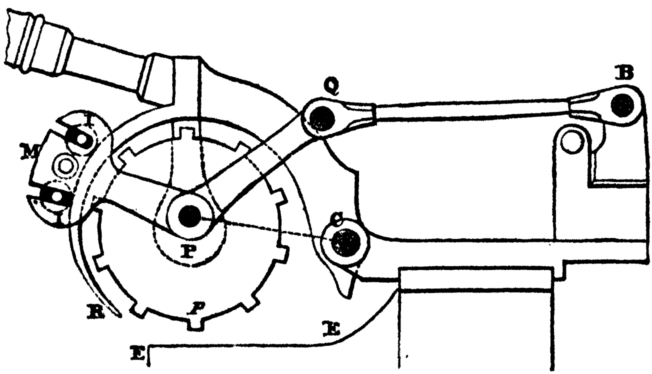
RR, upon which a supply of printing ink is spread out. In the working of the machine, the rollers, I, I, run upon the table, RR, and receive from it a supply of ink; they then pass on to the

faces of the printing type, and supply them with sufficient ink for an impression.

5. There is an impression table, EE, on which the operation of printing is performed.

6. There are two ratchet wheels in combination, being an ordinary and masking ratchet wheel respectively, which advance the numerals according to some required rate of progression. For a description of such wheels we refer the student to Art. 138; they are not inserted in the present diagrams.

FIG. 292.



By comparing the annexed sketch with that given previously, the operation of the machine will be clearly understood.

When the handle H is depressed to the full extent, the numbering wheels are brought down to the printing table, EE, and an impression is taken. At the same time the inking rollers run back upon RR, and take up a supply of ink.

During the time that the handle is being raised, the ratchet wheels do their work, and advance certain numbers as may be required. The inking rollers, in their turn, run from the table, RR, to the type, and supply the numerals with sufficient ink for the next impression, and thus the process goes on with a degree of ease and certainty which it is one of the triumphs of mechanical art to accomplish.

ART. 225.—As regards the operation of the ratchet wheels, it will be remembered that an ordinary masked ratchet suffices to suspend the operation of advancing the unit wheel until two strokes have been made, and thus it becomes easy to print each number in a series, such as 101, 102, 103, &c., twice over. Also after the unit ratchet has done its work, some method embodying the principle set forth in previous articles will be employed for carrying on the motion to the wheels printing tens, hundreds, and so on.

But there is yet something more to be provided for, inasmuch as it is often an advantage to print the odd numbers, as 101, 103, 105, &c., in one column, and the even numbers, as 102, 104, 106, &c., in another column.

Or the same machine may be required to print consecutive numbers.

The arrangement for effecting this double purpose will be understood by referring back to Art. 138, and it will be there seen that the numbering wheels are carried by an arm in a circular sweep from the inking apparatus to the printing table; in travelling along they encounter the paul, which is fixed to the framework, and if the circle should simply graze, as it were, against the paul during its travel, one tooth only would be taken up; whereas by setting the paul so that it meets the circle at a point nearer its centre, and strikes it more directly, two teeth may be taken up, and thus either *one* or *two* units may be advanced at each impression.

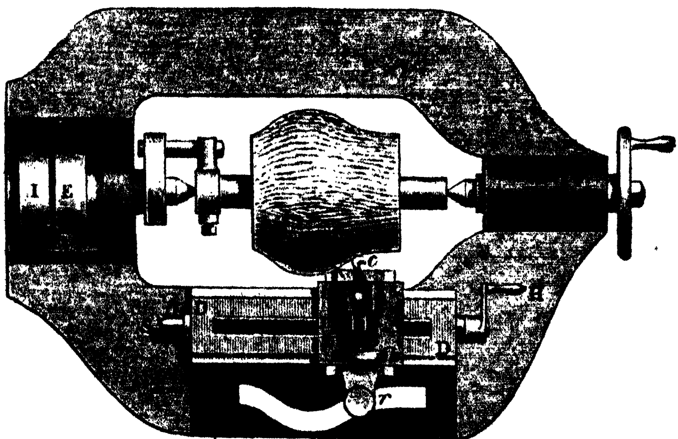
ART. 226.—The ninth chapter was devoted to illustrations of the importance of truth of surface, and we may now refer to a lecture diagram of a complete machine, which is an ordinary example of a combination of elementary surfaces, viz., the true plane, the screw, and the cylinder.

The machine is in use at Woolwich for turning out rapidly the bosses or naves of wheels, and is also a direct application of the copying principle.

The block of wood, intended for the nave, is supported between centres, as in a lathe, and is rotated rapidly by a belt passing over a driving pulley E. The adjacent pulley I rides loose on the shaft, and is an idle pulley.

The cutter *c* is carried by a slide rest, which resembles the ordinary slide rest of a lathe. There is a true plane surface DD, with inclined sides, supporting the saddle *fe*. The handle H operates a screw which traverses this saddle to and fro along the line parallel to the line of centres of the lathe. The form of the nave is determined by the cam-groove in the plate AB in which a roller *r* runs as shown in the diagram. A slide carrying the cutter *c* is attached to *r*, and it follows that the traversing of the saddle *fe* along DD will cause the cutter *c* to advance or recede in exact accordance with the outline of the cam-groove.

FIG. 293.



A B

When the block is first put in the lathe it is rough and uneven, and the cutter *c* should be advanced slowly and cautiously, in order that it may commence by paring off the principal inequalities. The cutter is therefore brought in or out by a screw actuated by a hand wheel *h*, and at the same time is traversed longitudinally by the handle H, the definite position of the cutter on the saddle being, of course, quite independent of its motion as due to the cam-groove.

In this way the machine does its work rapidly and effectually, the cutter runs to and fro along the outline of the block, and

removes the material while copying and preserving the outline of the guiding curve. The amount removed is also under the control of the operator, and is regulated by the hand wheel *h*.

Art. 227.—We have reserved for the concluding chapter an account of the principal conditions which obtain in the construction of the escapements of watches, and have to show that the principle of the '*dead beat*' is recognised in the three forms which are in common use.

And here we may remark that the pendulum of a clock appears as the *balance wheel* in a watch.

A wheel, pivoted on very small steel pivots, and working in jewelled supports, is attached to a flat spiral steel spring in pocket watches, or to a more powerful helical steel spring in marine chronometers. This wheel vibrates under the action of the pull of the spring just as a pendulum would do under the pull of the earth, but under better conditions theoretically, for the force of the spring increases with the angle through which the balance wheel swings, and in direct proportion to that angle; the result, therefore, is that the swing of the watch pendulum is always performed in very nearly equal times whether the arc of swing be increased or diminished.

We have what is technically called an *isochronous pendulum* in the balance wheel, and this is important, because the time is not affected by small changes in the arc of swing. Further, it should be noted that the balance wheel swings through an angle which is enormous as contrasted with the swing of the pendulum, being more than a whole revolution in the case of the chronometer or lever watch.

Consider now the construction of the chronometer escapement, which fulfils our conditions with an exactness that may well surprise us, and which exhibits in its arrangement a marvellous amount of mechanical skill and forethought.

The detent, which corresponds to the anchor pallets, consists of four principal parts :—

1. The locking-stone, D, a piece of ruby, upon which the tooth of the escape wheel rests.

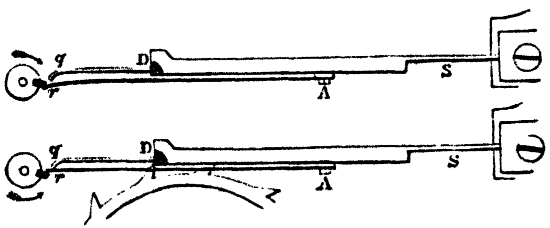
2. The discharging spring, A*r*, which is a very fine strip of hammered gold.

3. A screw at A to fix A*r* to the stem of the piece SD.

4. The shank of the detent, consisting of a projecting arm D*q*, the part DA, and the portion at S, which is cut away to form a spring which may bend and act as a pivot on which the whole detent can be moved a little.

The small circle on the left hand has a projecting piece which keeps the escapement in action, and it is a part of the stem of the balance wheel.

FIG. 294.



As the balance wheel swings to and fro, this roller also vibrates, and when passing downwards it encounters the spring A*r*, and pushes it aside without any perceptible effort, because the spring bends from the distant point A.

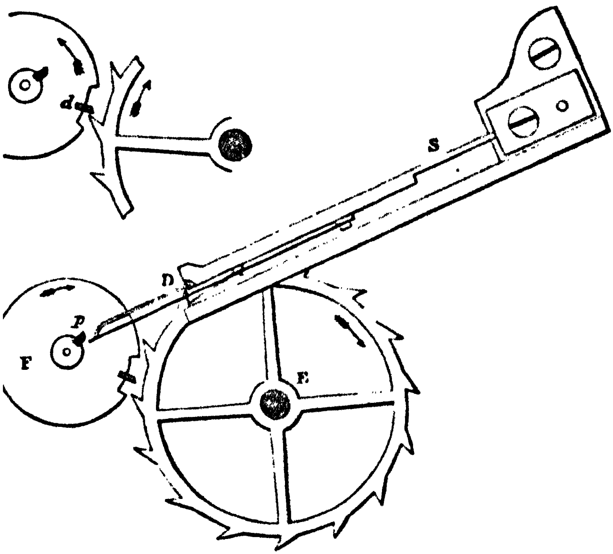
On its return the projection finds the spring to be capable of bending, not from the distant point A, but only from the point *q* against which it rests. The roller therefore takes the spring and the whole detent with it and raises the locking stone D from the point of the escape wheel, the escape wheel at once flies round, and before it can be caught upon the next tooth by the return of the detent to its normal position, is enabled to give an impulse to the balance wheel by striking against the point *d* in the manner shown in fig. 295.

The whole arrangement can now be studied from the drawings, and is complete with the exception of a banking screw which supports the detent when coming back to the position of rest. It will be seen that the large circle F is fixed to the smaller one, and that the projection marked *d* is quite clear of the escape wheel while a tooth is resting against the detent.

The advance of the escape wheel is so instantaneous that it is not seen to move : it appears to tremble a very little, but it comes to

rest again so quickly that the eye cannot follow and can scarcely detect the motion. It is, of course, made evident by watching the spokes of the wheel.

FIG. 295.



What, then, has been the action? In the first swing of the balance the only obstacle has been the bending aside of the spring *Ar*, which is no more than bending a light feather. In the second swing the pendulum or balance wheel has had to lift the detent: this is a momentary and very small action against it, but as quick as thought the action is compensated and the balance receives its impulse through equal distances on each side of the middle of its swing, according to the principle of the dead-beat escapement.

Here, then, theory and practice are in exact accord.

It should be noted that the impulse is given at every alternate swing of the balance, and not with every swing as in the case of the clock pendulum.

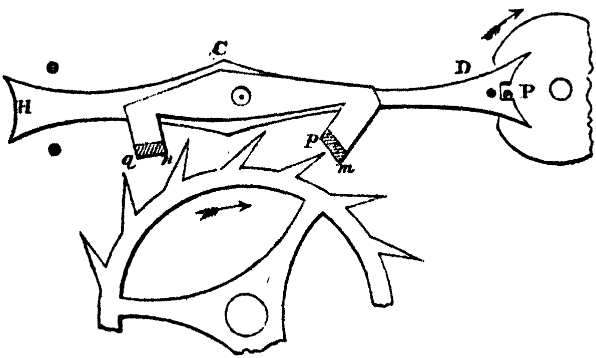
ART. 228.—The *Lever Escapement* comes next in order, and here we return to the anchor pallets. The escape wheel is locked

by these pallets, and gives its impulse upon their oblique edges in the manner described in Art. 55.

The balance wheel is free during the greater part of the swing (hence the name of *Detached Lever*), and oscillates through a considerable angle. The unlocking occupies an angle of about 3°, and the impulse is given through about 9°.

These are just the conditions which prepare us for the principle of the dead beat.

FIG. 296.



The pallets, *mp*, *qn*, are jewels inlaid into the arms, the light steel bar DH is the lever movable about C as a centre. An open jaw at one end is capable of receiving a ruby pin, P, attached to the roller, which is on the axis of the balance wheel and moves with it. There are also banking pins, and a small guard pin to prevent the lever from falling out of position.

As the balance vibrates the pin P swings to and fro with it: in doing so it enters the open cut at the end of the lever, and removes the locking portion of the pallet from the point of the escape wheel. Instantly the escape wheel flies forward, and by pressing against one oblique edge suddenly pushes on P, and the lever is no longer moved by the balance wheel but imparts an impulse to it. Very soon, that is after the nine degrees of the swing are consumed, the ruby leaves the lever behind, and the wheel goes on detached and unchecked in its swing.

On its return the pin finds the lever where it had left it, carries

it forward, unlocks the escape wheel, receives its impulse, leaves the lever behind, and the balance is free for the rest of the swing.

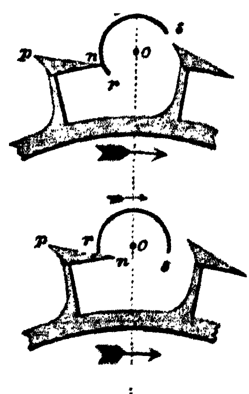
The only action against the balance is that of unlocking the escape wheel so as to enable it to give the impulse. This is very brief in duration as compared with the whole swing, and the watchmaker takes care that it shall be as little as possible. The impulse is given just at the middle of the vibration, and the construction follows out the theory very closely.

ART. 229.—Lastly, we may refer to the escapement of the so-called *Geneva Watches*, which is Graham's cylinder movement.

Here the balance is attached to a very thin cylinder centred at *o*, and the point of a tooth rests upon either the outside or the inside of this cylinder during a part of the swing. In this respect the action corresponds exactly to the friction of the escape tooth against the circular part of the pallets in the dead beat.

As the cylinder vibrates round its centre *o*, the tooth *pn* comes under the edge at *r*, and pushes the cylinder onward: this gives an impulse. The tooth soon passes *r*, flies into the cylinder, and is stopped by the concave surface near *s*; the cylinder now vibrates in the opposite direction, *pn* escapes, and in doing so gives another impulse at *s* to the cylinder in the opposite direction, and thus the action goes on.

FIG. 297.



The impulse would not be given in the middle of the swing, but through small arcs equally distant from the middle point, and equal in length to each other. Hence this combination is nearly identical with the dead-beat escapement, although inferior to it in this latter particular.

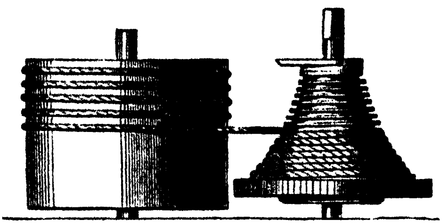
The manner in which the effects of the expansion and contraction of the material of the pendulum rod and the balance wheel, due to changes of temperature, are rendered innocuous, forms a separate branch of the subject.

ART. 230.—*The fusee* is adopted in chronometers, and in most English watches, in order to maintain a uniform force upon the train of wheels, and to compensate for the decreasing power of the spring.

The spring is enclosed in a cylindrical barrel, and sets the wheels in motion by the aid of a cord or chain wound partly upon the barrel and partly upon a sort of tapering drum called a fusee.

As the spring uncoils in the barrel, the pull of the cord decreases in intensity; at the same time, however, the cord unwinds

FIG. 298.



itself from the fusee, and continually exerts its strain at a greater distance from the axis, that is, with a greater leverage, and with more effect.

The theoretical form of the fusee is a hyperbola, being the section of a right cone made by a plane parallel to the axis of the cone.

To prove this statement we must first recognise the law according to which an elastic body under extension or compression exerts a force of restitution whereby it tends to recover its original form.

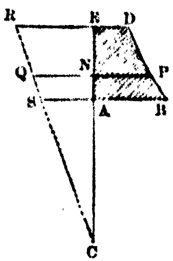
This law was stated by Dr. Hooke as being contained in the maxim *ut tensio sic vis*, by which it is intended to convey that when a body is extended or compressed in a degree less than that which produces a permanent derangement of form, the force necessary to keep it extended or compressed is proportional to such extension or compression.

Take a spiral steel spring balance, for example; hang upon it successive weights of 1, (2), (2 + 1), (3 + 1) lbs., the index point

will descend through equal spaces for each additional pound weight, and will rise by equal spaces as each pound is successively removed.

Assuming the law to hold exactly when the spiral spring is subject to a force of torsion instead of one of direct extension, we shall have the pull of the spring proportional to the angle through which the barrel has been made to turn.

FIG. 299.



Let DPBA represent one-half of the section of a fusee, DPB being the curve whose equation is to be found.

Draw DE, PN, BA perpendiculars on EA; take ER, QN, SA to represent the pull of the spring when the chain is at the points D, P, and B, respectively.

According to Hooke's law, the force of the spring will decrease uniformly as the chain passes from D to B, therefore RQS must be a straight line inclined to EA. Produce RQS to meet EA in C.

Then $\frac{QN}{CN} = \frac{SA}{CA}$, which is a constant ratio, by reason of the law of elasticity.

Assume that this ratio is represented by m,

$$\therefore \; QN = m \,.\, CN.$$

In order that the fusee may accomplish its object, the product of the pull of the spring into the arm NP must remain constant for every position of P.

Hence, calling $CN = x$, $NP = y$, we have
(pull of spring) $\times NP = m \,.\, CN \times NP = mxy$.

But this product is not to vary,

$$\therefore \; mxy = \text{a constant quantity,}$$
$$\text{or } xy = \text{a constant,}$$

which is the equation to a hyperbola.

In practice, where great accuracy is required, the strength of the spring is tested by fixing a light lever to the winding square of the fusee, and observing whether the pull of the spring is

balanced in every position by the same weight hung at the end of the lever. The fusee would be cut away a little where it was necessary to do so.

ART. 231.—In mechanism the fusee is frequently employed to transmit motion instead of to equalise force, and enables us to derive a continually increasing or decreasing circular motion from the uniform rotation of a driving shaft.

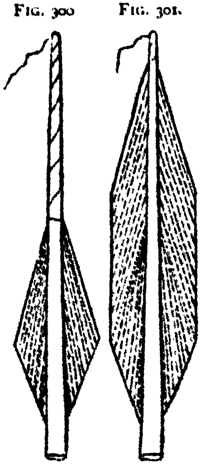
The groove of the fusee may be traced upon a cone or other tapering surface, or it may be compressed into a flat spiral curve: in all cases the effect produced will be that due to a succession of arms which radiate in perpendicular directions from a fixed axis, and continually increase or decrease in length.

The fusee can of course only make a limited number of turns in one direction.

A *flat spiral fusee* occurs in spinning machinery, and serves to regulate the velocity of the spindles, and to ensure the due winding of the thread in a succession of conical layers upon a bobbin or *cop*.

The formation of the *cop* is a problem upon which a vast amount of mechanical ingenuity has been expended; and without entering too much into details, we may observe that there are two distinct stages in the process of winding the yarn upon a spindle so as to produce a finished cop.

FIG. 300 FIG. 301.



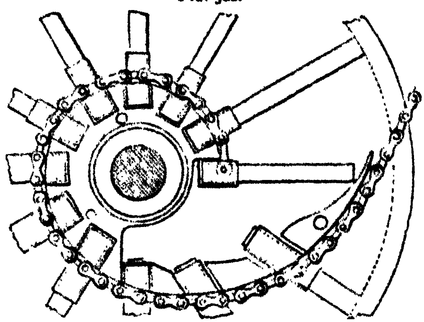
The *copbottom* (fig. 300) is first formed upon a bare spindle by superposing a series of conical layers with a continually increasing vertical angle.

The body of the cop is then built up by winding the yarn in a series of equal conical layers. (Fig. 301.)

The winding-on of the yarn begins at the base of the cone and proceeds upwards to the vertex; the spindles are driven by a drum which rotates under the pull of a chain, and they may be made to revolve with increasing rapidity by placing a fusee upon the driving shaft and causing the chain to coil upon it.

Such an arrangement as shown in fig. 302 will be adapted to the winding of a uniform supply of thread upon a conical surface ; and we can easily comprehend that a fusee of fixed dimensions will do very well for building up the body of the cop after the foundation is made. The main difficulty occurs in producing the copbottom, where the series of conical layers of continually increasing vertical angle demands a fusee whose dimensions shall gradually contract towards the centre.

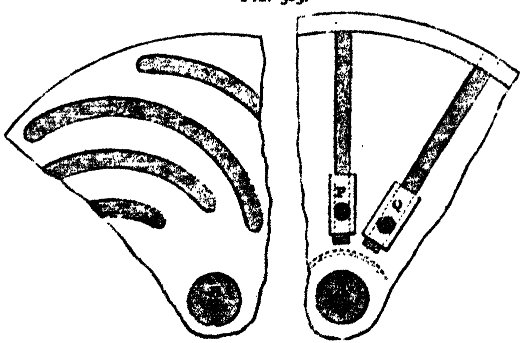
FIG. 302.



The method of contracting the form of the fusee may be explained as follows :—

Fig. 303 represents portions of two flat discs having axes at A and B, and upon which are cut radial and curved grooves in the

FIG. 303.



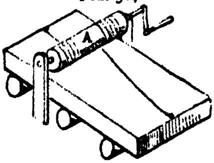
manner indicated ; it being arranged that when one plate is placed upon the other, the pins P and Q shall travel in both sets of grooves at the same time.

We can easily see that the blocks which carry the pins will

move along the radial grooves as the disc B turns relatively to A, and that by this combination we can obtain a spiral fusee of any required form, and can contract or enlarge its dimensions at pleasure.

ART. 232.—If two cords be wound in opposite directions round a drum, A, and the ends of the cords be fastened to a movable carriage, it is evident that the rotation of A in alternate directions will cause a reciprocating movement in the carriage.

FIG. 304.

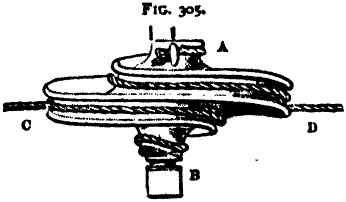


This is a mangle in its simplest form, and the objection that the handle must be continually turned in opposite directions may be obviated by the use of the mangle wheel.

It is clear that if the drum were divided into two portions, and that each half instead of being cylindrical were formed into a fusee, the motion of the piece driven by the rope would be no longer uniform but would vary with the dimensions of the fusee.

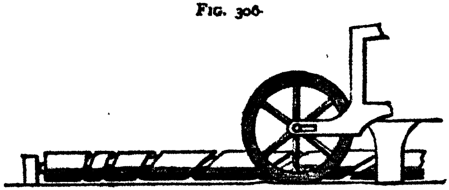
Hence the drum A has been replaced by a spiral fusee in the self-acting mule of Mr. Roberts, and thus the motion of the carriage is gradually accelerated until it has reached the middle of its path, and then decreases to the end of the movement.

FIG. 305.



It must be understood that the cord fastened at A goes off at C, while that fastened at B passes on to D.

FIG. 306.



A helical screw of a varying pitch traced upon a cylinder would produce a similar variable motion of the mule-carriage, and has

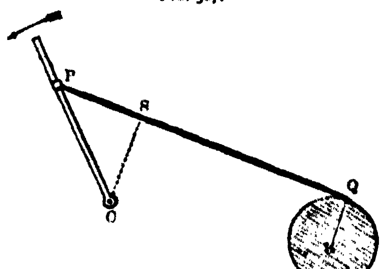
been applied in a machine constructed upon a different principle, in order to obtain a continually decreasing motion of the carriage. It replaces the fusee.

ART. 233.—*Mr. Roberts's winding-on motion* reposes upon the principle of the fusee, though in a modified form.

Let one end of a rope which is coiled round a drum be attached to a point, P, in the movable arm CP; it is evident that the rotation of CP about the centre of motion C will cause some portion of the rope to be unwound from the barrel (fig 307).

Draw CS perpendicular to the direction of the rope; then, at any instant of the motion, the arrangement supplies the jointed rods, CP, BQ, mentioned in Art. 93, and it is manifest that the rate at which the string is unwound will vary as the perpendicular CS.

FIG. 307.



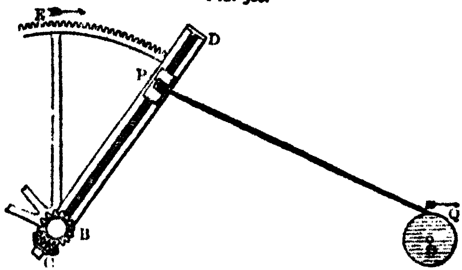
This rate is greatest when CP is perpendicular to PQ, but decreases to nothing when CS vanishes, and here, therefore, the varying arm of the fusee exists in a latent form.

Next conceive that the conditions are changed, and that the drum B moves to the right hand through a moderate space, while CP remains fixed. The cord will unwind from the drum with a nearly uniform velocity.

If, finally, the arm CP be not fixed, but be made to move from a position a little to the left of the vertical into one nearly horizontal during one journey of the drum, it is abundantly clear that we shall subtract from the uniform motion of unwinding that amount which is due to the action of a fusee, and that if the spindles derive their motion from the rotation of the drum they will continually accelerate as the drum recedes from CP. In this way we can make up the body of the cop. To form the cop-bottom, it is necessary that the winding on should begin more rapidly, and should gradually diminish. This character of motion

is produced by causing the nut P to traverse CD in successive steps during each journey of the drum. As soon as the cop has

FIG. 308.



attained its full diameter the nut ceases to travel along CD, and the thread is wound in uniform conical layers.

ART. 234.—Harrison's *going fusee* is employed in watches or clocks having a fusee for the purpose of keeping the timepiece going while the spring is being wound up.

The principle on which it is constructed will be readily understood. Referring to the small sketch in fig. 309, conceive that a force applied in direction of the arrow at x is to be communicated to y. Let x and y be connected by a spring S, and suppose further that some resistance to motion is felt at y, then the force at x would compress the spring until y began to yield, after which x and y would move together as if they were one piece.

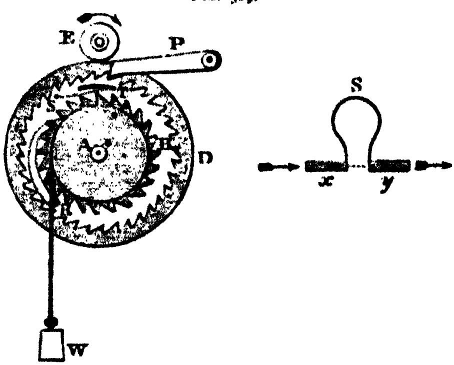
Next suppose that x becomes fixed, then it is apparent that the elasticity of the compressed spring S will cause a push to be felt at y, and that for a short interval the spring would urge y onward just as if the driving pressure on x had been maintained unimpaired. In other words, if we interpose a spring between the driver and the thing driven, it is possible to obtain some amount of working pressure from the spring after the driving force has ceased to act.

We pass on to explain the drawing, which is taken from a model arranged for making the contrivance intelligible to a learner. If the construction adopted in a watch were more closely followed it would be difficult to show the working parts.

A weight W, hanging upon a string wound round the disc A,

supplies the pull of the chain on the fusee, and the disc is provided with a ratchet, called the winding ratchet, having an ordinary detent at R. On the same axis as A is a circular plate B, having a ratchet, called the going ratchet, which is prevented from recoiling by a detent P. Connected with A, and forming part of it, is the great wheel D, which is shown in the model as a pitch circle without teeth, and which drives the pinion E.

FIG. 309.



The object is to keep E rotating in the direction of the arrow, and from what has been stated it is apparent that so long as W acts upon A it will act also on D and will drive E. The difficulty occurs when the weight is being wound up, in which case A is turned in the reverse direction, and D would be powerless to drive E, unless some new agent were called into play.

In the model this agent is supplied by the strong indiarubber cord ST, and in a watch it takes the form of a curved strip of steel, whose ends are brought near together. A circular slot marked by a dark thick line terminating in T determines the amount through which the recoil of the spring can act, and it should be noted that ST is attached at S to a pin on the plate B, and at T to a pin in the plate D.

The first action of W is to stretch ST, and when the spring is sufficiently stretched D begins to move and travels round, the going ratchet slipping tooth by tooth under the detent P. When

the winding-up takes place and the pressure of W is taken off, the action of the spring begins, and there is a pull of S towards T, and of T towards S. The detent P, acting on the going ratchet, prevents any motion in the direction ST, and therefore T approaches towards S, but in doing so ST pulls D in the same direction as that in which W previously moved it, and the result is that the motion of E continues, notwithstanding that the weight W has ceased to act.

ART. 235.—The mechanism of a keyless watch, so far as the winding of the spring and the setting of the hands are concerned, may be of interest to the student.

There are three separate things to be provided for :—

1. The spring is to be wound up without opening the case or inserting a key.

2. The same button or handle which turns the spring should be available for setting the hands.

3. No injury should happen if the winding button be turned in the wrong direction.

FIG. 310.

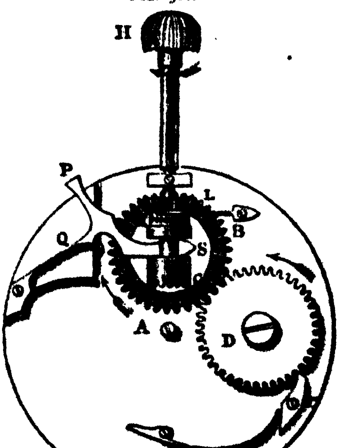
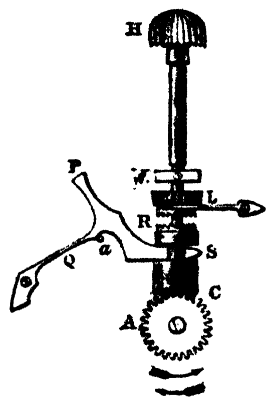


FIG. 311.



Taking number 3 first, the contrivance adopted is a very old

one in mechanism, being a ratchet cylindrical coupling, whereof the two parts are held together by a spring.

It is shown at R, in figs. 310 and 311, the spring being marked Q, and terminating in a tail, which holds the coupling by the groove S. When the spring is in action the two halves of the coupling are locked together, but upon depressing the stud P the tail S is forced down, and the halves of the coupling are separated. The button H, which is rotated for winding the watch-spring, or for setting the hands, is connected directly with RC, the lower half of the coupling, and when the spring is in action the turning of RC causes L also to rotate.

Thus, when the button is rotated in the direction of the arrow, the lower half of the coupling drives the upper half, but when the button is rotated in the opposite direction, the ratchet teeth on the lower half slip over those in the upper half, and all that happens is that RC vibrates up and down by the depth of a tooth. The movement is harmless, for no part of the mechanism which does any work is affected by it.

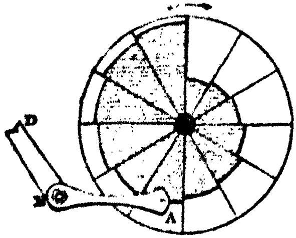
The winding square in an ordinary watch is replaced by the spur wheel D, having a detent *r*, actuated by a spring, whereby the toothed wheel serves the same purpose as a ratchet wheel with saw-cut teeth. This contrivance is borrowed from the engineers, as will be remembered.

The spur wheel D gears with another spur wheel B, on the face of which is a flat bevel wheel which engages with another bevel wheel L attached to the upper half of the ratchet coupling. The rotation of H in the direction of the arrow drives RC, which again drives L, and so causes B and D to rotate, and thereby to wind up the watch.

The setting of the hands is accomplished by a wheel A which operates on the minute hand. The lower half of the coupling RC terminates in a cylindrical toothed wheel, technically known as a crown or contrate wheel, which is brought into gear with the spur wheel A when the stud P is depressed. It will be apparent that when RC is lowered the winding-up stops, and at that time the hands can be set. Whereas when RC is allowed to rise the crown wheel is thrown out of gear with A, and the winding-up can begin. Each action is shown separately in the sketches.

ART. 236.—The *snail* is chiefly found in the striking part of repeating clocks. It is a species of fusee, and is used to define the amount of angular deviation of a bent lever ABD, furnished at the end A with a pin which is pressed against the curve of the snail by a spring, and is attached at the other end to a curved rack, whose position determines the number of blows which will be struck upon the bell.

FIG. 312.



In order to form the snail, a circle is divided into a number of equal parts (twelve, for example), and a series of steps are formed by cutting away the plate and leaving a circular boundary in each position.

As the snail revolves, ABD passes by jerks into twelve different positions, and the clock strikes the successive hours.

Since the point A describes a circle about B, it is clear that the depth of each step must vary in order to obtain a constant amount of angular motion in the arm BA during each progressive movement. It will be seen that the circular arc described by the end of the small lever has its tangent at A, when in the position sketched, parallel to the vertical diameter which divides the snail into two equal parts, and this reduces the inequality between the steps.

ART. 237.—The *disc and roller* is equivalent to the fusee, and is now but little used, on account of the probability that the roller will occasionally slip.

This arrangement consists of a disc A, revolving round an axis perpendicular to its plane, and giving motion to a rolling plate B, fixed upon an axis which intersects the axis of the disc A at right angles.

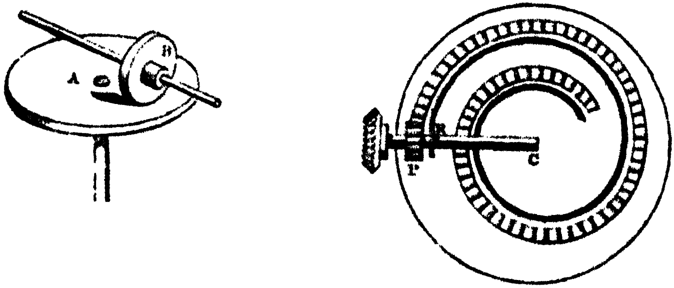
Supposing the rotation of the disc to be uniform, that of the roller B will continually decrease as it is shifted towards the centre of A, and conversely.

This is precisely the effect produced by a fusee.

The roller may be a wheel furnished with teeth, and may roll upon a spiral rack, as shown in the diagram.

As the disc revolves the pinion P slides upon the square shaft, and is kept upon the rack by the action of a guide-roller, R, which travels along the spiral shaded groove.

FIG. 313.



This example is by no means put forward as a good mechanical contrivance, for indeed the disc and roller possesses an inherent defect which should be diminished as much as possible in practice and not exaggerated. The bounding circles of the roller run with the *same* linear velocity, whereas the circular paths upon which they are both respectively supposed to roll move with *different* linear velocities, by reason of their being concentric circles of unequal size traced upon a plate which rotates with a uniform angular velocity about an axis through the common centre.

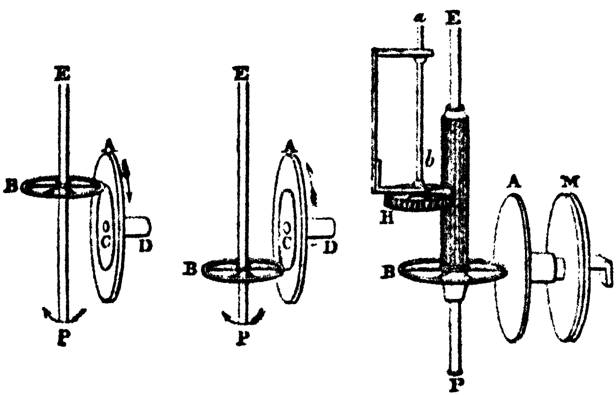
It is geometrically impossible that the bounding circles of the roller can both roll together, or the combination will fail in exactness as a piece of pure mechanism, and thus two rollers running round upon a flat surface or bed-stone in the manner suggested form an excellent pulverising or grinding apparatus. These rollers are called edge-runners: they are of large size and very weighty, and are placed near to the vertical axis about which they run.

It is apparent, without any proof, that the disc B will change the direction of its rotation as soon as it has passed over the centre of the plate A. This follows from the nature of circular motion, and, as we have indicated, the disc B will come to a standstill when its circumference reaches the centre of the driving plate A.

In like manner, if the axis of B be carried along in a straight

line passing over the centre of A, the student will understand that if A rotates in one direction while B is approaching its centre, and in the opposite direction, as soon as B has crossed the centre, it necessarily follows that the reciprocation of A will impart a continuous rotation in one direction to the roller B.

FIG. 314.



Such a result is exhibited in the sketch on the left-hand side of fig. 314, where the arrow shows that A is rotating in one direction when B is above the centre C, and in the opposite direction when B is below the same centre; while it becomes evident that the direction of rotation of B is correctly indicated by the arrow at P, and remains invariable.

ART. 238.—This movement has been applied in the construction of a continuous indicator. It has been shown that Watts' indicator gives the amount of work done in any stroke of a steam engine, and it follows that the general performance of the engine would be inferred from a comparison of several indicator cards taken at intervals.

The continuous indicator by Messrs. Ashton and Storey will, however, furnish a complete register of the work done in a given time. In this apparatus the piston of the indicator is attached to the roller B, and the plate A takes up, on a diminished scale, the reciprocating motion of the piston of the engine.

A general idea of the mechanism of the instrument may be gathered from the diagram. A grooved wheel, M, is connected directly with the disc A, and receives a reciprocating motion from the piston of the engine just as if it were the barrel of an ordinary indicator. The roller B is connected on the side marked P with the indicator piston, and travels up and down in the vertical line PE, the motion of B being subject to the opposing forces of a steel spring and the steam pressure, just as if it were the pencil in an ordinary indicator. The axis EP is provided with an elongated pinion, as shown, which gears into a wheel H in every position, the intent being that the rotation of B shall be carried by H to a recording or counting apparatus at the top of the instrument, and in the line *ba* produced.

The principle relied on is that the work done by the engine in a given time will be directly proportioned to the number of revolutions made by H in the same time.

Taking a condensing engine as an example, it is apparent that when the indicator piston is in the position which accords with the tracing of an atmospheric line in an ordinary diagram, the disc B lies exactly over the centre of the plate A and does not rotate. In such a state of things the wheel H also remains at rest and no work is done. Whereas any increase of the steam pressure carries B above the centre of A, and causes H to rotate with a velocity which increases as the pressure rises. Or, if the vacuum be improved, B sinks to a greater distance below the centre of A, and H rotates more rapidly during the return stroke, the result being to register work done in both cases. Also, if at any time there should be a subtraction of area in an ordinary indicator diagram, there will be a like subtraction here by reason that B will cross over the centre before the direction of rotation of A is changed, whereupon H will begin to rotate in the reverse direction, and some of the work previously scored up will be, as it were, rubbed out. Hence, the instrument sums up, or integrates, as it is termed, the actual performance of the engine in any given interval of time.

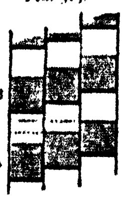
Art. 239.—*Step wheels* constitute a modification of toothed wheels; they are due to Dr. Hooke, and are used to ensure a smooth action in certain combinations of wheel-work.

It is evident that the action of two wheels upon each other becomes more even and perfect when the number of teeth is increased, but that the teeth at the same time become weaker and less able to transmit great force.

Dr. Hooke's invention overcomes the difficulty, and virtually increases the number of teeth without diminishing their strength.

Several plates or wheels are laid upon one another so as to form one wheel, and the teeth of each succeeding plate are set a little on one side of the preceding one, it being provided that the last tooth of one group shall correspond within one step to the first tooth of the next group. The principal part of the action of two teeth occurs just as they pass the line of centres, and there are now three steps instead of one from the tooth A to the tooth B.

FIG. 315.



A single oblique line might replace the succession of steps, but we should then introduce a very objectionable endlong pressure upon the bearings of the wheels.

Pinions of this construction are to be met with in planing machines, and are employed to drive the rack which is underneath the table; so again step wheels are used in marine steam engines where the screw shaft is driven from an axis considerably above it. They are valuable where strength and smoothness of action are to be combined.

ART. 240.—It has been shown that in the case of ordinary bevel wheels the pitch cones have a common vertex, and therefore of necessity the axes of rotation meet in a point. In some cases, however, it may be convenient that the axes of bevel wheels should pass close to each other without intersecting; the teeth have then a twisted form, and the wheels are known as *skew bevels*.

A general idea of the principle of construction adopted in wheels of this kind may be obtained by a simple experiment. Fasten a light rod of wood or a bright steel wire, such as a knitting pin, obliquely to the face-plate of a lathe, in such a manner that the direction of the rod intersects at an angle the axis or line of the lathe. Set the plate in rapid rotation, when the rod will appear to be replaced by a shadowy double cone. The effect is

due to the fact that any impression made on the retina of the eye endures for an appreciable time.

Now fix the rod in such a manner that it no longer cuts the line of centres, but passes obliquely on one side thereof, and set the face-plate in rapid rotation as before. The double cone will be replaced by a surface resembling a dicebox, which is curved in every part, and yet is generated by a straight line. This surface is known to mathematicians as a 'hyperboloid of revolution.' From the mode of generation it is clear that the surface in question is symmetrically disposed about the line of centres of the lathe, and we may regard that line as its axis.

If two such hyperboloids be brought into contact along a pair of generating lines they will roll together, and will serve to communicate a motion of rotation from the axis of one to that of the other; that is, they will enable us to communicate motion between two axes which are not parallel and do not meet, the directions of the generating lines in each surface determining the general direction of inclined teeth which might lie on the respective surfaces. Thus two hyperboloid surfaces replace, in the case of skew bevels, the pitch cones of ordinary bevelled wheels.

ART. 241.—Root's Blower, a mechanical equivalent for a fan, is a special contrivance which has been modified and varied by the ingenuity of patentees, and has been extensively used.

FIG. 316.

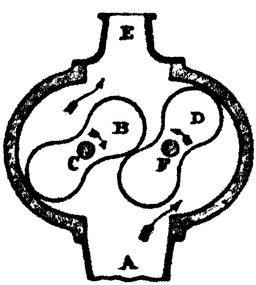
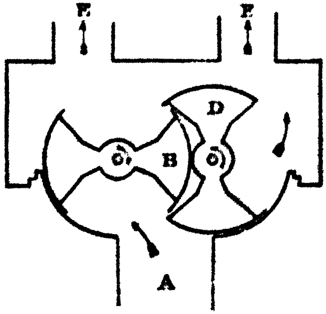


FIG. 317.



The sketch fig. 316 is from a model in the collection of the

School of Mines, and shows the principle of the action of this form of air-compressing machine.

There are two rotating pistons, B and D, centred on axes at C and F, and driven by a pair of equal spur wheels in opposite directions with equal velocities. A circular case surrounds the pistons, and it is provided that there shall be no actual contact of the surfaces of the rotating pistons, either with each other or with the casing, but that the working parts shall approach as closely as may be without any rubbing action or internal friction.

In the diagram the arrows show the manner in which the air enters the casing at A, and is swept forward to the exit at E, the two small arrows on the pistons being employed to indicate their relative directions of rotation.

In fig. 317 the blower is converted into a ventilating apparatus for a colliery, and is in use at a pit belonging to the South Durham Coal Company; it consists of two pistons, each 25 feet in diameter and 13 feet wide, driven by engines having a pair of 28-inch cylinders with 4-feet stroke.

The clearance between the periphery of one of the pistons and the central circle of the other is $\frac{1}{8}$ inch, and it is stated that when the ventilator runs at 21 revolutions per minute the amount of air passing through the machine is about 118,000 cubic feet per minute.

ART. 242.—The *Governor of a steam engine* usually appears under the form invented by Watt, and has proved of the greatest possible value in steam machinery.

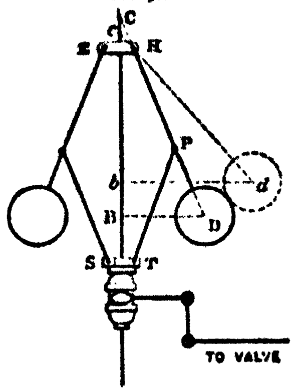
The diagram shows this well-known piece of apparatus, and the principle of its action may be described very briefly as follows:—

The engine imparts rotation to the balls of a heavy conical pendulum, and maintains them at a certain inclination to the vertical; if the velocity of the engine be increased, the balls open out more widely; if it be diminished they collapse, and in doing so they set in motion a system of levers which is connected with a throttle valve, and thereby regulate the supply of steam to the cylinder.

1. A common method of constructing the governor has been that shown in fig. 318. The balls are suspended at the points E

and H, a little on either side of the central vertical spindle CB. Each arm, as HD, is connected by a link to a sliding block ST. As the rate of rotation increases the balls fly out, ST rises, and in doing so actuates a lever which controls a steam valve and diminishes the supply of steam.

FIG. 318.



The arm DH is produced to meet the vertical axis in C, and DB is drawn perpendicularly to CB, whence the balls and suspending arms lie upon the surface of a cone whose axis is CB. The chief point to notice is that the number of revolutions made per minute by the balls depends upon the height of the cone, viz., CB.

The effect of placing E and H at a little distance from the axis CB is to cause the variation in the height of the cone to become greater for any given rise of the balls, and thereby to render the governor less sensitive. Thus the heights of the cone in the two positions shown are CB and *cb* respectively, the variation being equal to $Cc + Bb$.

Special methods of construction have been originated for reducing the amount of this variation; as to which the reader is referred to the author's text book on the Steam Engine.

An engineer can easily arrange that the variation in speed admitted by the governor shall not exceed one-tenth of the mean velocity, but it is of the essence of the invention that some change in the speed should be admissible: the balls cannot alter their position unless the time of a revolution changes, and they cannot accumulate such additional momentum as may be sufficient to move the valve until the rate of the engine has sensibly altered.

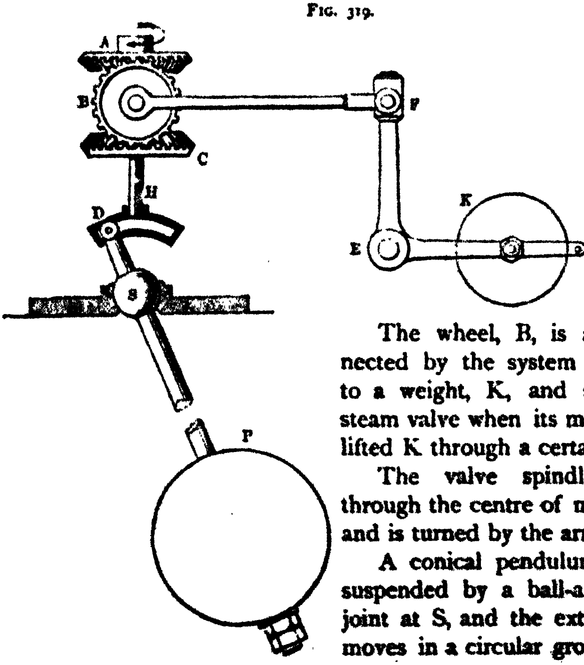
In some cases, as where the engine drives machinery for very fine spinning, it may be desirable to obtain an almost absolute uniformity of motion; or, again, it may be an object to avoid the fluctuations in speed to which the common governor is liable when any sudden change occurs in the load upon the engine.

ART. 243.—In order to control the engine with almost theoretical exactness, and to provide against the objections to which Watt's governor is exposed in certain extreme cases, Mr. Siemens has put forward a remarkable adaptation of epicyclic trains to the conical pendulum; and we shall proceed to an examination of his invention.

The original construction of this governor is exhibited in the diagram, and is better adapted for the purposes of explanation than a more recent arrangement (fig. 319).

An epicyclic train of three equal wheels, A, B, C, is placed between the driving power and the conical pendulum; of these, A is driven by the engine, C is connected with the pendulum, and B is capable of running round A and C to a small extent defined by stops, the joints at F and B being so constructed as to permit of such a motion.

FIG. 319.



The wheel, B, is also connected by the system of levers to a weight, K, and shuts the steam valve when its motion has lifted K through a certain space.

The valve spindle passes through the centre of motion, E, and is turned by the arm FE.

A conical pendulum, DP, is suspended by a ball-and-socket joint at S, and the extremity D moves in a circular groove, DH.

In this way the rotation of C is communicated directly to the pendulum.

It will be seen that a certain amount of energy is absorbed in preserving the pendulum at a constant angle with the vertical, and it is a part of the contrivance to increase artificially the friction which opposes the motion of the pendulum, and thus finally to make the pressure exerted by the weight, K, an actual measure of the amount of the maintaining force.

The governor is at work when the velocity of the engine is just sufficient to keep K raised through a small space.

In order to understand the peculiar action introduced by the epicyclic train we should remember that one of these two things will happen: either A and C will turn at the same rate, or else B will shift its position and run round the axis AH; there can be no departure from the rigid exactness of this statement.

Now, the wheel C is connected with the pendulum, and its rotation cannot be maintained without a constant expenditure of energy; in other words, the tendency of C is to lag behind A, and to cause B to run round the axis AH.

This indisposition in C to accept the full velocity of A is artificially increased by the friction until B shifts its position and raises the weight K permanently, and then of course it follows that the pull of K evidences itself as a constant pressure tending to drive the wheel C.

The pendulum being in this manner retained in permanent rotation, suppose that any increase were to occur in the velocity of A: the wheel C is in connection with a heavy revolving body, and can only change its velocity gradually, but K is already lifted, in the sense of being counterpoised, and the smallest increase of lifting power can therefore raise it higher; thus the tendency to an increase in the velocity of A will at once cause B to change its position, and will control the steam valve.

So sensitive is this form of governor to fluctuations in speed, that an alteration of $\frac{1}{50}$th of a revolution may suffice to close the throttle valve altogether. It is in its power to move the valve, as well as in its sensitiveness, that this arrangement presents so remarkable a contrast to Watt's governor, where the moving force on the valve spindle is only the difference between the momenta

stored up in the two positions of the balls. In this form of governor the power is only limited by the strength of the rods and levers; for it is apparent that the whole momentum stored up in the revolving pendulum would in an instant be brought to bear upon the valve spindle if any sudden alteration were to occur in the velocity of the wheel A.

In the method which has been adopted at Greenwich for registering the times of transits of the stars by completing a galvanic circuit at the instant of observation, a drum carrying a sheet of paper is made to revolve once in two minutes. A pricker actuated by an electro-magnet, and moving slowly in a lateral direction, is set in motion at the end of each beat of the seconds pendulum of a clock, and thereby makes a succession of punctures in a spiral thread running round the drum. The observer touches a spring at the estimated instant of the time of transit of a star across a wire of the telescope, and, producing a puncture intermediate to those caused by the pendulum, does in fact record the exact period of the observation. The regularity of motion in the drum is a matter of vital importance, and was at one time ensured by the employment of a clock train moving under the control of this pendulum of Mr. Siemens.

But, inasmuch as the tendency to simplify mechanical movements is always leading to new results, it has been found advantageous to replace the more complicated governor, with its train of wheels, by a conical pendulum controlled by a so-called dipper. The pendulum is driven by an ordinary clock train, and the dipper is carried round with the ball, and is merely a small flat plate dipping partially into a bath of glycerine and water. When the pendulum is accelerated the dipper enters the liquid more deeply, and the drag is greater, whereas when the velocity of the pendulum diminishes the dipper rises, and the motion is less retarded. It is said that this apparatus fulfils its purpose extremely well.

ART. 244.—*The Double Eccentric for Reversing an Engine.* When the piston is near the middle of its stroke in a locomotive or marine engine, the slide-valve will have moved over the steam-ports in the manner pointed out in fig. 320.

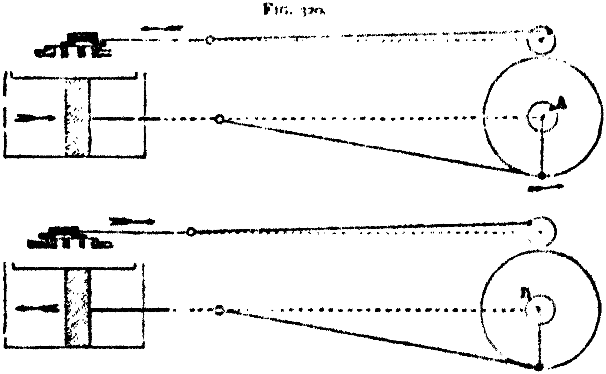
The slide-valve is connected with a point in the circumfer-

ence of a small circle which represents the path of the centre of the eccentric pulley, and the piston is connected with a point in a larger circle, representing the path of the centre of the crank-pin.

The piston and valve are shown as separated in the drawing, but the small circle is repeated in the position which it actually occupies, and the method of reversal is the following :—

In the upper diagram the piston is supposed to be moving to the right, and the valve to the left, the piston having travelled so far in its stroke that the valve is returning to cut off the steam : in order, therefore, to change the motion, we must drive the piston back by admitting the steam upon the opposite side, and by letting out that portion of the steam which is urging the piston forward. Hence we must move the valve into the position shown in the lower diagram, and shift the centre of the eccentric

FIG. 326.



pulley from A to B : the piston will then return before it reaches the end of the cylinder, and the movement of the engine will be reversed.

In examining the diagram it should be understood that the crank which works the slide-rod is inclined at an angle somewhat greater than 90° to the crank which is attached to the piston ; and also that the crank of the slide-rod is always in advance of the larger one in its journey round. The engine would not work if

the larger crank were to turn in the opposite way to that shown in the sketch.

This explanation shows that in reversing an engine we must either shift the eccentric from the one position into the other, or else we must employ two eccentrics, and provide some means of connecting each of them in turn with the slide-valve.

ART. 245.—The link motion commonly appears under three forms. (1) There is the shifting link, having its concave side towards the axle or crank shaft: this arrangement was introduced by the celebrated Stephenson, and is known as *Stephenson's Link Motion.* (2) There is the stationary link, where the curvature is in the opposite direction. (3) There is the straight link, which is derived from a combination or moulding together of the two former contrivances.

1. Stephenson's link motion is shown in fig. 321.

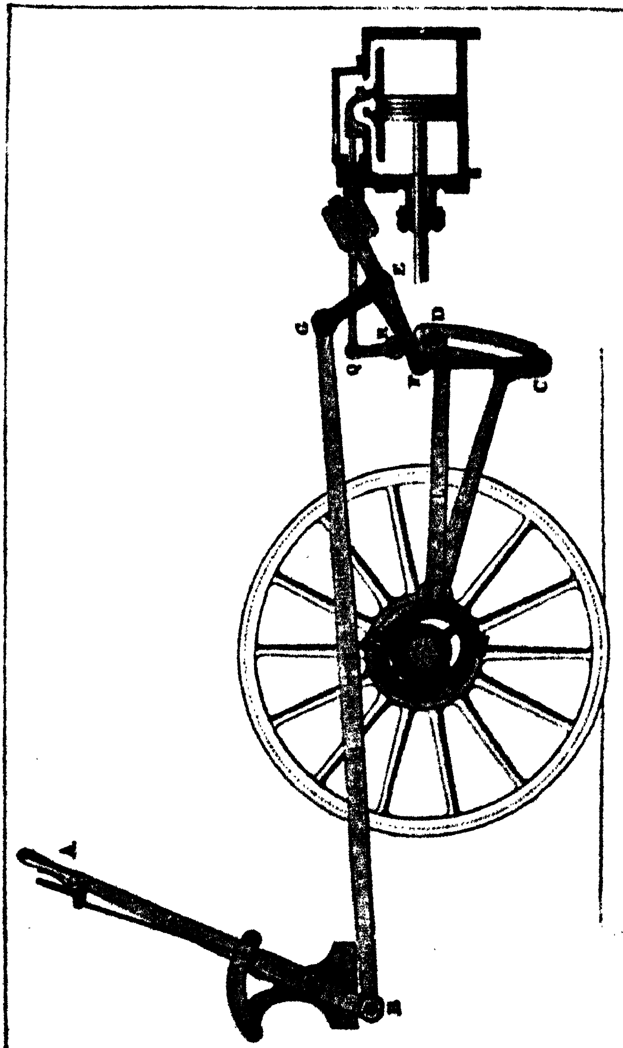
AB is the starting lever, under the control of the engine-driver, and is represented as being pushed forward in the direction in which the engine is moving; CD is the *link*, provided with a groove, along which a pin can travel; a short lever, centred at R, is connected at one end, Q, with the slide-valve, and at the other end with the pin which moves in the link.

It is clear that so long as the pin remains near the point D, the lever centred at R will be caused to oscillate just as if the pin were attached to the extremity of the outer eccentric bar, and that the outer eccentric alone will be concerned in the motion of the valve.

If now the engine-driver wishes to reverse his engine, he pulls back the lever AB, and by doing so he raises the link CD until the pin comes opposite to the end of the inner eccentric bar. The raising of the link is caused by the motion imparted to the bell-crank lever, GEF, which is centred at the point E. A counterpoise to the weight of the link is attached to the axis passing through E at some little distance behind the bell-crank FEG, so as to be out of the way of the moving parts, and the object of this counterpoise is to enable the engine-driver to raise the heavy link and bars easily.

The inner eccentric bar now alone comes into play, and the two eccentrics being fastened to the crank axle at the angles

Fig. 321.



Stephenson's Link Motion.

indicated in the first part of the article, it is apparent that the valve will be shifted, and that the action of the engine will be reversed.

2. The stationary link shown in fig. 322 was invented by Mr. Gooch, of the Great Western Railway. It will be seen that the link RS is here suspended by an arm *b*C, so as to be stationary so far as any up-and-down movement is concerned, and that it is circular in form, being struck by a radius equal to DR, whereby also its concavity lies towards the cylinder and away from the axle or shaft of the engine. In the sketch the forward eccentric is in operation, and the motion is readily traced from the axle to the slide, which is shown as having partly uncovered the steam port marked A. On pulling the rod H, which is in connection with the starting lever, or its equivalent, the bell crank K*e*L is moved, and the jointed rod DR is brought down by the pull of LM into a lower position, whereby it imparts to the slide the motion due to the back eccentric, and the engine is consequently reversed.

3. A third method is Allan's straight link motion, in which the link and the valve rod are both shifted in opposite directions at the same time. When the link is shifted it must of necessity be curved towards the eccentric rods, and when the slide rod is jointed as at D, and shifted up or down, the curvature of the link must be towards the slide, from which it follows that if both the link and the slide rod shift in a vertical plane the concavity and convexity may neutralise each other, and a straight link may serve to give the motion. Link motions prove to be rather complicated pieces of mechanism when any attempt is made to analyse them thoroughly, and therefore it may suffice to say that with a stationary link the lead of the slide is maintained constant under all changes in the position of the sliding block, whereas with the shifting link the lead increases a little towards the central position.

A special advantage of the several link motions consists in the power which it gives to the engineer of regulating the supply of steam admitted into the cylinder. By moving the starting lever or its equivalent into intermediate positions, the amount of travel of the valve is reduced at pleasure, for it is evident that no steam can enter the cylinder when the lever is half-way between its extreme positions, and that varying amounts of opening of the steam

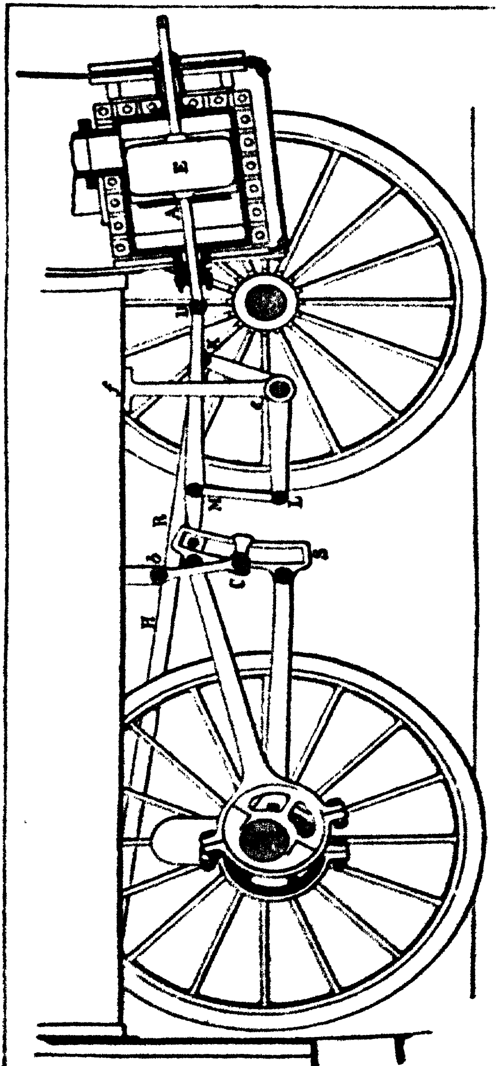


Fig. 322.

Gooch's Stationary Link.

ports, increasing to the maximum value, will occur when the lever is pushed over by successive steps.

ART. 246.—*Rolling curves* have been employed to vary the relative angular velocity of two revolving pieces.

The mathematician Euler appears to have been the first to refer to a class of curves which, when caused to turn about fixed centres, should communicate motion to each other by rolling contact. He deduced the property that the line of contact remains always in the straight line joining the centres of the curves.

The subject made no further progress until it was taken up by the Rev. H. Holditch, of Cambridge, who investigated the forms of several rolling curves and worked out the mathematical theory. (See 'Cambridge Phil. Trans.' vol. vii. A.D. 1838.)

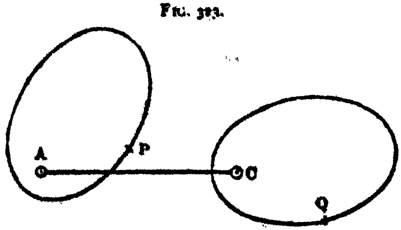
Prop. Where two curved plates, centred upon fixed axes, roll together, the point of contact must always lie in the line of centres.

In order to establish this proposition we may reason as follows :—

Let two curves have centres at A and C, and suppose that P and Q represent two points which will come together when the curves move each other by rolling contact.

Then P describes a circular arc round A as the plate revolves, and Q describes a circular arc round C. Hence P and Q will come into contact whenever these circles meet each other. Now P and Q are only in contact at one point, because the curves roll, and do not slide on each other, and therefore the circular arcs can only meet once, that is, they touch each other : but they can only touch in the line AC, therefore the point of rolling contact must lie in AC.

FIG. 323.



This is equivalent to saying that AP + CQ = a constant quantity.

Further than this, the curves will have a common tangent at the point where they roll upon each other.

Ex. Two equal ellipses which are centred on opposite foci will roll together.

It is the property of an ellipse that the tangent at any point P makes equal angles with the focal distances SP and HP, that is, that the angles SPT and HPt are equal to one another at every point of the curve ; and again, that the sum of the lines SP and HP is a constant quantity (fig. 324).

FIG. 324.

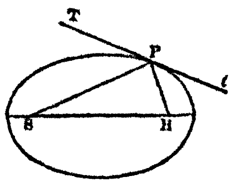
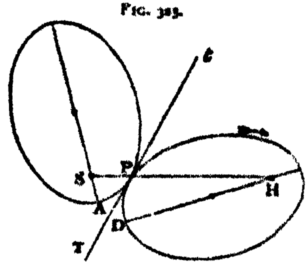


FIG. 325.



The two equal ellipses centred upon opposite foci are represented in contact at P (fig. 325).

Let PT be the tangent to the ellipse A at P, and Pt the tangent to the ellipse D at P.

Then SPT $=$ tPH by the property above stated, $\therefore$ TPt is a straight line, or the curves have a common tangent at the point P, also SP $+$ HP $=$ a constant quantity, and the two conditions of rolling are fulfilled.

ART. 247.—The theory is simple, and is the following :—

Let SP $= r$, ASP $= \theta$, HP $= r'$, DHP $= \theta'$, the curves AP, DP, representing any pair of rolling curves with centres at S and H respectively (fig. 325). Then $r + r' = c$.

Also the curves have a common tangent, $\therefore\ r\dfrac{d\theta}{dr} = r'\dfrac{d\theta'}{dr'}$.

Let the differential equation to the first curve be $r\dfrac{d\theta}{dr} = f(r)$,

then the differential equation to the second curve will be

$$r'\frac{d\theta'}{dr'} = f(r) = f(c - r'),$$

whence the relation between θ' and r' may be found by integration.

Ex. There is a curve known as the logarithmic spiral, which will roll without sliding upon a second similar and equal spiral.

The equation to the logarithmic spiral is $r = ae^{\frac{\theta}{m}}$, and the mode of setting out its form is extremely ingenious.

Draw a number of lines radiating from S, and inclined to SA at angles α, 2α, 3α, &c., respectively.

Then it will be readily seen that, if r_1, r_2, r_3, &c., are the corresponding values of SP, we have the following relations set up, viz. :—

$$r_1 r_3 = r_2^2,\ r_2 r_4 = r_3^2, \text{ and so on.}$$

In order to find the lengths of r_1, r_2, r_3, &c., we draw a straight line Sx, inclined at an angle α to SA, and draw Ax perpendicular to Sx, xy perpendicular to SA, yz perpendicular to Sx, zv perpendicular to SA, and so on.

FIG. 326.

Then SA × Sy = Sx^2, by similar triangles,
also, Sv × Sy = Sz^2, and so on.

∴ Sx, Sy, Sz, &c., are the respective values of r_1, r_2, r_3, &c., whence the curve can be set out.

The curve has the property that the tangent at every point is inclined at the same angle to the radius vector at the point considered.

Taking the equation $r = ae^{\frac{\theta}{m}}$, we have

$$dr = \frac{a}{m} \times e^{\frac{\theta}{m}}\, d\theta = r\frac{d\theta}{m},$$

$$\therefore\ r\frac{d\theta}{dr} = m, \text{ a constant quantity.}$$

But $r\dfrac{d\theta}{dr}$ is the tangent of the angle which the curve makes with the radius vector at any point. Let this angle be ϕ, therefore $\tan\phi$ = a constant, or ϕ is invariable.

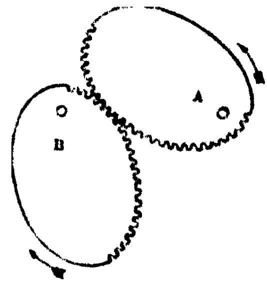
Also $$r'\frac{d\theta'}{dr'} = r\frac{d\theta}{dr} = m,$$

whence the second rolling curve is identical with the first spiral.

In like manner we could prove that two equal ellipses centred upon opposite foci would roll together.

ART. 248.—In practice rolling curves must be provided with teeth upon the retreating edge, otherwise the driver would leave the follower, and the revolution would not be completed (fig. 327).

FIG. 327.



As is usual in all cases where segmental wheels are employed, a guide must direct the teeth to the exact point where they commence to engage each other.

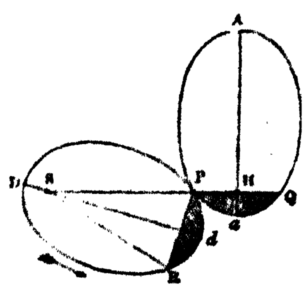
The guide may be dispensed with by carrying the teeth all round the curves: this construction is usually adopted in practice, although, strictly speaking, it destroys the rolling action entirely.

A quick return of the table in small planing machines has been effected by the aid of rolling ellipses.

The table is driven by a crank and connecting rod, and the crank exists under the form of a flat circular plate, centred on one of the foci, and having a groove radiating from the axis as a line of attachment for one end of the connecting rod. As the plate may be set in any position upon the elliptical wheel, we propose to inquire what will be the effect of a change of direction in the groove or crank.

Let the ellipses have the position shown in fig. 328, S and H being the centres of motion, and SPHQ being perpendicular to A*a*, the axis of one ellipse. Draw PR perpendicular to D*d*, and let the ellipse DR*d*P be the driver, rotating as shown.

FIG. 328.



While P*d*R is rolling upon P*a*Q, the ellipse A*a* makes half a revolution; and while RDP is rolling upon QAP, it makes the remaining half-revolution.

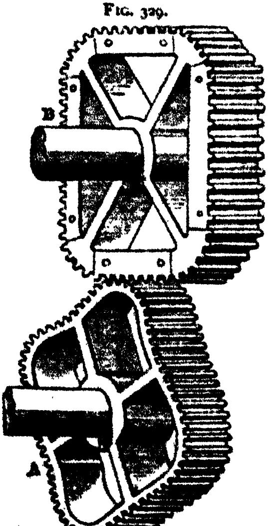
Suppose Dd to revolve uniformly, then the times of a half revolution of Aa will be in the same proportion to each other as the angle PSR to the angle $360°-PSR$. The quick half revolution occurs when the shaded segments are rolling upon each other. If, therefore, the table be made to move in the line HS produced, and the crank be placed in a direction perpendicular to Aa, we shall obtain the greatest possible difference between the periods of advance and return.

The practical difficulty with rolling wheels exists during that part of the revolution where the driver tends to leave the follower, and it can only be obviated by making the teeth unusually deep; also the wheels should work in a horizontal plane.

ART. 249.—An instance of rolling curves is exhibited in the sketch, and occurred in one of the many attempts made to improve the printing press before the invention of Mr. Cowper enabled the newspapers to commence a real and vigorous existence.

The type was placed upon each of the four flat sides of a rectangular prism, to which the wheel B corresponded in shape, and the paper was passed on to a *platten* corresponding in form and size with the pitch-line of the wheel A.

FIG. 329.



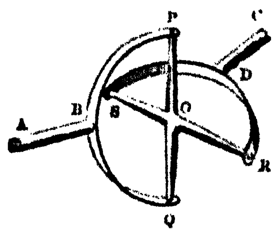
The prism and platten being in the same relative position as the wheels B and A, we can understand that the type would be in the act of impressing the paper while the convex edge of the wheel A rolled upon the flat side of B, and that in this way we should obtain four impressions for each revolution of the wheels.

By this construction, the patentees, Messrs. Bacon and Donkin, intended to introduce the principle of continuous rotation as opposed to the reciprocating movement in a common press; and the object of imitating exactly upon the wheels A and B the form of the printing prism and of the platten, was to

ensure that the paper and type should roll upon one another with exactly equal velocities at their opposing surfaces, and that no slipping or inequality of motion should destroy the sharpness of the impression.

ART. 250.—*Hooke's Joint* is a method of connecting two axes, whose *directions meet in a point*, in such a manner that the rotation of one axis shall be communicated to the other.

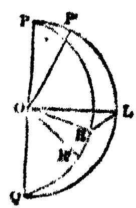
FIG. 330.



Here AB and CD represent two axes whose directions meet in the point O; the extremities of AB and CD terminate in two semicircular arms which carry a cross, PQSR; the arms of this cross are perfectly equal, and the joints P, Q, S, and R permit the necessary freedom of motion.

As the axis AB revolves, the points P and Q describe a circle whose plane is perpendicular to AB, and at the same time the points S and R describe another circle whose plane is perpendicular to CD.

FIG. 331.



These two circles are inclined at the same angle as the axes, and are represented in fig. 331; thus the arm OP starts from P, and moves in the circle PP'L, while the arm OR starts from R, and describes the circle RR'Q inclined to the former.

Let OP', OR' be corresponding positions of the two arms, then P'R' is constant, but changes its inclination at every instant, and as a consequence the relative angular velocities of OP' and OR' are continually changing.

To find the relative angular velocities of the axes AB and CD, we proceed as follows:—

Let the circle *prq* (fig. 332) represent the path of P, *ptq* being the projection upon this circle of the path of R, and suppose a to be the angle between AB and CD; then the dimensions of the curve *ptq*, which will be an ellipse, can be at once deduced from the equation $Ot = Or \cos a$.

Draw R*m* perpendicular to O*r*, then R*m* will be the actual

vertical space through which OR has descended while OP describes the angle pOP. But the path of R is really a circle, and only appears to be an ellipse by reason of its being projected upon

Fig. 332.

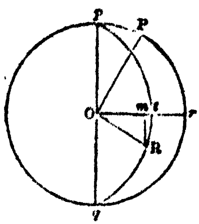


Fig. 333.

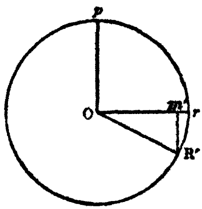
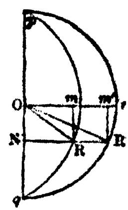


Fig. 334.



a plane inclined to its own plane. In order, therefore, to estimate the actual angular space through which OR has moved, we must refer this motion to the circle which R really describes (fig. 333), and thus by making $R'm' = Rm$, we can infer that m'OR$'$ will be the angle which OR describes while OP moves through the angle pOP.

But the angle pOP = angle mOR, and hence we can represent the motion of both axes upon one diagram by combining the ellipse pRq with the circle prR$'q$ (fig. 334). This being done, we may draw R$'$RN perpendicular to pOq, and join OR, OR$'$: it will at once appear that the angles ROr, R$'$Or are those described in the same time by the axes AB and CD.

Hence the axes AB and CD revolve together, but unequally, and the angles which they describe in the same time can always be found by construction.

First draw the circle prq in a plane perpendicular to one axis, and having O for its centre, next construct the ellipse whose major axis is the diameter pOq equal to POQ, and whose minor axis is the product of POQ × the cosine of the angle between the axes. Then take OR$'$ any position of OP, draw R$'$RN perpendicular to pOq, join OR$'$ and OR. It now appears that R$'$Or and ROr will represent the angles described by the axes AB and CD in the given time.

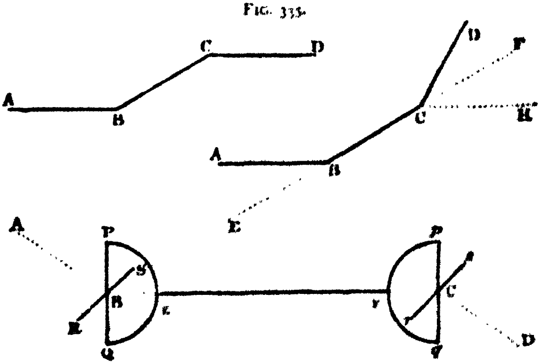
Furthermore, OR and OR$'$ coincide when R is at the end of an axis of the ellipse pRq, an event which must happen four

times as the cross goes round once; and there is therefore this curious result, that however unequal may be the rate at which the axes are at any time revolving, they will coincide in relative position four times in one revolution.

The single joint may often be very useful in light machinery which is required to be movable, and the parts of which do not admit of very accurate adjustment; but it will be understood that the friction, and especially that irregularity which we have just proved to exist, would render it necessary to confine the angle between the shafts within narrow limits in actual practice.

ART. 251.—Now that the general character of the movement is understood, we shall be in a position to comprehend the change which is effected by interposing a *double joint* between the axes.

FIG. 335.



1. Take the case where AB and CD are parallel axes connected by an intermediate piece BC, having a Hooke's joint at both the points B and C.

Conceive that the arms of the crosses at B and C are placed in the manner shown in the sketch, or let each vertical arm be connected with the forks at B and C.

If AB revolves uniformly, BC will also revolve with a varying velocity dependent upon the angle ABC, but the variable velocity which BC receives from AB is precisely the same as that which it would receive from DC if the latter axis were the driver and were to revolve uniformly.

It follows therefore that the motion which AB imparts to CD will be a uniform velocity of rotation exactly equal to that of AB.

Hence a double Hooke's joint may be used to communicate uniform motion between two parallel axes whose directions nearly coincide.

If, however, the construction were varied and the vertical arm PQ of the first cross were connected with E, while the horizontal arms, *s*, *r*, were connected with F, we should communicate no doubt a motion of rotation between the axes, but it would no longer be uniform but variable, by reason that we could not return by the same course reversed under like conditions. The deviations from uniform rotation would no longer oppose and correct each other, but they would act together and increase the inequality. This is seen at once upon constructing the diagrams which represent the relative rotation between each pair of axes.

2. Let AB and CD be inclined to each other, and be connected by the piece BC jointed at B and C, and so placed that the angle ABC is equal to the angle BCD.

As in the former case we must be careful to connect B and C with the corresponding arms of the crosses, and we have seen that the inequality produced by DC in the motion of CB depends both upon the angle BCD and the position of the cross; it is therefore the same whether CD lies in the direction shown, or in the dotted line CH parallel to AB. In both cases the angle between the axes and the position of the cross will respectively coincide.

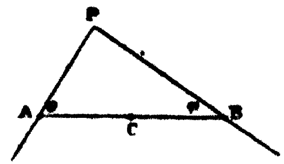
But we have seen that when the parallel axes AB and CH are connected by joints at B and C in the manner stated, the axes AB and CH will rotate with equal uniform velocities, and we conclude, therefore, that they will also rotate in a similar manner when placed in the position ABCD.

Hence a double Hooke's joint may be employed to communicate a *uniform rotation* between two axes inclined at a given angle.

Art. 252.—It will be found that well-known propositions of Euclid obtain a new significance when applied to movable combinations of the lines of figure.

Referring again to the triangle from which we deduced the law of motion of the crank and connecting rod :—

Fig. 336.



1. Let APB represent such a triangle, A and B being fixed points, and the angle APB being a rigid angle. Also, let the sides PA, PB, be produced indefinitely in order that the dimensions of the triangle may vary by the sliding of PA, PB through the points A and B respectively.

$$\text{Let PAB} = \theta,\ \text{PBA} = \phi,$$
$$\text{then } \theta + \phi = 180 - \text{APB} = \text{a constant.}$$
$$\therefore\ \delta\theta + \delta\phi = 0,\ \text{or } \delta\theta = -\delta\phi,$$

whence the angular velocity of PA about A is equal and opposite to the angular velocity of PB about B.

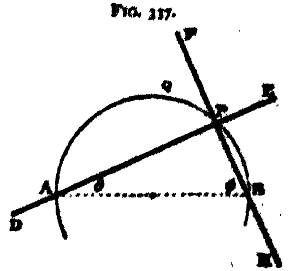
Also since APB is a rigid angle, and since the angles in the same segment of a circle are equal to one another, we infer that the point P lies always in a circular arc passing through A and B.

It will simplify the result if we take APB$=90°$, as we can then apply the property that the angle in a semicircle is a right angle, also AB will in that case be the diameter of the circle traced out by the point P.

In fig. 337 take A and B to represent two fixed axes, and let DEFH be a rigid rectangular cross whose arms can slide through the points A and B.

The motion will only be possible so long as the arms of the cross have perfect liberty to slide through the points A and B as well as to rotate about them.

Fig. 337.



Let this be arranged, and join AB ; then we have

PAB+PBA$=90°$ in every position of the cross.

Hence if the angle PAB increase by the rotation of PA, the angle PBA must diminish equally by the rotation of PB, or ED and FH must revolve with equal angular

Also P, the angle of the cross, will describe a circle whose diameter is AB, and our proposition follows directly from Euclid, for if P move on to any point Q, the angles QAP, QBP are angles in the same segment of a circle, and are therefore equal to each other.

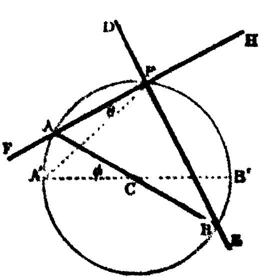
This movement was put into a practical shape by Mr. Oldham, and used in machinery at the Bank of England. The student may easily construct a model after the manner of Hooke's Joint, when the centre of the cross will be seen to describe a circle whose diameter is the perpendicular distance between the axes while the arms of the cross slide to and fro through holes in the forked arms that spring from the axes and support them.

2. Bisect AB in C, and take P and C as fixed centres of motion. As before, let APB be a right angle, then the rotation of AB about C will set up a rotation in both PA and PB, whereby each of the latter lines will tend to rotate with half the angular velocity of AB.

In order to make the motion continuous, the lines AP, BP must be produced so as to form a rectangular cross, and they may be conveniently formed as straight grooves in a plain board whose axis passes through P. The bar AB will then be provided with pins working in the grooves.

Describe a circle with centre C and radius equal to CA or CB; draw any fixed diameter A'CB', and join A'P.

FIG. 338.



Then it is proved in Euclid that the angle at the centre of a circle is double the angle at the circumference when both angles stand upon the same arc,

that is angle ACA' = 2 angle APA',
or the angular velocity of CA is twice that of the cross.

As the driver ACB revolves the pins A and B will oscillate to and fro along their respective grooves and will traverse through the centre P.

ART. 253.—*The differential worm wheel and tangent screw* is a combination which will be understood without any drawing.

Here two worm wheels, differing by one tooth in the number which they carry, are placed side by side and close together, so as to engage with an endless screw. As the wheels are so very nearly alike the endless screw can drive them both at the same time, and it is evident that one wheel will turn relatively to the other, through the space of the extra tooth, in a complete revolution, and that a very slow relative motion will thus be set up.

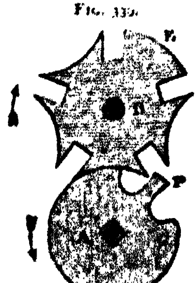
In this way, if one wheel carries a dial plate, and the other a hand, we may obtain the record of a very large number of revolutions of the tangent screw.

ART. 254.—Where a train of wheels is set in motion by a spring enclosed in a barrel it becomes of consequence not to overwind the spring. The *Geneva stop* has been contrived with the view of preventing such an occurrence, and will be found in all watches which have not a fusee.

Here a disc A, furnished with one projecting tooth, P, is fixed upon the axis of the barrel containing the mainspring, and is turned by the key of the watch.

Another disc, B, shaped as in the drawing, is also fitted to the cover of the barrel, and is turned onward in one direction through a definite angle every time that the tooth P passes through one of its openings, being locked or prevented from moving at other times by the action of the convex surface of the disc A.

FIG. 332.



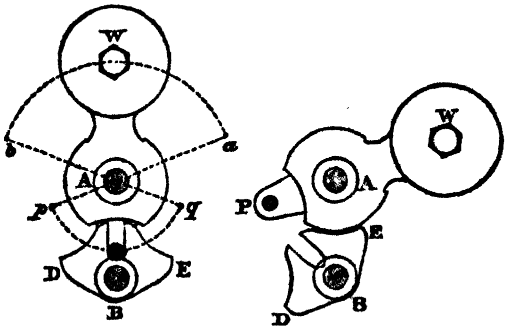
In this manner each rotation of A will advance B through a certain space, and the motion will continue until the convex surface of A meets the convex portion E, which is allowed to remain upon the disc B, in order to stop the winding up.

The winding action having ceased, the discs will return to their normal positions as the mechanism runs down.

Instead of supposing A to make complete revolutions let it oscillate to and fro through somewhat more than a right angle; then B will oscillate in like manner and will be held firmly by the opposition of the convex to the concave surface except during the time that P is moving in the notch.

ART. 255.—The Geneva stop has been applied by Sir J. Whitworth in his planing machine, in order to give a definite vibration to a piece from which the feed motion is derived.

FIG. 340.



The drawing shows a lever centred at A, and having a pin P at one end. The other end of the lever is weighted at W, the object of the weight being to cause the lever to fall over suddenly, and with sufficient power to carry the driving belt from one working pulley to the next in order.

A pinion on a shaft terminating at A gears into a rack formed on a traversing rod which is moved longitudinally in alternate directions by tappets on the table of the machine. The rod, in its turn, actuates a bell crank lever which is connected with a fork employed for passing the driving belt from one pulley to another.

As far as this explanation has gone it would appear that the weighted lever was designed simply for controlling the driving belt, but it will be seen that a portion of a Geneva stop is superadded, and this extra piece enables the lever to actuate also the feed motion. The axis A of the lever is surrounded by a circular plate, corresponding with A in fig. 339, and the piece BED has two concave circular cheeks at E and D exactly fitting the plate A. Also the pin P works in the open jaws as shown.

The result is that the lever PA carries BED as it swings over, and locks it in either position, whether to the right or the left. In the diagram the pin P is shown—(1) in a vertical posi-

tion, while in the act of driving BED, and also, (2) after having fallen over to the right, at which time BED is securely locked.

The extreme positions of the weighted lever are shown by the dotted lines *pa*, *qb*, and it only remains to point out that the feed motion is taken from the axis B by means of a grooved pulley and a catgut band. This band runs round another pulley which has already been described in Art. 132, and is there marked as F, and by a comparison with the previous description the general arrangement will be readily understood. The locking of BED is essential in order to prevent any motion of the cutter, before the completion of the cut.

Art. 256.—The *star wheel* is used in cotton-spinning machinery, and is analogous to the Geneva stop.

Fig. 341.

If the convex portion E were removed, so as not to interfere with the rotation of A, we should virtually possess a star wheel in the disc B. See Art. 254, and fig. 339.

In that case each rotation of A would advance B by the space of one tooth, or we should convert a continuous circular motion into one of an intermittent character.

The usual form of the star wheel is given in the sketch, where the revolving arm encounters and carries forward a tooth at each revolution. The action is the same as if a wheel with one tooth were to drive another with several teeth.

Art. 257.—It is well known to mathematicians that the angular velocity of a rigid body about an axis may be properly represented by a straight line in the direction of that axis, whence it follows that angular velocities may be combined according to the law which gives us the parallelogram of linear velocities or the parallelogram of forces.

A remarkable illustration of the compounding of angular velocities has been afforded in the construction of the so-called *Plimpton* or roller skate for use on artificial ice.

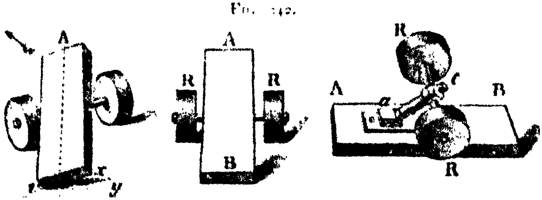
This skate runs like a wagon upon four wheels, but instead of the perch-pin being vertical, as in an ordinary wagon, it inclines

inwards, and dips towards the centre of the skate. Indeed, each pair of wheels is provided with an inclined axis, and it will be presently seen that the skate will not serve its purpose unless the respective axes converge downwards to a point underneath the centre of the footboard.

Everyone is aware of the manner in which curves are described by a skater upon ice. For example, when tracing out a circular sweep on, say, the right foot, technically distinguished as 'an outside edge,' the plan is to keep the leg and body quite straight and to lean over a little towards the centre of the circle.

With a roller skate the same movement of the body produces the same result, but in a very different manner. The act of tilting over the footboard causes the fore wheels to deviate a little towards the right, and the hind wheels to deviate a little towards the left, the respective axles of the front and hind pair meeting, as they ought to meet, in the centre of the circular path described by the skater, and our object is to exhibit a method of construction which necessarily produces this result.

Fig. 142.



In order to simplify the explanation it will be better to confine our attention to the front pair of wheels, and the drawing shows a model which may illustrate the movement of the wheels as consequent upon the tilting of the footboard.

The right-hand figure gives a perspective view of one half of the footboard AB, together with the inclined axis *ac*, and a pair of rollers attached to an axle standing at right angles to a pipe or hollow tube, which is threaded upon the immovable inclined axis.

The next sketch shows the footboard resting on its wheels near the edge of a horizontal table, and it is apparent that if the

skate were pushed forward a little it would advance in a line perpendicular to the edge of the table.

Now raise one edge of the footboard without in any way altering the direction in which its central dotted line points, and let it be noted that the direction of that line is at right angles to the edge of the table.

In the drawing the footboard is shown as tilted through an angle *xry*, and the immediate result is that the axle of the rollers turns to the right in the manner indicated by the arrow, and that if the skate were pushed forward it would immediately move in a curved line pointing to the left hand.

It is extremely easy to construct the model, and the movement may then be studied with advantage.

In applying general reasoning we say that there are three axes of rotation before us, and it will be better to take the case where the board in the model is held in a horizontal position parallel to the plane of the table. In exhibiting the model it is more convenient to hold it in this way, but the drawing is clearer when the board is allowed to drop with one end on the table.

The three axes of rotation are :—

1. A horizontal axis through the horizontal footboard AB.
2. The inclined axis *ac*.
3. A vertical axis through *a*.

Here two simultaneous rotations about the vertical and horizontal axes may give a resultant rotation about the inclined axis, just as horizontal and vertical forces acting on a point have an inclined resultant.

But again, in the case of forces, the combination of the inclined resultant with the horizontal component would give the other vertical component ; so here, the combination of the rotation about the horizontal axis AB with another rotation about the inclined axis *ac*, gives a rotation about an imaginary vertical axis, viz., that passing through *a*, and hence the axle of the rollers does in effect rotate about a vertical axis, just as if it were the axle of an ordinary carriage provided with a vertical perch-pin. Such a rotation causes the rollers to deviate on one side of the normal direction.

The specification of Mr. Plimpton's patent, granted to A. V.

Newton and numbered 2190 of the series for 1865, contains drawings of the skate, and shows the foot stand running upon four wheels, two on each side of the roller axle. The respective axles are supported in frames capable of turning to a small extent limited by stops, and are directed downwards to a point half-way between the heel and toe of the skate. These ledges perform the function of inclined axes. The invention is described as relating to an improvement in attaching rollers to the foot stand of a skate, whereby the rollers are made to turn by the rocking of the foot stand so as to cause the skates to run in a curved line either to the right or left.

It has been stated by experts on the subject that the arrangement of two inclined axes for causing the roller axles to converge towards the centre of curvature of the path of the skater was a completely new invention at the date of the patent, and that no machine existed at that time in which a like motion had been arrived at in so simple a manner.

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